

A NEW HIERARCHY OF SDP-RELAXATIONS FOR POLYNOMIAL PROGRAMMING

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References

- [1] R. Ash, *Real Analysis and Probability*, Academic Press, San Diego, 1972.
- [2] A. Barvinok, *A Course in Convexity*, American Mathematical Society, Providence Rhode Island, 2002.
- [3] S. Basu, R. Pollack and M-F. Roy, *Algorithms in Real Algebraic Geometry*, Springer, Berlin, 2003.
- [4] C. Berg, The multidimensional moment problem and semi-groups, *Proc. Symp. Appl. Math.* 37 (1980) 110–124.
- [5] G. Blekherman, There are significantly more nonnegative polynomials than sums of squares, Dept. of Mathematics, University of Michigan, ArXiv #0309130, 2003.
- [6] B. Hanzon and D. Jibeteau, Global minimization of a multivariate polynomial using matrix methods, *J. Glob. Optim.* 27 (2003) 1–23.
- [7] D. Henrion and J.B. Lasserre, GloptiPoly : Global optimization over polynomials with Matlab and SeDuMi, *ACM Trans. Math. Soft.* 29 (2003) 165–194.
- [8] D. Henrion and J.B. Lasserre, Solving nonconvex optimization problems, *IEEE Control Systems Magazine* 24 (2004) 72–83.
- [9] T. Jacobi and A. Prestel, Distinguished representations of strictly positive polynomials, *J. Reine. Angew. Math.* 532 (2001) 223–235.
- [10] D. Jibeteau and M. Laurent, Semidefinite approximations for global unconstrained polynomial optimization, *SIAM J. Optim.* 16 (2005) 490–514.
- [11] M. Kojima, S. Kim and H. Waki, Sparsity in sums of squares of polynomials, *Math. Program.* 103 (2005) 45–62.

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- [12] M. Kojima, S. Kim and H. Waki, Generalized Lagrangian duals and sums of squares relaxations of sparse polynomial optimization problems, *SIAM J. Optim.* 15 (2005) 697–719.
- [13] J.B. Lasserre, Global optimization with polynomials and the problem of moments, *SIAM J. Optim.* 11 (2001) 796–817.
- [14] J.B. Lasserre, Polynomials nonnegative on a grid and discrete optimization, *Trans. Amer. Math. Soc.* 354 (2002) 631–649.
- [15] J.B. Lasserre, T. Prieto-Rumeau and M. Zervos, Pricing a class of exotic options via moments and SDP-relaxations, *Math. Finance* 16 (2006) 469–494.
- [16] J.B. Lasserre, Bounds on measures satisfying moment conditions, *Ann. Appl. Prob.* 12 (2002) 1114–1137.
- [17] J.B. Lasserre, S.O.S. approximations of polynomials nonnegative on an algebraic set, *SIAM J. Optim.* 16 (2005) 610–628.
- [18] M. Laurent, Semidefinite representations for finite varieties, *Math. Program.* 109 (2007) 1–26.
- [19] J. Nie, J.W. Demmel and B. Sturmfels, Minimizing polynomials via sum of squares over the gradient ideal, *Math. Program.* 106 (2006) 587–606.
- [20] A.E. Nussbaum, Quasi-analytic vectors, *Ark. Mat.* 6 (1966) 179–191.
- [21] P.A. Parrilo, Semidefinite programming relaxations for semialgebraic problems, *Math. Program. Ser. B* 96 (2003) 293–320.
- [22] P.A. Parrilo, An explicit construction of distinguished representations of polynomials nonnegative over finite sets, IfA Technical Report AUT02-02, Zurich, Switzerland, March 2002.
- [23] P. Parrilo and B. Sturmfels, Minimizing polynomial functions, in *Algorithmic and Quantitative Real Algebraic Geometry*, S. Basu and L. Gonzales-Vega (eds.) DIMACS Series in Discrete Mathematics and Theoretical Computer Science, Vol 60, 2003.
- [24] A. Prestel and C.N. Delzell, *Positive Polynomials*, Springer, Berlin, 2001.
- [25] M. Putinar, Positive polynomials on compact semi-algebraic sets, *Indiana Univ. Math. J.* 42 (1993) 969–984.
- [26] C. Scheiderer, Positivity and sums of squares: A guide to some recent results, Department of Mathematics, University of Duisburg, Germany.
- [27] K. Schmüdgen, The K -moment problem for compact semi-algebraic sets, *Math. Ann.* 289 (1991) 203–206.
- [28] M. Schweighofer, Optimization of polynomials on compact semialgebraic sets, *SIAM J. Optim.* 15 (2005) 805–825.
- [29] L. Vandenberghe and S. Boyd, Semidefinite programming, *SIAM Review* 38 (1996) 49–95.

- [30] H. Waki, S. Kim, M. Kojima and M. Maramatsu, Sums of squares and semidefinite programming relaxations for polynomial optimization problems with structured sparsity, *SIAM J. Optim.* 17 (2006) 218–242.

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