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Preface: Special Issue on Tensor Optimization and Data Analysis

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Tensors (also called hypermatrices) are natural extensions of vectors and matrices, and have been shown to be a powerful tool for capturing multiple interactions and inherent hierarchies in data sets from wide applications in scientific and engineering communities. The tensor-based optimization modeling and data analysis are then emerging, with new opportunities to gain better accuracy and efficiency comparing to traditional vector-based and matrix-based approaches, and deep research challenges that require scalable, tensor-based algorithms.

In the past decades, data-driven tensor optimization theory and algorithms have been widely used in many applications. Although a remarkable number of papers have been contributed to tensor computation in recent years, it is still challenging and particularly important to develop structure-exploiting tensor optimization models, theory and algorithms for higher-order tensor related problems.

This special issue is designed to solicit high-quality papers on tensor optimization and their applications in data analysis. A total of 11 papers accounted below, which were reviewed according to the usual high standards of the journal, cover a wide range of theoretical, practical, and applied topics in tensors and tensor optimization.

C. M. Li, Z. L. Jiang and X. F. Duan [1] investigate the numerical methods for two kinds of the best Hankel tensor approximation problems. Based on the Vandermonde decomposition of Hankel tensors, the Hankel tensor approximation problem with missing data is transformed into an unconstrained optimization problem, and then the BFGS method is used to solve it. Furthermore, the authors propose Dykstra's algorithm and its acceleration versions for Hankel tensor approximation problems with the interval constraint and box constraint. Numerical examples illustrate that these methods are feasible and effective.

J. Z. Li, K. Usevich and P. Comon [2] propose a Jacobi-type algorithm to solve the low rank orthogonal approximation problem of symmetric tensors, and study the global convergence of this algorithm under a gradient based ordering for a special case: the best rank-2 orthogonal approximation of 3rd order symmetric tensors. This algorithm includes as a special case the well-known Jacobi CoM2 algorithm for the approximate orthogonal diagonalization problem of symmetric tensors. Furthermore, they also propose a proximal

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variant of this algorithm in general case, and prove its global convergence without any further condition.

X. Y. Li and Y. N. Yang [3] consider the problem of orthogonal low-rank approximation to higher-order tensors, and view the coefficients of the latent rank-1 terms as a vector to be sparsified. Based upon this, the authors propose a reweighted alternating least squares algorithm to solve the model, and its convergence is established without any assumption. Numerical experiments show that the proposed model and algorithm can provide a valid estimate of the number of rank-1 terms.

M. L. Liang, L. F. Dai, and R. J. Zhao [4] study the solvability of the tensor inverse eigenvalue problem based on Moore-Penrose inverses of tensors, and the tensor nearness problem is considered as an application.

D. D. Liu, W. Li, and Y. N. Chen [5] utilize stochastic tensors to address a system of cyclic stationary probability distribution equations for a second order Markov chain process in case where all states are independent each other. Moreover, the authors investigate some theoretical properties of their proposed stationary equation, which can be viewed as a unified framework to study some second order Markov chain with independent or non-independent states.

Q. S. Wang, C. F. Cui and D. R. Han [6] present a momentum accelerated version of the block-randomized stochastic gradient descent algorithm for low-rank tensor CP decomposition. Under some mild conditions, the authors show the global convergence to the stationary point. Compared with the algorithms without momentum, the preliminary numerical experiments for the synthetic and real data demonstrate that the proposed accelerated algorithms are efficient.

C. Y. Wang, M. Wang, H. B. Chen, and Y. J. Wang [7] introduce a nonnegative triple decomposition for third order nonnegative tensors, which decomposes a third order nonnegative tensor to three third order low rank nonnegative tensors in a balanced way, and propose a nonnegative tensor completion method based on such a low rank nonnegative triple decomposition.

W. W. Yang, H. Liu, and Q. Ni [8] propose a quarticly convergent method for solving a system of nonlinear equations, which is used to find the largest H eigenvalue of irreducible nonnegative tensor and the Z eigenvalues of general tensors. The computational complexity is slightly greater than Newton method. The global and quartic convergence of the new method are proved. Numerical results indicate that the proposed method is efficient on some tensor problems.

G. H. Yu, L. Q. Wang, S. C. Wan, L. Qi, and Y. W. Xu [9] present a novel network traffic data imputation model based on low rank tensor completion and TV regularization to well characterize the global structure and local smoothness features of network traffic data, and design an easy-to-operate and relatively effective algorithm with convergence analysis.

T. Zhang, Z. H. Huang, and Y. F. Li [10] extend three classes of copositive-type matrices to higher order tensors, with a refined study on the solvability of tensor complementarity problems where the underlying tensor is of the type among the above three classes.

A. W. Zhou, Y. R. Wu, and J. Y. Fan [11] discuss some properties of the (real) Hermitian completely positive tensors and (real) Hermitian copositive tensors. Furthermore, the authors propose a semidefinite algorithm for checking whether a Hermitian tensor is Hermitian completely positive or not.

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