Yokohama Publishers
ISSN 1349-8169 ONLINE JOURNAL

Pacific Journal of Optimization
Vol. 9, No. 4, pp. 709–719, October 2013

COORDINATION OF COOPRATIVE ADVERTISING MODELS IN SUPPLY CHAIN WHEN MANUFACTURER OFFERS TRADE-CREDIT*

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Abstract: This paper considers the issue of cooperative (co-op) advertisement in a manufacturer-retailer supply chain, in which the manufacturer offers complete/partial trade credit (T-C) to the retailer, and the retailer may be capital constrained. Three game structures, namely, Nash game, Stackelberg game and Partnership game, are used to discuss four different models: complete T-C with capital adequate retailer, complete T-C with capital constrained retailer, partial T-C with capital adequate retailer, and partial T-C with capital constrained retailer. We present the optimal brand investment, and local advertising expense associated with the decision of the manufacturer on T-C type and the condition of initial fund of retailer. By comparing the optimal co-op advertising decision without T-C, we show that the higher the ratio of manufacturer willing to offer T-C is, the higher both the retailer and the manufacturer spend on local advertising and brand investment in the Nash equilibrium and the Stackelberg equilibrium for the four models, except the local advertising investment of the retailer in the Nash equilibrium when the retailer has sufficient capital. We show that the combination of T-C and co-op advertising policy is more effective in coordinating the supply chain.

Key words: *supply chain, cooperative advertising, trade-credit, capital constraint*

Mathematics Subject Classification: *91A40, 91B60, 90B50*

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1 Introduction

Cooperative (co-op) advertising is an interactive relationship between two members of a supply chain, in which the retailer implements local advertisement and the manufacturer pays a portion of the cost. Co-op advertising provides consumers with needed information when they move though the final stages of a purchase. It also provides consumers a congruence of information and other needed information that would be impossible to obtain if there is only national advertising (Young & Greyser, 1983). Co-op advertising can strengthen the image of a brand, motivate immediate sales at the retail level, increase the competitiveness of the brand in markets, and change the relationship of the manufacturer and the retailer from a win-lose situation into a win-win situation in terms of saving costs and increasing revenue, such as what occurred between P&G and Wal-Mart (Huang et al., 2002).

In order to maximize individual gain, knowing how to determine the optimal co-op advertising policy of each member in a supply chain is natural and important. Many studies

*The National Natural Science Foundation(71272085), The Ministry of Education of Humanities and Social Science Planning Fund(12YJA630135).

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have discussed the co-op advertising policy of a supply chain. However, to the best of our knowledge, only a few consider trade credit (T-C) or the budget constraint of retailer. In developing economies where firms have limited access to credit and financing services, budget constraint will influence their operational decisions as well as the decisions of correlative firms within the same supply chain. The portion of local advertising expense taken by manufacturer can relieve the budget pressure on retailer. However, the co-op advertising policy is finite in handling those problems associated with capital constraint. To solve these problems, the manufacturer needs to provide some different patterns for retailer to ease its budget pressure. T-C is a short-term business loan used by a buyer to purchase goods from a seller. The seller finances the purchase by allowing the buyer to delay the payment for a bill with a rate. T-C has been the largest source of working capital for a majority of business-to-business firms in the United States, and a critical source of capital for many businesses, especially for those startups and growing businesses (Berlin, 2003).

In this paper, we focus on co-op advertising between two supply chain members (i.e., a manufacturer and a retailer) with T-C. Some studies on T-C are presented from the financial perspective (see, Schwartz, 1974; Biais & Gollier, 1997; Brennan et al., 1988; Lang and Nakamura, 1995; Petersen & Rajan, 1997; Lee & Rhee, 2009, 2011; Chen & Wang, 2012). In these studies, T-C is an important way for retailer with constrained capital to improve ordering quantity. Alternatively, T-C is also a vital issue in supply chain management, because of the time value of money (see, Beranek, 1967; Aggarwal & Jaggi, 1995; Jamal et al., 1997; Gupta & Wang, 2009). In this study, the main objective of the manufacturer providing T-C to the retailer, is to obtain a coordination mechanism that increases the local advertising expenditure of the retailer regardless of whether the retailer has sufficient capital.

Co-op advertising is a coordination mechanism for advertising activities in a supply chain, in which to motivate the immediate sales of product, the manufacturer needs to not only pay brand name investments, but also should take part of local advertisement cost incurred by the retailer. Huang et al. (2002) examined and compared both traditional leader-follower co-op advertising and incorporated partnership co-op advertising coordination models, and determined the optimal brand name investment and the optimal local advertising expenditure, including the manufacturer's allowance in both cases. Yue et al.(2006), who assumed that demand relied on retail price and co-op advertising efforts, developed the work of Huang et al.(2002) by using a price discount model to coordinate the advertising expenses of two supply chain members. Moreover, using game theory, Xie & Neyret (2009) identified the optimal pricing and co-op advertising strategies for four classical relationship types between manufacturer and retailer. Xie & Wei (2009), in a study that differed from Xie & Neyret(2009), further investigated the co-op advertising and pricing problems in a one-manufacturer one-retailer channel, by employing a sales response function with respect to the advertising expenditure and the selling price of product. Taking the effect of advertising on the reference price into account, Zhang et al.(2013) proposed a dynamic cooperative advertising model for a manufacturer-retailer supply chain and analyzed how the reference price effect would influence the decisions of supply chain members. Wang et al.(2011) considered the co-op advertising issues of a monopolistic manufacturer with competing duopolistic retailers, and discussed the optimal co-op advertising policies under four possible game structures: Stackelberg-Cournot, Stackelberg-Collusion, Nash-Cournot and Nash-Collusion.

In co-op advertising literature, T-C has not been discussed much. Nevertheless, T-C is a significant coordination mechanism and an effective approach to improve the order quantity of a retailer and ease its budget shortage. In this paper, we will consider the co-op advertising issues for a two-stage supply chain, in which the manufacturer only sells its product to the retailer only, in turn, the retailer sells the product to customers at a constant retail price. The manufacturer likes to offer complete/partial T-C to the retailer, whose budget may be constrained. We explore two types of vertical game structures (i.e., Stackelberg game and Nash game) between the two echelons of a supply chain, and a partnership game for each model (i.e., complete T-C for a retailer with sufficient capital, complete T-C for a retailer with capital-constrained, partial T-C for a retailer with sufficient capital, and partial T-C for a retailer with capital-constrained). Our main objective of this paper is to obtain the optimal co-op advertising strategies for the four models, and introduce a more effective coordination mechanism.

The rest of the paper is organized as follows: Section 2 introduces the basic models and notations. Section 3 shows the co-op advertising decisions of two members in four two-echelon supply chain models using the Nash game, Stackelberg game and Partnership game. Section 4 presents a comparison between the results obtained in this paper and the optimal co-op advertising decisions without T-C. The conclusion is give in Section 5.

2 Basic Models and Notations

Consider a supply chain that consists only of a single manufacturer and a single retailer, in which the manufacturer intends to offer complete/partial trade-credit (T-C) mechanisms to the retailer, and the capital of the retailer may be constrained according to different preliminary fund conditions. Complete T-C indicates that payment for the bill of the whole product can be delayed until the net date agreed upon with the manufacturer. For partial T-C, it indicates that there only part of the bill can be delayed until the net date, the other products require immediate payment. There are four models discussed in this paper according to what are the different conditions of retailer's initial capital. They are (i) complete T-C for a retailer with sufficient capital; (ii) complete T-C for a retailer with capital constraint; (iii) partial T-C for a retailer with adequate capital; (iv) partial T-C for a retailer with capital constraint.

Let S denote the demand volume function of product, defined by

$$S(a, A) = \alpha - a^{-\gamma} A^{-\delta},$$

where $a \geq 0$, $A \geq 0$ and α, γ and δ are positive constants. It is affected by both the local advertising expenditure of the retailer and the brand/national investment of the manufacturer. (Huang et al., 2002).

r_b is the interest rate of bill that the retailer delayed payment. Let w be the wholesale price, and let p be the retailer price which is exogenously determined. c is the unit production cost to the manufacturer. Let $\rho_{mb} = w(1 + r_b) - c$ and $\rho_{rb} = p - w(1 + r_b)$. They represent the dollar marginal profits for the manufacturer and the retailer, when the manufacturer offers complete T-C, respectively. Furthermore, let $\rho_m = w - c$ and $\rho_r = p - w$ denote, respectively, the marginal profits for the manufacturer and the retailer when there is no T-C. Clearly, the marginal profit for the manufacturer when it offers T-C is higher than the case when there is no trade-credit. On the other hand, the marginal profit of the retailer is lower, as $r_b \geq 0$.

Let $t \in [0, 1]$ denote the fraction of the total local advertising expenditure, which the manufacturer agrees to share with the retailer. $B > 0$ is the initial budget of the retailer, according to the amount of B, the retailer may be a capital-constrained enterprise. Let λ be the ratio that the bill can be delayed when the manufacturer offers partial T-C. Let $\rho_{m\lambda} = \lambda\rho_{mb} + (1 - \lambda)\rho_m$ and $\rho_{r\lambda} = \lambda\rho_{rb} + (1 - \lambda)\rho_r$ denote the marginal profits for the manufacturer and the retailer, respectively. π_{mb}, π_{rb} and π are profit functions for the manufacturer, the retailer and the supply chain system with complete T-C, while $\pi_{mb}(\lambda)$, $\pi_{rb}(\lambda)$ and $\pi(\lambda)$ denote the profit functions with partial T-C. Clearly, we have $\rho_{mb}+\rho_{rb}=\rho_m+\rho_r=\rho_{m\lambda}+\rho_{r\lambda}$.

Assumption : The manufacturer has the right to decide the T-C form, which is either complete or partial.

Model i: complete T-C, and the initial capital of the retailer is sufficient. The profit functions of the manufacturer, the retailer and the supply chain system can be expressed as follows:

$$\pi_{mb}=\rho_{mb}(\alpha-a^{-\gamma}A^{-\delta})-ta-A \tag{2.1}$$

$$\pi_{rb}=\rho_{rb}(\alpha-a^{-\gamma}A^{-\delta})-(1-t)a \tag{2.2}$$

$$\pi=(\rho_{mb}+\rho_{rb})(\alpha-a^{-\gamma}A^{-\delta})-ta-A \tag{2.3}$$

Model ii: complete T-C, and the retailer is capital constrained. For the manufacturer and the retailer and the system, we have the same profit functions similar to (2.1), (2.2) and (2.3), but with limitation conditions $(1-t)a\leq B$ and $a\leq B$.

Model iii: partial T-C, and the retailer has sufficient capital. The profit functions of the manufacturer, the retailer and the supply chain system can be expressed as follows:

$$\pi_{mb}(\lambda)=(\lambda\rho_{mb}+(1-\lambda)\rho_m)(\alpha-a^{-\gamma}A^{-\delta})-ta-A \tag{2.4}$$

$$\pi_{rb}(\lambda)=(\lambda\rho_{rb}+(1-\lambda)\rho_r)(\alpha-a^{-\gamma}A^{-\delta})-(1-t)a \tag{2.5}$$

$$\pi(\lambda)=[\lambda(\rho_{mb}+\rho_{rb})+(1-\lambda)(\rho_m+\rho_r)](\alpha-a^{-\gamma}A^{-\delta})-ta-A \tag{2.6}$$

Model iv: partial T-C, and the retailer is capital constrained. The profit functions of the manufacturer, the retailer and the supply chain system are similar to (2.5) and (2.6), but with the limitation conditions $(1-\lambda)w(\alpha-a^{-\gamma}A^{-\delta})+(1-t)a\leq B$ and $a\leq B$.

In order to discriminate the different game equilibria for four models, in the rest of this paper, let superscript indexes n,s,c denote the equilibrium decisions of the Nash, Stackelberg and Partnership games, and subscript indexes i,ii,iii,iv denote four different models, respectively.

3 Three Kinds of Equilibrium for the Four Models

3.1 Nash equilibrium

In a Nash game, the manufacturer and the retailer have same deciding power, and they decide their optimal decisions independently and simultaneously. In this subsection, we determine the Nash equilibrium points for the four supply chain models introduced in the previous section. To determine the Nash equilibrium, the decision problems of the manufacturer and the retailer are solved separately.

Proposition 3.1. *The four models in the Nash game all have unique equilibriums. They are given by*

Model i: $t_i^n=0$, $a_i^n=(\rho_{mb}^{-\delta}\rho_{rb}^{\delta+1}\delta^{-\delta}\gamma^{\delta+1})^{\frac{1}{\delta+\gamma+1}}$, $A_i^n=(\rho_{mb}^{\gamma+1}\rho_{rb}^{-\gamma}\delta^{\gamma+1}\gamma^{-\gamma})^{\frac{1}{\delta+\gamma+1}}$;

Model ii: $t_{ii}^n=0$, $a_{ii}^n=B$,$A_{ii}^n=(\rho_{mb}\delta B^{-\gamma})^{\frac{1}{\delta+1}}$;

Model iii: $t_{iii}^n=0$, $a_{iii}^n=((\lambda\rho_{mb}+(1-\lambda)\rho_m)^{-\delta}(\lambda\rho_{rb}+(1-\lambda)\rho_r)^{\delta+1}\delta^{-\delta}\gamma^{\delta+1})^{\frac{1}{\delta+\gamma+1}}$, $A_{iii}^n=((\lambda\rho_{mb}+(1-\lambda)\rho_m)^{\gamma+1}(\lambda\rho_{rb}+(1-\lambda)\rho_r)^{-\gamma}\delta^{\gamma+1}\gamma^{-\gamma})^{\frac{1}{\delta+\gamma+1}}$;

Model iv: $t_{iv}^n = 0$, a_{iv}^n *satisfies the equation:*

$$B = a_{iv}^n + (1-\lambda)w\{\alpha - [(a_{iv}^n)^{-\gamma}\delta^{-\delta}(\lambda\rho_{mb} + (1-\lambda)\rho_m)^{-\delta}]^{\frac{1}{\delta+1}}\},$$

and A_{iv}^n *satisfies the equation:*

$$A_{iv}^n = [\delta(\lambda\rho_{mb} + (1-\lambda)\rho_m)(a_{iv}^n)^{-\gamma}]^{\frac{1}{\delta+1}}\}.$$

Proof. Take the first partial derivative of π_{mb} with respect to t, it follows that, $\frac{\partial \pi_{mb}}{\partial t} = -a < 0$, meaning that the manufacturer has a negative coefficient in its objective function. That is, the more fraction of the local advertising cost the manufacturer is willing to pay, the less profit it will make, therefore, the optimal decision of t in the Nash game is $t = 0$ for all the four models.

Now, we analyze the Nash equilibrium for model i, with $t = 0$. The profit functions (2.1) and (2.2) can be rearranged as:

$$\pi_{mb} = \rho_{mb}(\alpha - a^{-\gamma}A^{-\delta}) - A,$$

$$\pi_{rb} = \rho_{rb}(\alpha - a^{-\gamma}A^{-\delta}) - a.$$

Taking the first derivative of π_{mb} and π_{rb} with respect to A and a, respectively, and then setting them to be zero, it gives

$$\rho_{mb}\delta a^{-\gamma}A^{-\delta-1} - 1 = 0, \rho_{rb}\gamma a^{-\gamma-1}A^{-\delta} - 1 = 0.$$

Solving these two equations, we can obtain the Nash equilibrium for model i, i.e., $t_i^n = 0$, $a_i^n = (\rho_{mb}^{-\delta}\rho_{rb}^{\delta+1}\delta^{-\delta}\gamma^{\delta+1})^{\frac{1}{\delta+\gamma+1}}$, and $A_i^n = (\rho_{mb}^{\gamma+1}\rho_{rb}^{-\gamma}\delta^{\gamma+1}\gamma^{-\gamma})^{\frac{1}{\delta+\gamma+1}}$. Similarly, we can obtain the Nash equilibrium for model iii as described in the proposition.

For model ii, as π_{rb} is concave with respect to a, it increases as the local advertising cost is increased while the cost of local advertising is less than the optimal decision. Thus, if the retailer is capital constrained ($a < (\rho_{mb}^{-\delta}\rho_{rb}^{\delta+1}\delta^{-\delta}\gamma^{\delta+1})^{\frac{1}{\delta+\gamma+1}}$), then, the best decision of a is $a = B$. Here, the optimal brand investment of the manufacturer is $A = (\rho_{mb}B^{-\gamma})^{\frac{1}{\delta+1}}$ in accordance with $\rho_{rb}\gamma a^{-\gamma-1}A^{-\delta} - 1 = 0$. Similarly, for model iv, in order to obtain more revenue the retailer will spend all the capital so as to creat profit, i.e. $B = a + (1-\lambda)w(\alpha - a^{-\gamma}A^{-\delta})$. With the decision of the manufacturer $A = [(\lambda\rho_{mb} + (1-\lambda)\rho_m)\delta a^{-\gamma}]^{\frac{1}{\delta+1}}$, we obtain the Nash equilibrium for model iv. □

According to the Nash equilibrium formulations, if the retailer has an ample operational fund, T-C is positive in improving the national investment of the manufacturer. Moreover, the higher the interest rate of T-C is, the more fervent is the brand advertising. However, the retailer experiences the opposite. If the initial capital is constrained, the retailer has the chance to utilize limited finances more effectively by taking advantage of T-C. Therefore, T-C is useful for increasing both the local and national advertising investments. In particular, if we do not consider the time value of money, because of higher marginal profit, the manufacturer will always offer T-C to the retailer. However, a capital sufficient retailer will not like this policy in the Nash game because it will give rise to a lower marginal profit gain.

3.2 Stackelberg equilibrium

The Stackelberg game is an interactive two-stage non-cooperative game between a manufacturer and a retailer. The manufacturer is the leader, and the retailer is the follower.

The solution of this structure is called Stackelberg equilibrium. The manufacturer first declares the level of brand investment, its reimbursement policy for local advertising, and the choice of T-C type. According to these factors, the retailer then determines the quantity of products to be purchased and the cost of local advertising. To obtain the Stackelberg equilibrium, we use the reverse decision-making sequence.

Consider the Stackelberg equilibriums for models i and iii. We analyze the reaction function in the second stage of the game for the model i. The optimal value of the local advertising expenditure is determined by setting the first derivative of π_{rb} with respect to a to zero:

$$\frac{\partial \pi_{rb}}{\partial a} = \gamma\rho_{rb}a^{-\gamma-1}A^{-\delta} - (1-t) = 0.$$

We obtain

$$a = \left(\frac{\gamma\rho_{rb}}{(1-t)A^{\delta}}\right)^{\frac{1}{\gamma+1}} \tag{3.1}$$

Then, by substituting (3.1) into the objective function (2.1) of the manufacturer, we obtain

$$\max \pi_{mb} = \rho_{mb}\left(\alpha - \left(\frac{\gamma\rho_{rb}}{(1-t)A^{\delta}}\right)^{\frac{-\gamma}{\gamma+1}}A^{-\delta}\right) - t\left(\frac{\gamma\rho_{rb}}{(1-t)A^{\delta}}\right)^{\frac{1}{\gamma+1}} - A \tag{3.2}$$

Solving the first-order conditions of (3.2) with respect to t and A, and substituting the optimal decisions t and A into (3.1), we have the unique equilibrium for model i. Similarly, we can get the Stackelberg equilibrium for model iii.

Proposition 3.2. *Stackelberg equilibrium for model i can be described as:*

$$a_i^s = [\delta^{-\delta}\gamma^{\delta+1}(\rho_{mb} - \gamma\rho_{rb})]^{\frac{1}{\delta+\gamma+1}},$$

$$t_i^s = \begin{cases} \frac{\rho_{mb}-(\gamma+1)\rho_{rb}}{\rho_{mb}-\gamma\rho_{rb}}, & \frac{\rho_{mb}}{\rho_{rb}} > \gamma+1 \\ 0, otherwise, \end{cases}$$

$$A_i^s = [\delta^{\gamma+1}\gamma^{-\gamma}(\rho_{mb} - \gamma\rho_{rb})]^{\frac{1}{\delta+\gamma+1}}.$$

Furthermore, with ρ_{mb} and ρ_{rb} replaced, respectively, by $\lambda\rho_{mb}+(1-\lambda)\rho_m$ and $\lambda\rho_{rb}+(1-\lambda)\rho_r$, Stackelberg equilibrium $(t_{iii}^s, a_{iii}^s, A_{iii}^s)$ is obtained for model iii.

Observe that

$$\frac{\partial a_i^s}{\partial(\rho_{mb} - \gamma\rho_{rb})} = \frac{1}{\delta+\gamma+1}[\delta^{-\delta}\gamma^{\delta+1}(\rho_{mb} - \gamma\rho_{rb})^{-\gamma-\delta}]^{\frac{1}{\delta+\gamma+1}} > 0,$$

$$\frac{\partial A_i^s}{\partial(\rho_{mb} - \gamma\rho_{rb})} = \frac{1}{\delta+\gamma+1}[\delta^{\gamma+1}\gamma^{-\gamma}(\rho_{mb} - \gamma\rho_{rb})^{-\gamma-\delta}]^{\frac{1}{\delta+\gamma+1}} > 0,$$

$$\frac{\partial t_i^s}{\partial r_b} = \frac{w(\rho_{mb} + \rho_{rb})}{(\rho_{mb} - \gamma\rho_{rb})^2} > 0.$$

As $(\rho_{mb} - \gamma\rho_{rb}) - (\rho_m - \gamma\rho_r) = w(1+\gamma)r_b \geq 0$, we observe that a T-C policy will lead to more marginal profit for the manufacturer. Therefore, the manufacturer has a higher enthusiasm on the national advertising and the participation of the local advertising, and the retailer would like to spend more on the local advertising. Thus, T-C can induce more immediate sales. In contrast, to capitalize the enhanced brand awareness created by a higher investment on the national advertising effectively, the retailer may want to increase local advertising expenditure, which transforms the awareness into an immediate need of the product. Furthermore, by analyzing Stackelberg equilibrium for model *iii*, we can see that a higher ratio of T-C offered by the manufacturer will induce both higher local and brand advertising investments, as well as a higher fraction t^s_{iii}. In other words, T-C acts as a coordination mechanism for the supply chain co-op advertising, is available.

When the capital is insufficient, the retailer is restricted to alleviate the negative influence of insufficient capital with only a co-op advertising policy. Specially, if the shortage is serious, a new coordination mechanism is required to increase the investment of the local advertising. The delayed payment for a bill can improve the utilization of limited budget more effectively and ease the financial pressure on the retailer. In models *ii* and *iv*, with different capital constraint conditions $(1-t)a \le B$, $(1-\lambda)w(\alpha - a^{-\gamma}A^{-\delta}) + (1-t)a \le B$, the retailer can not get its optimal strategies like models *i* and *iii*. It needs to consider the capital conditions. In both models *i* and *iii*, the profit functions of the retailer are all concave in a. Thus, it follows that the profit function increase in a, for $a \le a^s_i$ and $a \le a^s_{iii}$, respectively. The optimal choice for the retailer is to invest all its initial fund on local advertising and ordering. Then, the optimal decision for the retailer on the advertising expense is such that $(1-t)a = B$ for model *ii* and $(1-\lambda)w(\alpha - a^{-\gamma}A^{-\delta}) + (1-t)a = B$ for model *iv*. Substituting this two limitation conditions into (2.1) and (2.4), respectively, the profit functions of the manufacturer for models *ii* and *iv* can be rearranged as:

$$\pi_{mb} = \rho_{mb}(\alpha - (\frac{B}{1-t})^{-\gamma}A^{-\delta}) - \frac{tB}{1-t} - A, \tag{3.3}$$

$$\pi_{mb}(\lambda) = (\lambda\rho_{mb} + (1-\lambda)\rho_m)\frac{B-(1-t)a}{(1-\lambda)w} - ta - A. \tag{3.4}$$

Analyze both objective functions (3.3) and (3.4), we obtain Stackelberg equilibriums for models *ii* and *iv*.

Proposition 3.3. *Stackelberg equilibrium for model ii is:*

$$A^s_{ii} = [\rho_{mb}\delta^{\gamma+1}\gamma^{-\gamma}]^{\frac{1}{\delta+\gamma+1}},$$

$$t^s_{ii} = 1 - B(\rho_{mb}\gamma^{\delta+1}\delta^{\delta})^{-\frac{1}{\delta+\gamma+1}},$$

and all the expenses of the retailer for the local advertising is B.

As $\rho_{mb} \ge \rho_{mb} - \gamma\rho_{rb}$, with limited local advertising budget, the manufacturer, product to be purchased, would try its best to enhance its publicity budget.

Proposition 3.4. *Stackelberg equilibrium for model iv can be described as:*

(1) *Suppose that the partial T-C interest rate is r_b and that the proportion of T-C that the manufacturer would offer satisfies $\lambda(1+r_b) \ge \frac{c}{w}$. Then, the equilibrium is: $t^s_{iv} = 1$, $A^s_{iv} = [(\alpha - \frac{B}{(1-\lambda)w})^{-1}\delta^{\gamma}\gamma^{-\gamma}]^{\frac{1}{2\delta\gamma+\delta+1}}$ and $a^s_{iv} = [(\alpha - \frac{B}{(1-\lambda)w})^{2\delta+1}\delta^{\delta}\gamma^{-\delta}]^{\frac{-1}{2\delta\gamma+\delta+1}}$.*

(2) *Suppose that the partial T-C interest rate is r_b and that the proportion of T-C that the manufacturer would offer satisfies $\lambda(1+r_b) < \frac{c}{w}$. Then, the equilibrium is: $t^s_{iv} = 0$, and a^s_{iv} and A^s_{iv} are, respectively, determined by $(1-\lambda)w(\alpha - a^{-\gamma}A^{-\delta}) + a = B$ and $(\lambda\rho_{mb} + (1-\lambda)\rho_m)\delta = a^{\gamma}A^{\delta+1} + w(1-\lambda)\gamma a^{-1}A^{2\delta+1}$.*

Proof. Taking the first partial derivation of (3.4) with respect to t yields

$$\frac{\partial \pi_{mb}}{\partial t} = a(\frac{\lambda w(1+r_b)-c}{(1-\lambda)w}).$$

Clearly, suppose that the partial T-C interest rate is r_b and that the proportion of T-C that the manufacturer would offer satisfies $\lambda(1+r_b) \geq \frac{c}{w}$, $\frac{\partial \pi_{mb}}{\partial t} \geq 0$. Then, the optimal value of t is $t^s_{iv} = 1$; otherwise, $t^s_{iv} = 0$. Substituting the above two different cases into (3.4), and analyzing, respectively, the two corresponding optimization problems, the conclusion of the proposition follows readily. □

Consider the relations of a^s_{iv} and A^s_{iv} with λ. It is seen that the higher the ratio of T-C that the manufacturer would offer, the more the local advertising investment and the brand advertising expenditure. In contrast, the amount of initial capital of the retailer will directly restrict the co-op advertising expense of the supply chain. With more capital, the capital-constrained retailer will invest more in the local advertising and order more products. As a consecution, it encourages the manufacturer to spend more on brand cost to develop brand knowledge and preference.

3.3 Partnership equilibrium

In the last three decades, many studies on marketing show that the bargaining power of retailers in many industries have increased. The altered position of P&G and Wal-Mart mentioned in many studies is frequently cited as an example (such as, Li et al., 2002; Huang et al., 2002). For Wal-Mart, its relationship with P&G has changed from being dominated by P&G to engaging it in a partnership and full coordination. This partnership or coordination has turned a win-lose situation between P&G and Wal-Mart into a win-win situation because of lower cost and higher revenue for both firms. Similarly, for many industries, a well-organized supply chain leads to increased efficiencies, faster response to market changes, better design and manufacturing processes, increased productivity, and increased competitiveness of both the manufacturer and the retailer. By considering the manufacturer and the retailer as partners in co-op advertising, the profit of the total system, as well as individual profits, is much increased.

To implement the partnership game, we relax the relationship of the supply chain firms as a system. The central decision-maker will seek to maximize the expected profit of the system. Since $\rho_{mb} + \rho_{rb} = \rho_m + \rho_r = \rho_{m\lambda} + \rho_{r\lambda}$, it follows that the same system profit functions for the four models (i, ii, iii, iv) showed appear as:

$$\pi_c = (\rho_m + \rho_r)(\alpha - a^{-\gamma}A^{-\delta}) - a - A. \tag{3.5}$$

So, the optimal decision of the system, for which the capital of the retailer is adequate, can be expressed as:

$$a^c_i = a^c_{iii} = \delta^{-\delta}\gamma^{\delta+1}(\rho_m+\rho_r)^{\frac{1}{\delta+\gamma+1}},$$
$$A^c_i = A^c_{iii} = \delta^{\gamma+1}\gamma^{-\gamma}(\rho_m+\rho_r)^{\frac{1}{\delta+\gamma+1}}.$$

We can see that $A^c_i, A^c_{iii}, a^c_i, a^c_{iii}$ are identical with the results obtained in Huang et al. (2002). That is, no matter what kind of T-C (complete, partial) that the manufacturer wants to offer, if both enterprises are capital sufficient, the optimal decision of the system will be constant.

Here, we consider the cases of models ii and iv. Unlike the different limitation conditions for model i and model iv in Nash and Stackelberg games, the objective functions for models *ii* and *iv* are similar, and the limitation conditions are identical as well in the partnership game, i.e. $a \leq B$. Therefore, obtaining the optimal decision is easy, which is given as follows:

$$A_{ii}^c = A_{iv}^c = [(\rho_m + \rho_r)\delta B^{-\gamma}]^{\frac{1}{\delta+1}}, a_{ii}^c = a_{iv}^c = B.$$

4 Comparison

In this section, we mainly present a comparison between the optimal decisions when the manufacturer offers complete/partial T-C and the case when there is no T-C, and give the proof for the case where the action of T-C is useful in the co-op advertising supply chain system as a coordinating mechanism.

First, in terms of Nash equilibriums, if any kind of T-C does not exist, the best local advertising decision for models *ii* and *iv* are lower than those when the manufacturer offers T-C, because the retailer needs to utilize part of its initial capital on the payment of the bill at the beginning of the selling season. T-C can effectively relieve the budget pressure on the retailer. Regarding the results in Proposition 3.1, we can easily show that the brand investment of the manufacturer with T-C is always higher than when there is no T-C given in models *ii* and *iv*, because of $\rho_{mb} > \rho_m$.

Let w_b denote $w(1 + r_b)$. Then, it follows readily that $\frac{\partial a_i^n}{\partial w_b} < 0$ and $\frac{\partial A_i^n}{\partial w_b} > 0$, meaning that if the manufacturer and the retailer decide their advertising investments separately, the manufacturer will be more vigorous in advertisement than when there is no T-C given. However, the enthusiasm of the retailer will be reduced. Because the marginal profit of the manufacturer will be improved when it offers T-C (complete/partial). However, the marginal profit of the retailer will decrease. This conclusion also applies to model *iii*.

Second, in term of Stackelberg equilibrium, as described in Proposition 3.2, both the optimal brand and local advertising expenses are higher than the case when there is no T-C given in models *i* and *iii*, as $\rho_{mb} - \gamma\rho_{rb} > \rho_m - \gamma\rho_r$ and $(\lambda\rho_{mb} + (1-\lambda)\rho_m) - \gamma(\lambda\rho_{rb} + (1-\lambda)\rho_r) > \rho_{mb} - \gamma\rho_{rb}$. Moreover, if the retailer is capital constrained, the analysis process is similar to the analysis process of Nash equilibrium in the precious paragraph. In other words, the presence of T-C (complete/partial) in Stackelberg game, regardless of the condition of the retailer's capital, the equilibrium of the investment by the supply chain firms in the national and local advertising will increase.

Finally, the T-C is useless in the case of Partnership game, because the firms belong to the same system.

5 Discussion

Most research on the cooperative advertising in the literature has focused on the relationship between manufacturers as a leader and retailers as followers. However, the market structure has altered in recent years, meaning that cooperative advertising as a coordination mechanism can not adequately handle new situations. The manufacturer must adopt a more appropriate approach.

The major contributions of the paper include three points:

(1) We investigate four supply chain co-op advertising models. In these models, some important market factors are considered, such as trade-credit, capital constraint and co-op advertising.

(2) We examine the optimal co-op advertising decision for the four models, in Nash game, Stackelberg game and Partnership game, and obtain the equilibrium point for the four models in different games.

(3) We show the proof of that the combination of T-C and co-op advertising policy is more effective in coordinating the supply chain.

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Manuscript received 28 February 2013
revised 7 August 2013, 2 September 2013
accepted for publication 18 September 2013

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