Preface

While preparing the manuscript of this book, we were informed the very sad news that Dr. J. Nagata died in November, 2007 and finished his eighty two years lifetime. This is the biggest grief to the Japanese researchers of general topology as well as all researchers in the world.

To my memory, it is about thirty years ago that I personally met this world figure in general topology, and it was at Tsukuba University, where he gave us a lecture and afterward at the party I have had a little conversation with him. This is the first contact with him. Since that time, I have been given many helpful suggestions not only on study on general topology but also on other matters, for example, on doing mathematics.

To be frank, the content of this book is very related with the Nagata's result. In fact, let us recall the famous Nagata-Smirnov theorem that a regular space X is metrizable if and only if there exists a σ-locally finite base for X. It is needless to say that this is one of the most valuable contributions that he has made to general topology.

Our topic treated here is about the M_1-spaces, M_3-spaces. But they are defined by relaxing the condition "locally finite" in the theorem to "closure-preserving" or, "base" to "quasi-base".

On the other hand, Nagata himself proposed the following problem: For an M_1-space X, is $\dim X \leq n$ (or $\operatorname{Ind} X \leq n$) characterized by the existence of a σ-closure-preserving base $\mathcal{U}$ for X such that $\dim \mathrm{Bd}(U) \leq n-1$ (or $\operatorname{Ind} \mathrm{Bd}(U) \leq n-1$, respectively) for each $U \in \mathcal{U}$. (stated later as (P10)) . This remains still open.

Thinking that way, this publication is deeply influenced by his existence.

Here, I again thank him for all the contact with him and sincerely condole him.

Takemi Mizokami , December, 2007

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