



## PERFORMANCE OPTIMIZATION OF A DYNAMIC CHANNEL BONDING STRATEGY IN COGNITIVE RADIO NETWORKS\*

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**Abstract:** In a conventional cognitive radio network serving primary and secondary users, all channels are active, even when there are no packets to be transmitted. Obviously, this leads to a waste of network resources, such as spectrum. In order to conserve the network resources and to guarantee the Quality of Service (QoS) for secondary users, we propose a dynamic channel bonding strategy, in which channels are bonded dynamically based on the level of traffic in cognitive radio networks. We consider the digital nature of modern communication and the pre-emptive priority of the primary users in cognitive radio networks. Based on the working principle of the dynamic channel bonding strategy, we build a discrete-time pre-emptive priority queueing model, regarding the time period when part channels are bonded as a working vacation period. To derive the steady-state distribution of the queueing model, we construct a three-dimensional Markov chain, and give the state transition probability matrix of the Markov chain. Correspondingly, we derive the exact solution for the performance measures in terms of the blocking ratio, the throughput, the average latency of the secondary users, and the closed channel ratio. By taking into account different performance measures, we develop a net benefit function to optimize both the proportion of closed channels during the Part Bonding Period, and the buffer capacity of the SUs in the dynamic channel bonding strategy.

**Key words:** *cognitive radio networks, channel bonding, discrete-time priority queue, working vacation, optimization*

**Mathematics Subject Classification:** *68M20, 49K35*

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### 1 Introduction

Increasing demand for radio spectrum has stimulated research into improving the efficient use of spectrum resources. Several research studies have indicated that the utilization of the spectrum is very low in practical networks [10]. For example, most spectrum utilization was found to be less than 6% [17]. Cognitive radio networks have thus emerged as a promising technology for improving spectrum utilization [4].

There are two types of users in cognitive radio networks, namely, primary users (PUs) and secondary users (SUs) [15]. The network spectrum is licensed to the PUs while the SUs access to the spectrum opportunistically when the spectrum is not occupied by any PUs.

Recently, cognitive radio networks have attracted considerable attention [9], [2]. Channel bonding strategy is one spectrum enhancement technology that has emerged from cognitive radio networks research. In this technology, available channels are aggregated into one

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channel [11]. There have been several research studies that have focused on the study of cognitive radio networks with channel bonding strategies. Lee et al. considered a kind of channel bonding scheme in which an SU can utilize the spectrum with multiple available channels [8]. The loss probability and throughput were obtained with a continuous-time Markov chain. Jiao et al. assumed the channel occupancy time of an SU was inversely proportional to the number of bonded channels, and investigated the loss probability and throughput of the SUs [5].

Additionally, in the studies of cognitive radio networks with channel bonding strategies, queueing theory is widely used [16]. Su et al. developed a Markov chain model and an M/G/1 queueing model to analyze the cognitive radio networks with two channel sensing policies, and obtained the aggregate throughput [12]. Konishi et al. built a continuous-time priority queueing model to investigate the effect of the number of sub-channels to be bonded in multi-channel cognitive radio networks [7], where the blocking probability, the forced termination probability and the throughput of the SUs were derived.

As shown above, we find that most research of channel bonding strategy with queueing theory was considered in a continuous-time field. However, as communication systems are now more often digital [1], it would be more accurate and efficient to use discrete-time queueing models rather than their continuous counterparts when analyzing and designing digital transmitting systems [13]. Moreover, most of the research into cognitive radio networks with channel bonding strategies was performed under the condition that all the channels are active even when there is no packet to be transmitted. Obviously, this will lead to a waste of network resources. By introducing a Part Bonding Period, in this paper, we propose a dynamic channel bonding strategy, in which only part channels are bonded to be active for a time period when there is no packet to be transmitted.

In this paper, taking into account the working principle of the dynamic channel bonding strategy and the digital nature of modern networks, we build a discrete-time queueing model with pre-emptive priority and a working vacation. To get the steady-state distribution of the queueing model, we construct a three-dimensional Markov chain and give the transition probability matrix of this Markov chain. Accordingly, we derive the performance measures of the system, such as the blocking ratio, the throughput, the average latency of the SUs, and the closed channel ratio. Furthermore, we develop a net benefit function to optimize the proportion of closed channels during the Part Bonding Period and the buffer capacity of the SUs, respectively. To the best of our knowledge, this is the first paper related to the channel bonding strategy with a dynamic closed channel scheme in a cognitive radio network.

The remainder of this paper is organized as follows. A dynamic channel bonding strategy in cognitive radio networks is proposed in Section 2. The system model and model analysis are given in Section 3. In Section 4, the formulas for the performance measures, such as the blocking ratio, the throughput, the average latency of the SUs and the closed channel ratio are obtained. Numerical results are provided in Section 5, and the optimizations for the proportion of closed channels during the Part Bonding Period and the buffer capacity of the SUs are shown in Section 6. Finally, conclusions are drawn in Section 7.

## **2 A Dynamic Channel Bonding Strategy in Cognitive Radio Networks**

We consider a cognitive radio network with a licensed spectrum that is equally divided into  $N$  channels. Normally, all these  $N$  channels will be aggregated into one bonding channel for the packet transmission. When there is no packet to be transmitted, a part of the channels will be closed, and the unclosed channels will be aggregated into one bonding channel for

a random period. We call the time period that part of the channels are closed a “Part Bonding Period”. During the Part Bonding Period, the packets will be transmitted with a lower transmission rate. On the other hand, the time period when all channels are bonded so as to become active is called a “Full Bonding Period”. During the Full Bonding Period, the packets can be transmitted with a higher transmission rate.

If there has not been any packet arrival during one Part Bonding Period, another Part Bonding Period will be initiated. If a PU packet arrives at the system during a Part Bonding Period, due to the priority of the PUs, the Part Bonding Period will terminate immediately, and a Full Bonding Period will begin. Then this PU packet will be transmitted with a higher transmission rate. During a Part Bonding Period, the SU packets will be transmitted with a lower transmission rate. If the transmission of an SU packet is not finished before the end instant of a Part Bonding Period, a Full Bonding Period will begin after this Part Bonding Period is over, and the SU packet will be sequentially transmitted with a higher transmission rate. If there are not any SU packets to be transmitted when a Part Bonding Period is over, another Part Bonding Period will begin.

Therefore, we can infer that the PU packets will always be transmitted during a Full Bonding Period. But the SU packets will be transmitted during both a Part Bonding Period and a Full Bonding Period. Specially, an SU packet may be transmitted with a lower transmission rate during a Part Bonding Period firstly, and then the transmission will be continued with a higher transmission rate during a Full Bonding Period.

As mentioned above, there are two transmission stages in the dynamic channel bonding strategy, namely, the Part Bonding Period and the Full Bonding Period. We describe the transition of these two stages in Fig. 1.

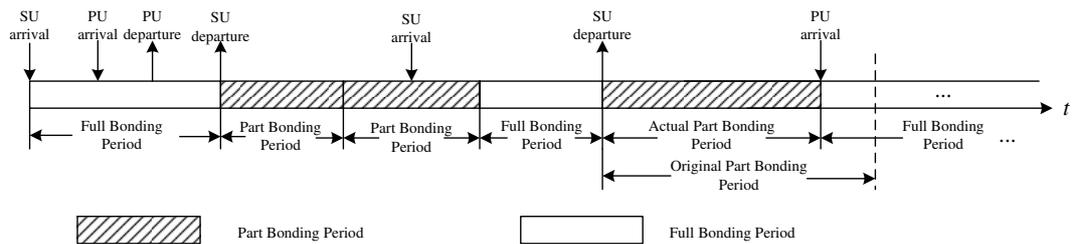


Figure 1: Transition between the Part Bonding Period and the Full Bonding Period.

In Fig. 1, the Part Bonding Period terminated due to the arrival of a PU packet is called an “Actual Part Bonding Period”, and the Part Bonding Period referred to previously is called an “Original Part Bonding Period”.

Moreover, we can describe the actions of the PU packets and the SU packets in the networks with the dynamic channel bonding strategy proposed in this paper as follows.

- (1) When an SU packet arrives at the system, if the bonding channel is being occupied by another packet (a PU packet or an SU packet), this SU packet will queue in the buffer. If the buffer is full, this SU packet will be blocked.
- (2) When a PU packet arrives at the system, if the bonding channel is idle, this PU packet will occupy the channel directly; if the bonding channel is being occupied by another PU packet, the newly arriving PU packet will be blocked; if the bonding channel is being occupied by an SU packet, this PU packet will interrupt the transmission of the SU packet and occupy the channel immediately.

- (3) When an SU packet is interrupted by a PU packet, this interrupted SU packet will return to the buffer of the SUs and queue at the head. But if the buffer of the SUs is full, this interrupted SU packet will be forced to leave the system.
- (4) Specially, we assume the interrupted SU packet have a higher priority than the newly arriving SU packets. This is because if the arrival of an SU packet and the interruption of an SU packet occur simultaneously, the interrupted SU packet will join the system, and the newly arriving SU packet will be blocked by the system.

### 3 System Model and Analysis

#### 3.1 System Model

In cognitive radio networks with a dynamic channel bonding strategy as proposed in this paper, we assume that the bonding channel acts as a server, the Part Bonding Period as the working vacation, and the PU packets and the SU packets as two types of customers in queueing theory. Then we can build a discrete-time pre-emptive priority queueing model with a working vacation. In the following paper, with the aim of avoiding ambiguity, we will equalize the terms “customer” and “packet”.

We assume the arriving intervals and transmission times of the packets are independent and identically distributed (i.i.d) random variables. The arriving intervals of the PU packets and the SU packets are supposed to follow geometrical distributions with parameters  $\lambda_1$  ( $\bar{\lambda}_1 = 1 - \lambda_1$ ) and  $\lambda_2$  ( $\bar{\lambda}_2 = 1 - \lambda_2$ ), respectively. The transmission time (slots) of a PU packet is assumed to follow a geometrical distribution with parameter  $\mu_1$  ( $\bar{\mu}_1 = 1 - \mu_1$ ). The transmission times (slots) of an SU packet during a Part Bonding Period and a Full Bonding Period are supposed to follow geometrical distributions with rates  $\mu_{2v}$  ( $\bar{\mu}_{2v} = 1 - \mu_{2v}$ ) and  $\mu_{2b}$  ( $\bar{\mu}_{2b} = 1 - \mu_{2b}$ ), respectively. We call  $\mu_1$ ,  $\mu_{2v}$  and  $\mu_{2b}$  the transmission rates (packets/slot) in this paper. Additionally, the time length  $T_V$  of a Part Bonding Period is assumed to follow a geometrical distribution with parameter  $\theta$ .  $\theta$  is called the “Part Bonding Rate” in this paper. During a Part Bonding Period, the proportion of closed channels is defined as the fraction of the number of closed channels with respect to the total number of channels in the system. That is denoted as  $\alpha$ . The buffer capacity of the SUs is defined to be finite with size  $H$  ( $H > 0$ ), and the PUs are supposed to have no buffer. As a result, we conclude that the PU packets constitute a pure disappearing queueing system, and the SU packets constitute a queueing system with both disappearing and waiting. Moreover, the transmissions of the SU packets are supposed to follow a First-Come First-Served (FCFS) strategy.

We assume the time axis is divided into slots with equal length. The slot boundaries are marked by  $t = 1, 2, \dots$ . The packets are supposed to arrive immediately after the beginning instant of a slot, and depart just prior to the end of a slot. We consider the instant  $t = n$  ( $n = 1, 2, \dots$ ) and suppose the arrivals of packets can only occur in  $(n, n^+)$ , and the departures of packets can only occur in  $(n^-, n)$ .

Let  $L_n = i$  ( $i = 0, 1, 2, \dots, H + 1$ ) be the total number of packets in the system at the instant  $t = n^+$ , and let  $L_n^{(1)} = j$  ( $j = 0, 1$ ) be the number of PU packets in the system at the instant  $t = n^+$ , respectively. Let  $K_n = k$  ( $k = 0, 1$ ) indicate the system stages.  $K_n$  can be described as follows:

$$K_n = \begin{cases} 0, & \text{the system is in the Part Bonding Period at the time } t = n^+, \\ 1, & \text{the system is in the Full Bonding Period at the time } t = n^+. \end{cases}$$



the system leaves. So the transition probability matrix  $\mathbf{P}_{1,0}$  is a column vector with three elements given by

$$\mathbf{P}_{1,0} = (\bar{\lambda}_1 \bar{\lambda}_2 \mu_{2v}, \bar{\lambda}_1 \bar{\lambda}_2 \mu_{2b}, \bar{\lambda}_1 \bar{\lambda}_2 \mu_1)^T \quad (3.5)$$

where  $T$  describes the transpose operator of the matrix.

The system level  $v = 1$  means there is one packet in the system at  $t = (n + 1)^+$ . For this case, there are no packet arrivals and the packet (one PU packet or one SU packet) in the system does not leave; or there is one PU packet arrival, and the PU packet in the system does not leave; or there is one packet arrival and the packet (one PU packet or one SU packet) in the system leaves. So the transition probability matrix  $\mathbf{P}_{1,1}$  is a  $3 \times 3$  matrix given by

$$\mathbf{P}_{1,1} = \begin{pmatrix} \bar{\lambda}_1(\bar{\lambda}_2 \bar{\mu}_{2v} + \lambda_2 \mu_{2v}) \bar{\theta} & \bar{\lambda}_1(\bar{\lambda}_2 \bar{\mu}_{2v} + \lambda_2 \mu_{2v}) \theta & \lambda_1 \bar{\lambda}_2 \mu_{2v} \\ 0 & \bar{\lambda}_1(\bar{\lambda}_2 \bar{\mu}_{2b} + \lambda_2 \mu_{2b}) & \lambda_1 \bar{\lambda}_2 \mu_{2b} \\ 0 & \bar{\lambda}_1 \lambda_2 \mu_1 & \bar{\lambda}_2(\lambda_1 \mu_1 + \bar{\mu}_1) \end{pmatrix}. \quad (3.6)$$

The system level  $v = 2$  means there are two packets in the system at  $t = (n + 1)^+$ . For this case, there is one packet arrival and the SU packet in the system does not leave; or there are two packet arrivals and the PU packet in the system does not leave; or there is one SU packet arrival and the PU packet in the system does not leave; or there are two packet arrivals and the packet (one PU packet or one SU packet) in the system leaves. So the transition probability matrix  $\mathbf{P}_{1,2}$  is a  $3 \times 3$  matrix given by

$$\mathbf{P}_{1,2} = \begin{pmatrix} \bar{\lambda}_1 \lambda_2 \bar{\mu}_{2v} \bar{\theta} & \bar{\lambda}_1 \lambda_2 \bar{\mu}_{2v} \theta & \lambda_1(\bar{\lambda}_2 \bar{\mu}_{2v} + \lambda_2 \mu_{2v}) \\ 0 & \bar{\lambda}_1 \lambda_2 \bar{\mu}_{2b} & \lambda_1(\bar{\lambda}_2 \bar{\mu}_{2b} + \lambda_2 \mu_{2b}) \\ 0 & 0 & \lambda_2(\lambda_1 \mu_1 + \bar{\mu}_1) \end{pmatrix}. \quad (3.7)$$

The system level  $v = 3$  means there are three packets in the system at  $t = (n + 1)^+$ . For this case, there are two packet arrivals and the SU packet in the system does not leave. So the transition probability matrix  $\mathbf{P}_{1,3}$  is a  $3 \times 3$  matrix given by

$$\mathbf{P}_{1,3} = \begin{pmatrix} 0 & 0 & \lambda_1 \lambda_2 \bar{\mu}_{2v} \\ 0 & 0 & \lambda_1 \lambda_2 \bar{\mu}_{2b} \\ 0 & 0 & 0 \end{pmatrix}. \quad (3.8)$$

- (3) At  $t = n^+$ , if the system level  $2 \leq u \leq H - 1$ , i.e., there are  $u$  ( $2 \leq u \leq H - 1$ ) packets in the system, the system level will be  $v$  ( $v = u - 1, u, u + 1, u + 2$ ) at  $t = (n + 1)^+$ .

The system level  $v = u - 1$  means there are  $(u - 1)$  packets in the system at  $t = (n + 1)^+$ . For this case, there are no packet arrivals and one of the packets in the system leaves. So the transition probability matrix  $\mathbf{P}_{u,u-1}$  is a  $3 \times 3$  matrix given by

$$\mathbf{P}_{u,u-1} = \begin{pmatrix} \bar{\lambda}_1 \bar{\lambda}_2 \mu_{2v} \bar{\theta} & \bar{\lambda}_1 \bar{\lambda}_2 \mu_{2v} \theta & 0 \\ 0 & \bar{\lambda}_1 \bar{\lambda}_2 \mu_{2b} & 0 \\ 0 & \bar{\lambda}_1 \bar{\lambda}_2 \mu_1 & 0 \end{pmatrix}. \quad (3.9)$$

When  $u \leq v \leq u + 2$ ,  $\mathbf{P}_{u,v}$  has the same form as the matrix  $\mathbf{P}_{1,v-u+1}$  obtained from Eqs. (3.6)-(3.8) for the system level  $u = 1$ . So  $\mathbf{P}_{u,v}$  can be given by

$$\mathbf{P}_{u,v} = \mathbf{P}_{1,v-u+1}, \quad u \leq v \leq u + 2. \quad (3.10)$$

- (4) At  $t = n^+$ , if the system level  $u = H$ , i.e., there are  $H$  packets in the system and there is only one vacancy in the buffer, the system level will be  $v$  ( $v = H - 1, H, H + 1$ ) at  $t = (n + 1)^+$ . Similar to the structures shown in Eqs. (3.9), (3.10),  $\mathbf{P}_{H,H-1}$ ,  $\mathbf{P}_{H,H}$  and  $\mathbf{P}_{H,H+1}$  can be given as follows:

$$\mathbf{P}_{H,v} = \begin{cases} \mathbf{P}_{2,1}, & v = H - 1, \\ \mathbf{P}_{2,2}, & v = H, \\ \mathbf{P}_{2,3} + \mathbf{P}_{2,4}, & v = H + 1 \end{cases} \quad (3.11)$$

where  $\mathbf{P}_{2,1}$ ,  $\mathbf{P}_{2,2}$ ,  $\mathbf{P}_{2,3}$  and  $\mathbf{P}_{2,4}$  can be obtained from Eqs. (3.9), (3.10).

- (5) At  $t = n^+$ , if the system level  $u = H + 1$ , i.e., there are  $(H + 1)$  packets in the system and there is no vacancy in the buffer, the system level will be  $v$  ( $v = H, H + 1$ ) at  $t = (n + 1)^+$ . Both  $\mathbf{P}_{H+1,H}$  and  $\mathbf{P}_{H+1,H+1}$  are  $3 \times 3$  matrices given as follows:

$$\mathbf{P}_{H+1,v} = \begin{cases} \mathbf{P}_{2,1}, & v = H, \\ \mathbf{P}_{2,2} + \mathbf{P}_{2,3} + \mathbf{P}_{2,4}, & v = H + 1 \end{cases} \quad (3.12)$$

where  $\mathbf{P}_{2,1}$ ,  $\mathbf{P}_{2,2}$ ,  $\mathbf{P}_{2,3}$  and  $\mathbf{P}_{2,4}$  can be obtained from Eqs. (3.9), (3.10).

### 3.3 Steady-State Distribution

The structure of the transition probability matrix  $\mathbf{P}$  indicates that the three-dimensional Markov chain  $\{L_n, L_n^{(1)}, K_n\}$  is non-periodic, irreducible and positive recurrent. The steady-state distribution  $\pi_{i,j,k}$  of the three-dimensional Markov chain is defined as follows:

$$\pi_{i,j,k} = \lim_{n \rightarrow \infty} P \{L_n = i, L_n^{(1)} = j, K_n = k\}. \quad (3.13)$$

Let  $\mathbf{\Pi}_i$  be the steady-state probability vector for the system being at level  $i$ .  $\mathbf{\Pi}_i$  can be given as follows:

$$\mathbf{\Pi}_i = \begin{cases} \pi_{0,0,0}, & i = 0, \\ (\pi_{i,0,0}, \pi_{i,0,1}, \pi_{i,1,1}), & 1 \leq i \leq H + 1. \end{cases} \quad (3.14)$$

$\mathbf{\Pi}_i$  can be calculated by solving the following equilibrium equations with the normalization condition [14]:

$$\begin{cases} (\mathbf{\Pi}_0, \mathbf{\Pi}_1, \dots, \mathbf{\Pi}_H, \mathbf{\Pi}_{H+1})\mathbf{P} = (\mathbf{\Pi}_0, \mathbf{\Pi}_1, \dots, \mathbf{\Pi}_H, \mathbf{\Pi}_{H+1}), \\ (\mathbf{\Pi}_0, \mathbf{\Pi}_1, \dots, \mathbf{\Pi}_H, \mathbf{\Pi}_{H+1})\mathbf{e} = 1 \end{cases} \quad (3.15)$$

where  $\mathbf{e}$  is a one's column vector.

By substituting Eq. (3.14) with Eq. (3.15) and using a Gaussian elimination method, we can obtain the steady-state distribution  $\pi_{i,j,k}$  of Eq. (3.13).

### 4 Performance Measures

We define the blocking ratio  $\delta$  of the SUs as the probability that a new arrival of SU packets will be blocked by the system. A pending SU packet will be blocked by the system when the number of packets in the system is  $(H + 1)$  at the arriving instant, i.e., the bonding channel

is occupied, and the buffer of the SUs is full. Therefore, the blocking ratio  $\delta$  of the SUs can be given as follows:

$$\begin{aligned} \delta = & \lambda_2((\bar{\mu}_{2v} + \mu_{2v}\lambda_1)\pi_{H+1,0,0} + (\bar{\mu}_{2b} + \mu_{2b}\lambda_1)\pi_{H+1,0,1} + (\bar{\mu}_1 + \mu_1\lambda_1)\pi_{H+1,1,1} \\ & + \bar{\mu}_{2v}\lambda_1\pi_{H,0,0} + \bar{\mu}_{2b}\lambda_1\pi_{H,0,1}). \end{aligned} \quad (4.1)$$

We define the throughput  $S$  of the SUs as the number of SU packets transmitted successfully per slot. An SU packet can be transmitted successfully if and only if that SU packet is neither blocked from the system, nor forced to leave the system before the transmission is successfully completed. Therefore, the throughput  $S$  of the SUs is given as follows:

$$S = \lambda_2 - \delta - \lambda_1(\bar{\mu}_{2v}\pi_{H+1,0,0} + \bar{\mu}_{2b}\pi_{H+1,0,1}). \quad (4.2)$$

We define the latency of an SU packet with successful transmission as the time period in slots from the arrival instant of an SU packet to the departure instant of this SU packet, i.e., the sojourn time of the SU packet.

Let  $L_n^{(2)}$  be the number of SU packets in the system at the instant  $t = n^+$ , and let  $L^{(2)} = \lim_{n \rightarrow \infty} L_n^{(2)}$  be the steady-state distribution of  $L_n^{(2)}$ . We can obtain the average number  $E[L^{(2)}]$  of SU packets in the steady-state of the system as follows:

$$E[L^{(2)}] = \sum_{j=0}^{H+1} jP\{L^{(2)} = j\} = \sum_{j=1}^{H+1} j(\pi_{j,0,0} + \pi_{j,0,1}) + \sum_{j=0}^H j\pi_{j+1,1,1}. \quad (4.3)$$

In this paper, the average latency  $E[T]$  of the SUs is in fact the average time period that a successfully transmitted SU packet stays in the system. By referencing Little's law, the average latency  $E[T]$  of the SUs can be given as follows [6]:

$$E[T] = \frac{E[L^{(2)}]}{S} \quad (4.4)$$

where  $E[L^{(2)}]$  can be obtained from Eq. (4.3).

We define the closed channel ratio  $\beta$  as the probability that one channel closed is during a Part Bonding Period. It is related to the proportion of closed channels during a Part Bonding Period and the probability that the system is in the Part Bonding Period. So the closed channel ratio  $\beta$  can be given as follows:

$$\beta = \alpha \sum_{i=0}^{H+1} \pi_{i,0,0} \quad (4.5)$$

where  $\alpha$  is the proportion of closed channels during the Part Bonding Period given as a system parameter.

## 5 Numerical Results

In order to investigate the influences of the proportion of closed channels during the Part Bonding Period and the buffer capacity of the SUs on the system performance, some system parameters are set as follows: the Part Bonding Rates are set as  $\theta = 0.05, 0.10, 0.15$ ; the transmission rate of the PU packets is set as  $\mu_1 = 0.21$ ; the arrival rate of the SU packets is set as  $\lambda_2 = 0.12$ , and the transmission rate of the SU packets during the Full Bonding

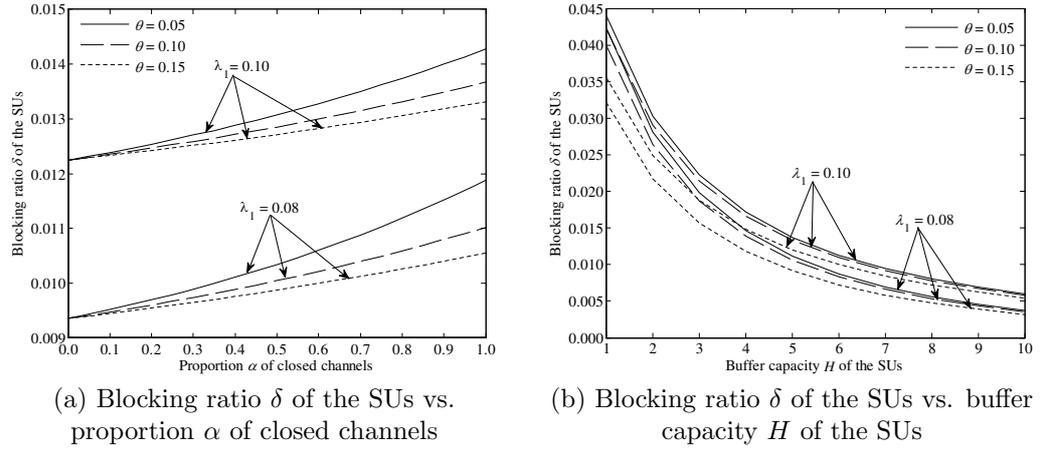


Figure 2: Change trend for the blocking ratio  $\delta$  of the SUs.

Period is set as  $\mu_{2b} = 0.2$ ; the data set for the proportions of closed channels is supposed as  $\alpha = \{0.00, 0.05, 0.10, \dots, 0.95, 1.00\}$ .

Figure 2 demonstrates the change trend for the blocking ratio  $\delta$  of the SUs.

From Fig. 2 (a), we find that, for the same Part Bonding Rate  $\theta$  and the same arrival rate  $\lambda_1$  of the PU packets, the blocking ratio  $\delta$  of the SUs will increase as the proportion  $\alpha$  of closed channels increases. The reason is that the larger the proportion of closed channels is, the lower the transmission rate during the Part Bonding Period is, and the more SU packets there are that will be blocked by the system, so the larger the blocking ratio of the SUs will be.

From Fig. 2 (b), we conclude that, for the same Part Bonding Rate  $\theta$  and the same arrival rate  $\lambda_1$  of the PU packets, the blocking ratio  $\delta$  of the SUs will decrease as the buffer capacity  $H$  of the SUs increases. The intuitive reason is that the larger the buffer capacity of the SUs is, the less likely it is that a newly arriving SU packet will be blocked by the system, so the lower the blocking ratio of the SU packets will be.

Figures 2 (a) and (b) both show that, for the same proportion  $\alpha$  of closed channels and the same arrival rate  $\lambda_1$  of the PU packets, or the same buffer capacity  $H$  of the SUs and the same arrival rate  $\lambda_1$  of the PU packets, the larger the Part Bonding Rate  $\theta$  is, the smaller the blocking ratio  $\delta$  of the SUs will be. This is because as the Part Bonding Rate increases, the time length of a Part Bonding Period will decrease, and so more packets will be transmitted at a high transmission rate. As a result, the blocking ratio of the SUs will decrease.

On the other hand, Figs. 2 (a) and (b) also show that, for the same proportion  $\alpha$  of closed channels and the same Part Bonding Rate  $\theta$ , or the same buffer capacity  $H$  of the SUs and the same Part Bonding Rate  $\theta$ , the higher the arrival rate  $\lambda_1$  of the PU packets is, and the greater the blocking ratio  $\delta$  of the SU packets will be. This is because the higher the arrival rate of the PU packets is, the more likely it is that the channel is occupied by a PU packet. As a result, more SU packets will queue in the buffer, and so more SU packets will be blocked by the system, thus the blocking ratio of the SU packets will increase.

In Fig. 3, we show the change trend for the throughput  $S$  of the SUs.

From Fig. 3 (a), we observe that, for the same Part Bonding Rate  $\theta$  and the same arrival rate  $\lambda_1$  of the PU packets, the throughput  $S$  of the SUs will decrease as the proportion  $\alpha$  of closed channels increases. The reason is that the larger the proportion of closed channels is, the lower the transmission rate during the Part Bonding Period is, and the smaller the

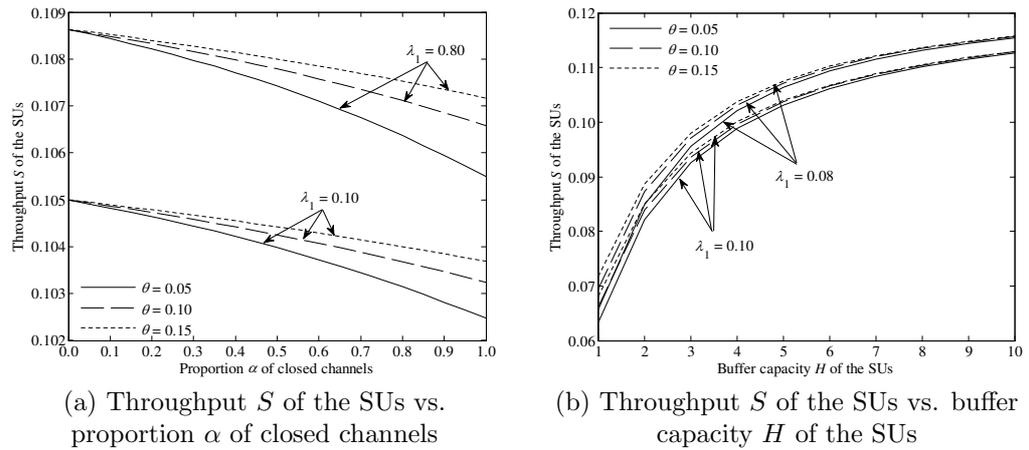


Figure 3: Change trend for the throughput  $S$  of the SUs.

throughput of the SUs will be.

From Fig. 3 (b), we see that, for the same Part Bonding Rate  $\theta$  and the same arrival rate  $\lambda_1$  of the PU packets, the throughput  $S$  of the SUs will increase as the buffer capacity  $H$  of the SUs increases. The intuitive reason is that the larger the buffer capacity of the SUs is, the more likely it is that an SU packet can join the system and is transmitted successfully, so the greater the throughput of the SU packets will be.

Figures 3 (a) and (b) both show that, for the same proportion  $\alpha$  of closed channels and the same arrival rate  $\lambda_1$  of the PU packets, or the same buffer capacity  $H$  of the SUs and the same arrival rate  $\lambda_1$  of the PU packets, the larger the Part Bonding Rate  $\theta$  is, and the greater the throughput  $S$  of the SUs will be. This is because as the Part Bonding Rate increases, the time length of a Part Bonding Period will decrease, so more packets will be transmitted with a high transmission rate. This will induce an increase in the throughput of the SUs.

On the other hand, Figs. 3 (a) and (b) also show that, for the same proportion  $\alpha$  of closed channels and the same Part Bonding Rate  $\theta$ , or the same buffer capacity  $H$  of the SUs and the same Part Bonding Rate  $\theta$ , the higher the arrival rate  $\lambda_1$  of the PU packets is, and the smaller the throughput  $S$  of the SU packets will be. This is because the higher the arrival rate of the PU packets is, the more likely it is that the channel is occupied by a PU packet, so the number of successfully transmitted SU packets will be reduced. This will lead to a decrease in the throughput of the SU packets.

We examine the change trend for the average latency  $E[T]$  of the SUs in Fig. 4.

In Fig. 4 (a), we find that for the same Part Bonding Rate  $\theta$  and the same arrival rate  $\lambda_1$  of the PU packets, the average latency  $E[T]$  of the SUs will increase as the proportion  $\alpha$  of closed channels increases. The reason is that the larger the proportion of closed channels is, the lower the transmission rate during the Part Bonding Period is, and the greater the average latency of the SUs will be.

In Fig. 4 (b), we conclude that for the same Part Bonding Rate  $\theta$  and the same arrival rate  $\lambda_1$  of the PU packets, the average latency  $E[T]$  of the SUs will also increase as the buffer capacity  $H$  of the SUs increases. The reason is that the larger the buffer capacity of the SUs is, the greater the number of SU packets in the system is, and the longer the average latency of the SU packets will be.

Figures 4 (a) and (b) both demonstrate that, for the same proportion  $\alpha$  of closed channels

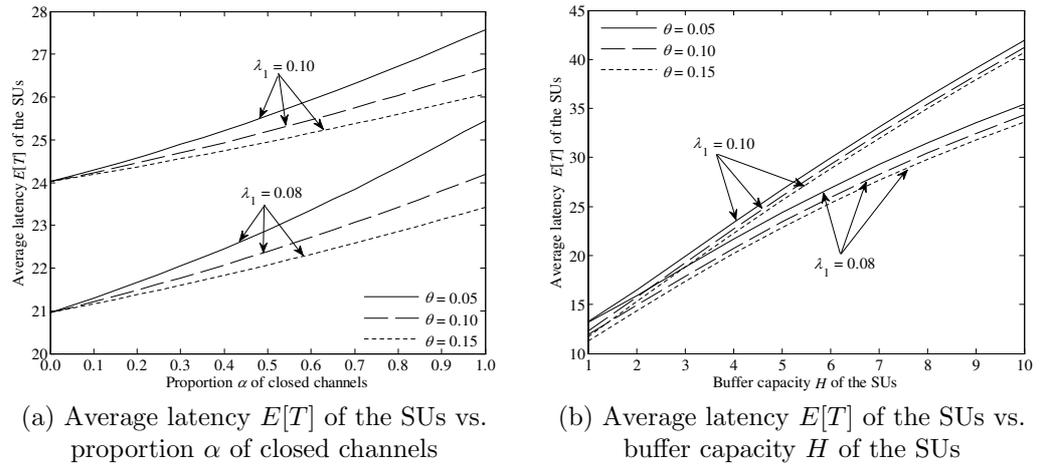


Figure 4: Change trend for the average latency  $E[T]$  of the SUs.

and the same arrival rate  $\lambda_1$  of the PU packets, or the same buffer capacity  $H$  of the SUs and the same arrival rate  $\lambda_1$  of the PU packets, the larger the Part Bonding Rate  $\theta$  is, and the shorter the average latency  $E[T]$  of the SUs will be. This is because as the Part Bonding Rate increases, the time length of a Part Bonding Period will decrease, so more packets will be transmitted during the Full Bonding Period. This will induce a decrease in the average latency of the SUs.

On the other hand, Figs. 4 (a) and (b) also show that, for the same proportion  $\alpha$  of closed channels and the same Part Bonding Rate  $\theta$ , or the same buffer capacity  $H$  of the SUs and the same Part Bonding Rate  $\theta$ , the higher the arrival rate  $\lambda_1$  of the PU packets is, the longer the average latency  $E[T]$  of the SU packets will be. This is because the higher the arrival rate of the PU packets is, the more likely it is that the channel is occupied by a PU packet, so the longer the time length for an SU packet waiting in the buffer is. This will lead to an increase in the average latency of the SU packets.

In Fig. 5, we show the change trend for the closed channel ratio  $\beta$ .

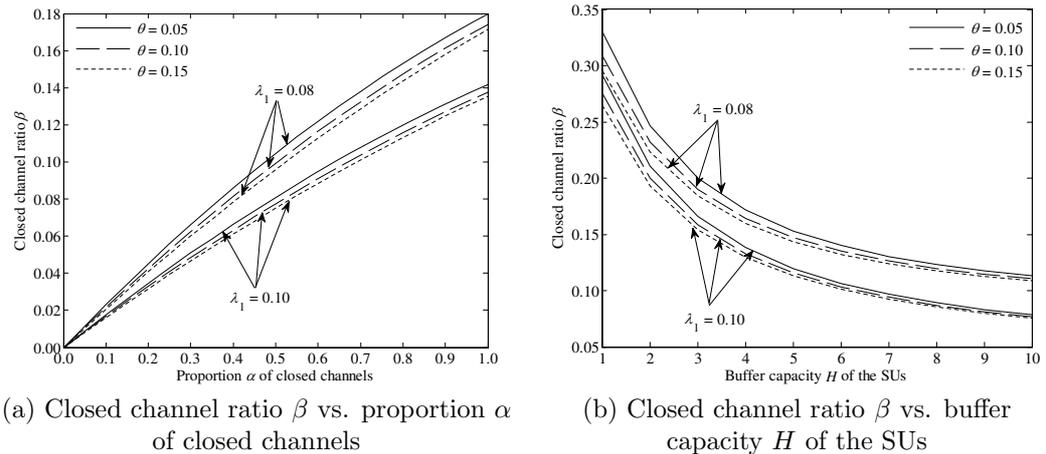


Figure 5: Change trend for the closed channel ratio  $\beta$ .

As illustrated in Fig. 5 (a), for the same Part Bonding Rate  $\theta$  and the same arrival rate  $\lambda_1$  of the PU packets, the closed channel ratio  $\beta$  will increase as the proportion  $\alpha$  of closed channels increases. The intuitive reason is that the larger the proportion of closed channels is, the greater number of closed channels there are, the higher the possibility is that a channel is closed during the Part Bonding Period, and therefore the closed channel ratio will be higher.

In Fig. 5 (b), for the same Part Bonding Rate  $\theta$  and the same arrival rate  $\lambda_1$  of the PU packets, the closed channel ratio  $\beta$  will decrease as the buffer capacity  $H$  of the SUs increases. The reason is that the larger the buffer capacity of the SUs is, the greater the number of SU packets that can join the system is, the less likely it is that the system is in the Part Bonding Period, and the lower the closed channel ratio will be.

Figures 5 (a) and (b) both show that, for the same proportion  $\alpha$  of closed channels and the same arrival rate  $\lambda_1$  of the PU packets, or the same buffer capacity  $H$  of the SUs and the same arrival rate  $\lambda_1$  of the PU packets, the larger the Part Bonding Rate  $\theta$  is, and the lower the closed channel ratio  $\beta$  will be. This is because as the Part Bonding Rate increases, the time length of a Part Bonding Period will decrease, the probability of the system being in a Part Bonding Period will decrease, and the closed channel ratio will be lower.

On the other hand, Figs. 5 (a) and (b) also show that, for the same proportion  $\alpha$  of closed channels and the same Part Bonding Rate  $\theta$ , or the same buffer capacity  $H$  of the SUs and the same Part Bonding Rate  $\theta$ , the higher the arrival rate  $\lambda_1$  of the PU packets is, and the lower the closed channel ratio  $\beta$  will be. This is because the higher the arrival rate of the PU packets is, the more likely it is that the channel is occupied by a PU packet, so the probability of the system being in a Part Bonding Period will decrease. This will lead to a decrease in the closed channel ratio.

Figures 2-5 clearly show that the system parameters such as the blocking ratio  $\delta$ , the throughput  $S$ , the average latency  $E[T]$  of the SUs and the closed channel ratio  $\beta$  are heavily dependent on the proportion  $\alpha$  of closed channels during the Part Bonding Period and the buffer capacity  $H$  of the SUs.

Especially, in Figs 2 (a)-5 (a), we conclude that, for the case of  $\alpha = 0$ , all of the channels will be active during the Part Bonding Period, i.e., there is no Part Bonding Period at all. In this case, the blocking ratio  $\delta$  and the average latency  $E[T]$  of the SUs are lower, and the throughput  $S$  of the SUs is greater, but the closed channel ratio  $\beta$  is zero. This means that there is no conservation of the network resources.

On the other hand, for the case of  $\alpha = 1$ , all of the channels will be closed during the Part Bonding Period. In this case, the closed channel ratio  $\beta$  will be significantly improved, but the blocking ratio  $\delta$  and the average latency  $E[T]$  of the SUs will increase, and the throughput  $S$  of the SUs will decrease. So we conclude that the results of the two extreme cases are both considerably separated from satisfactory. This indicates that the dynamic channel bonding strategy we have proposed is significant.

## **6** System Optimization

From the numerical results shown in Section 5, we draw the following conclusions.

From Figs. 2 (a)-5 (a), we find that as the proportion of closed channels increases, the closed channel ratio will increase. This is the result we hope to see in order to save network resources. On the other hand, the throughput of the SUs will decrease, and the blocking ratio and the average latency of the SUs will increase as the proportion of closed channels increases. This is what we do not want to see. Therefore, we conclude that there is a trade-off that needs to be made when assigning the proportion of closed channels during the Part

Bonding Period.

On the other hand, from Figs. 2 (b)-5 (b), we also see that as the buffer capacity of the SUs increases, the blocking ratio of the SUs will decrease and the throughput of the SUs will increase. These are good things obviously. On the other hand, the closed channel ratio will decrease and the average latency of the SUs will increase as the buffer capacity of the SUs increases. These results are counter productive from the point of view of saving network resources. Therefore, we can conclude that there is also a trade-off that needs to be made when assigning the buffer capacity of the SUs.

In order to provide the optimal proportion of closed channels and the optimal buffer capacity of the SUs, we construct a net benefit function  $B(X)$  given as follows:

$$B(X) = C_1S + C_2\beta - C_3\delta - C_4E[T] \tag{6.1}$$

where  $C_1$  and  $C_2$  are assumed to be the rewards per slot due to the throughput of the SUs and the conservation of networks resources, respectively;  $C_3$  and  $C_4$  are supposed to be the costs per slot due to the blocking ratio and the average latency of the SUs, respectively.

From Eq. (6.1), the optimal value  $X^*$  can be given as follows:

$$X^* = \arg \max\{B(X)\} \tag{6.2}$$

where “arg max” stands for the argument of the maximum [3]. That is to say, the set of points from which the given function  $B(X)$  attains its maximum value. When  $X$  is set to  $\alpha$ , we can obtain the optimal proportion  $\alpha^*$  of closed channels during the Part Bonding Period; when  $X$  is set to  $H$ , we can obtain the optimal buffer capacity  $H^*$  of the SUs.

**6.1 Optimization for Proportion of Closed Channels**

Because of the complexity of the net benefit function  $B(\alpha)$  in Eq. (6.1), we present an efficient iteration algorithm by using a steepest descent optimization method to give the optimal proportion  $\alpha^*$  of closed channels. The steepest descent optimization method is to resolve an unconstrained optimization problem, while the optimization problem in this paper has a constraint of  $\alpha \in [0, 1]$ .

In order to guarantee the approximated value of  $\alpha^*$  ranging from 0 to 1, we also employ an internal point penalty function below.

We first construct a penalty term as follows:

$$\gamma(\alpha) = \frac{1}{1 - \alpha} + \frac{1}{\alpha}.$$

Then we build an internal point penalty function  $F(\alpha)$  as follows:

$$F(\alpha) = -B(\alpha) + f\gamma(\alpha) \tag{6.3}$$

where  $f > 0$  is the penalty factor.

Combining both of the steepest descent optimization method and the internal point penalty function, we can give an iteration algorithm to estimate the optimal proportion  $\alpha^*$  of closed channels during the Part Bonding Period shown in Table 1.

In Table 1,  $\phi$  is the step factor which is set according to the iteration accuracy of  $\alpha$  (for example,  $\phi = 0.01$ ),  $\alpha_0$  is the initial value of  $\alpha$ ,  $\alpha_m$  is the  $m$ th order approximation of  $\alpha$ ,  $D$  is the decline coefficient of the penalty factor (for example,  $D = 0.1$ ), and  $\epsilon$  is an arbitrary

Table 1: Iteration algorithm to obtain  $\alpha^*$ .

---

**Input:**  $\epsilon, f$  and  $\alpha_0$   
**Output:**  $\alpha^*$

---

**Begin**  
 $m = 1;$   
 $Y_0 = F(\alpha_0);$   
 $\alpha_m = \alpha_{m-1} - \phi \frac{\partial F(\alpha_{m-1})}{\partial \alpha_{m-1}};$   
 $Y_m = F(\alpha_m);$   
**while**  $|Y_m - Y_{m-1}| > \epsilon$  or  $|\alpha_m - \alpha_{m-1}| > \epsilon$  **do**  
 $m = m + 1;$   
 $\alpha_m = \alpha_{m-1} - \phi \frac{\partial F(\alpha_{m-1})}{\partial \alpha_{m-1}};$   
 $Y_m = F(\alpha_m);$   
**end while**  
 $\alpha_0 = \alpha_m;$   
**while**  $f\gamma(\alpha_0) > \epsilon$  **do**  
 $f = fD;$   
 $Y_0 = F(\alpha_0);$   
 $m = 1;$   
 $\alpha_m = \alpha_{m-1} - \phi \frac{\partial F(\alpha_{m-1})}{\partial \alpha_{m-1}};$   
 $Y_n = F(\alpha_m);$   
**while**  $|Y_m - Y_{m-1}| > \epsilon$  or  $|\alpha_m - \alpha_{m-1}| > \epsilon$  **do**  
 $m = m + 1;$   
 $\alpha_m = \alpha_{m-1} - \phi \frac{\partial F(\alpha_{m-1})}{\partial \alpha_{m-1}};$   
 $Y_m = F(\alpha_m);$   
**end while**  
 $\alpha_0 = \alpha_m;$   
**end while**  
 $\alpha^* = \alpha_0;$   
**End**

---

sufficiently small number (for example,  $\epsilon = 10^{-6}$ ). Moreover, the differential operation for  $F(\alpha)$  can be numerically approximated as follows:

$$\frac{\partial F(\alpha)}{\partial \alpha} \approx \frac{F(\alpha + \Delta) - F(\alpha)}{\Delta}$$

where  $\Delta$  is an arbitrary sufficiently small number (for example,  $\Delta = 10^{-6}$ ).

Employing the parameters used in Section 5 and setting  $C_1 = 200$ ,  $C_2 = 9$ ,  $C_3 = 0.5$ ,  $C_4 = 0.35$  in the iteration algorithm as an example, the optimal proportion  $\alpha^*$  of closed channels and the corresponding maximum value of  $B(\alpha^*)$  with the different arrival rates  $\lambda_1$  of the PU packets and the different Part Bonding Rates  $\theta$  can be given in Table 2.

In Table 2, the estimates of  $\alpha^*$  and  $B(\alpha^*)$  are accurate to four decimal places.

From Table 2, we observe that as the Part Bonding Rate  $\theta$  increases, the optimal proportion  $\alpha^*$  of closed channels will increase. This is because the greater the Part Bonding Rate is, the shorter the time length of a Part Bonding Period is. So, to guarantee a larger closed channel ratio, the optimal proportion of closed channels during the Part Bonding Period will be greater.

Table 2: Optimal proportion  $\alpha^*$  of closed channels during the Part Bonding Period.

$\lambda_1$	$\theta$	$\alpha^*$	$B(\alpha^*)$
0.06	0.05	0.2964	16.1995
0.06	0.10	0.5570	16.4078
0.06	0.15	0.8512	16.6661
0.08	0.05	0.2728	14.4700
0.08	0.10	0.5236	14.6109
0.08	0.15	0.8097	14.7961
0.10	0.05	0.2358	12.6314
0.10	0.10	0.4785	12.7236
0.10	0.15	0.7577	12.8535
0.12	0.05	0.1862	10.6952
0.12	0.10	0.4219	10.7526
0.12	0.15	0.6951	10.8411

## 6.2 Optimization for Buffer Capacity of the SUs

We note that the possible sizes for the buffer capacities are countable. We can obtain the exact solution for the optimal buffer capacity  $H^*$  of the SUs by comparing the net benefits with all the possible buffer capacities.

Employing the parameters used in Section 5 and setting  $C_1 = 200$ ,  $C_2 = 9$ ,  $C_3 = 0.5$ ,  $C_4 = 0.35$  as an example, we plot how the net benefit  $B(H)$  changes with respect to the buffer capacity  $H$  of the SUs for the different arrival rates  $\lambda_1$  of the PU packets and the different Part Bonding Rates  $\theta$  in Fig. 6.

From Fig. 6, we conclude that for the same Part Bonding Rate  $\theta$  and the same arrival rate  $\lambda_1$  of the PU packets in this example, as the buffer capacity  $H$  of the SUs increases, the change trend for the net benefit will exhibit two stages. During the first stage, the net benefit  $B(H)$  will increase as the buffer capacity  $H$  of the SUs increases. In this stage, the throughput  $S$  and the blocking ratio  $\delta$  of the SUs are the main factors impacting on the net benefit. As the buffer capacity  $H$  of the SUs increases, the throughput will increase and the blocking ratio will decrease, and this will lead to an increase in the net benefit. During the second stage, the net benefit  $B(H)$  will decrease as the buffer capacity  $H$  of the SUs increases. In this stage, the average latency  $E[T]$  of the SUs and the closed channel ratio  $\beta$  are the main factors impacting on the net benefit. As the buffer capacity  $H$  of the SUs increases, the average latency of the SUs will increase, and the closed channel ratio will decrease, so the net benefit will decrease.

Conclusively, the optimal buffer capacities  $H^*$  of the SUs and the maximum net benefits  $B(H^*)$  for the different Part Bonding Rates  $\theta$  and the different arrival rates  $\lambda_1$  of the PU packets are shown in Table 3.

From Table 3, we observe that as the Part Bonding Rate  $\theta$  increases, the optimal buffer capacity  $H^*$  of the SUs show a declining tendency. This is because the larger the Part Bonding Rate is, the shorter the time length of a Part Bonding Period is. So, to guarantee a greater closed channel ratio, the optimal buffer capacity of the SUs will be smaller.

## 7 Conclusions

In this paper, in order to conserve network resources and to guarantee the QoS for the secondary users (SUs), we proposed a novel dynamic channel bonding strategy in cognitive radio networks. The time period was divided into a Full Bonding Period and a Part Bonding

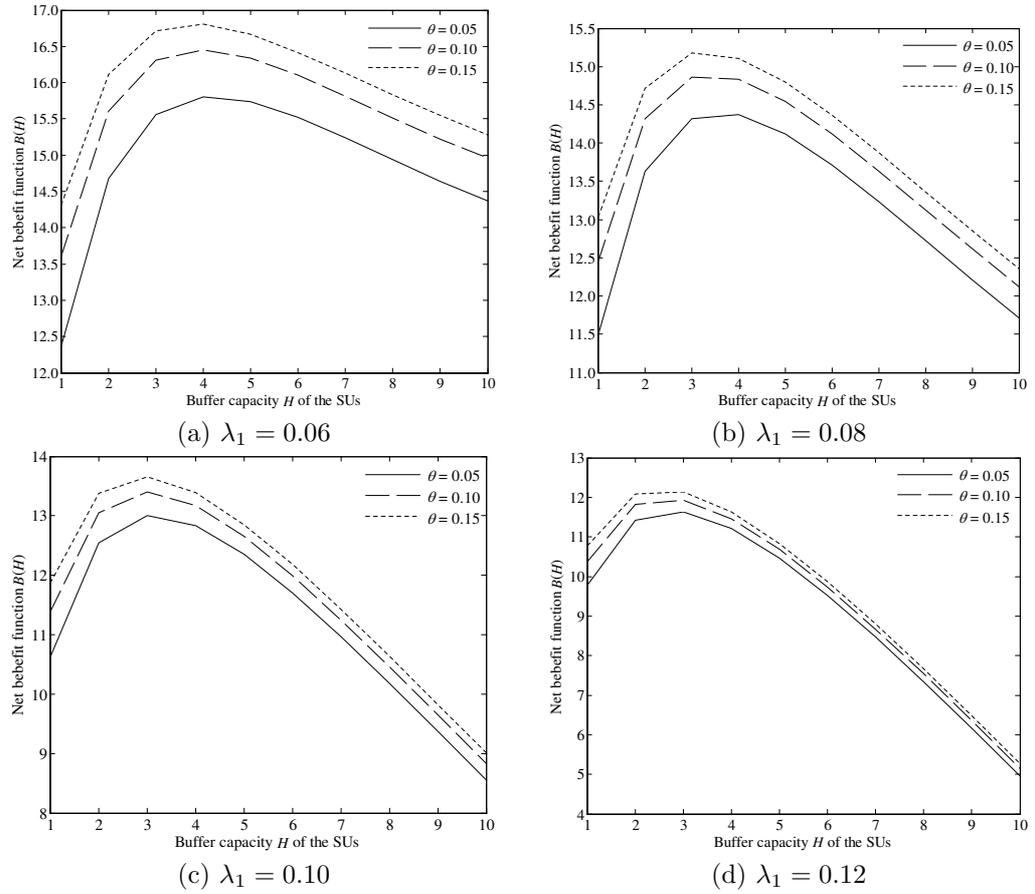


Figure 6: Change trend for the net benefit  $B(H)$  with the different buffer capacities  $H$  of the SUs.

Table 3: Optimal buffer capacity  $H^*$  of the SUs.

$\lambda_1$	$\theta$	$H^*$	$B(H^*)$
0.06	0.05	4	15.8060
0.06	0.10	4	16.4539
0.06	0.15	4	16.8057
0.08	0.05	4	14.3742
0.08	0.10	3	14.8607
0.08	0.15	3	15.1771
0.10	0.05	3	13.0030
0.10	0.10	3	13.4019
0.10	0.15	3	13.6522
0.12	0.05	3	11.6337
0.12	0.10	3	11.9353
0.12	0.15	3	12.0823

Period. During the Full Bonding Period, all the channels were bonded to transmit packets; during the Part Bonding Period, only a part of the channels were bonded to transmit packets. Based on the working principle of the dynamic channel bonding strategy and the priority of the primary users (PUs) in cognitive radio networks, a discrete-time pre-emptive priority queueing model with a working vacation was built. The steady-state distribution of the system model was analyzed with a three-dimensional Markov chain. The formulas for the blocking ratio, the throughput, the average latency of the SUs, and the closed channel ratio were derived to evaluate the system performance. The numerical results show that the dynamic channel bonding strategy proposed in this paper can effectively improve the system performance. Finally, the trade-off between different performance parameters was investigated. Accordingly, a net benefit function was constructed to give the optimal design for the proportion of closed channels during the Part Bonding Period and the buffer capacity of the SUs, respectively.

The research in this paper presented a theoretical basis for the channel allocation scheme in cognitive radio networks with a channel bonding strategy. This research work will have potential applications in the improvement of the network resource saving in cognitive radio networks.

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