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A METHOD OF TRACK SEEKING CONTROL FOR DUAL STAGE SYSTEM*†

Guang-Ren Duan, Shi Li and Yan-Ming Fu

Abstract: This paper proposes a strategy for the design of a track-seeking control system for a dual-stage hard disk drive (HDD) system. A proximate time optimal servomechanism (PTOS) has been widely used in primary actuator because of its simplicity and good performance. However, this strategy is unable to obtain the minimal settling time when the seeking length is long. We design the primary actuator controller by applying damping scheduling proximate time optimal servomechanism (DSPTOS) method which has faster settling time than the original PTOS does. Then, a parametric Lyapunov equation approach of low gain feedback to secondary actuator is designed to avoid input saturation. The simulation results show good short-span-seeking performance, as well as long-span-seeking performance with fast settling time.

Key words: *dual-stage HDD, DSPTOS, track seeking*

Mathematics Subject Classification: *58E25, 70Q05, 93B51*

1 Introduction

Improved performance of the head-positioning is required for continuous increase in the track density and storage capacity of HDD. Dual-stage HDD is regarded as a future alternative to single-stage VCM-based servo system, such as the "FUMA"-actuator in [11]. The so-called dual-stage HDD servo system consists of a voice coil motor (VCM) as the primary actuator and a piezo-electric transducer (PZT) as the secondary actuator. The primary one is of long range but with poor accuracy and slow response. The secondary one delivers much higher precision and faster response but has a constrained range. Therefore, the dual-stage HDD system is expected to provide large displacement, high precision and fast response. The system needs to complete two tasks: accurately maintain the head position along the center of the track (track following) and provide fast movement for the head from one track to another (track seeking). The main obstacles are that saturation of both secondary actuator and the primary actuator should be taken into consideration during design and coupling effect exists between the two actuators.

The methods to solve track-seeking problem can be largely classified into two groups: those based on classical single-input-single-output (SISO) design methodologies, and those

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based on multi-input-multi-output (MIMO) design methodologies. Most methodologies of the first type perform some form of decoupling control, followed by sequential multiple SISO compensator loop shaping design, so as to shape the overall closed-loop sensitivity transfer function frequency response. Examples of these methods include the master-slave method in [13], the PQ method in [12] and decoupled sensitivity design approach in [6]. On the other hand, as the DSA has dual input, it is natural to use modern state-based optimal and robust MIMO methodologies, such as LQG/LQR in [15], H_∞ -optimization in [7] and μ-synthesis in [16].A PTOS [10] has been widely used in track-seeking control for HDD. To obtain a better performance, DSPTOS [18], modified damping scheduling proximate time optimal servomechanism (MDSPTOS) [17] and 2-DOF PTOS [1] are proposed.

Recently, research on the following two aspects has attracted a lot of people. On one hand, the displacement range of both secondary actuator and primary actuator are limited and the input signals for the actuators are constrained to prevent damage. Methods dealing with the actuator constraints are necessary (see [5], [14], [8]). On the other hand, many results did not allow long-span-seeking. The methods for both short-span-seeking and long-span-seeking are to be researched. Related papers are [2], [3].

The basis for this approach is a well known decoupled dual-stage servo controller structure. In contrast to other work, this paper applied a DSPTOS method which has faster settling time than the PTOS does and maintains the simplicity of the PTOS for the primary actuator. A parametric Lyapunov equation approach of low gain feedback to secondary actuator was designed to avoid input saturation. Such an approach possesses the advantages of both the eigenstructure assignment approach and the ARE-based approach in [4].

The remainder of this paper is organized as follows. In section 2, we present the description of dual stage control systems. In section 3, we state the procedure for the design of the proposed control systems. A design example is shown in section 4. Finally, conclusions will be discussed in section 5.

2 Control System Description

The dual-stage HDD is generally treated as a dual-input-single-output (DISO) system. The structure of the control system in this paper is based on the decoupled master-slave method. We show the simplified block diagram of the proposed control system in Fig. 1. The output y is a combination of the primary actuator output y_1 and the secondary actuator output y_2. S_1 is a saturation block. y_r and e represent reference input signal and the controller error between y_r and y, respectively. C_{tr1} and C_{tr2} represent the control function for P_1 and P_2. P_1 and P_2 represent the description of the state space of the primary actuator and the secondary actuator, respectively. The representation of state space is as follows:

$$\begin{array}{l} \left\{ \begin{array}{ll} \Sigma_1: & \dot{x}_1 = A_1 x_1 + B_1 sat\left(u_1\right) \\ \Sigma_2: & \dot{x}_2 = A_2 x_2 + B_2 sat\left(u_2\right) \end{array} \right. \\ y = y_1 + y_2 = C_1 x_1 + C_2 x_2 \end{array} \tag{2.1}$$

with the initial conditions:

$$x_1\left(0\right) = 0, \quad x_2\left(0\right) = 0 \tag{2.2}$$

where

$$x_1 = \begin{bmatrix} y_1 & \dot{y}_1 \end{bmatrix}^T, \quad x_2 = \begin{bmatrix} y_2 & \dot{y}_2 \end{bmatrix}^T,$$
$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ b_1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0 & 1 \\ a_1 & a_2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ b_2 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

and the function $sat\,(u)$ is defined as

$$sat\,(u) = sgn\,(u) \min \{\bar{u}, |u|\}$$

where $\bar{u}$ is the saturation level of control input u. The definition of a_1, a_2, b_1 and b_2 are given in [9].

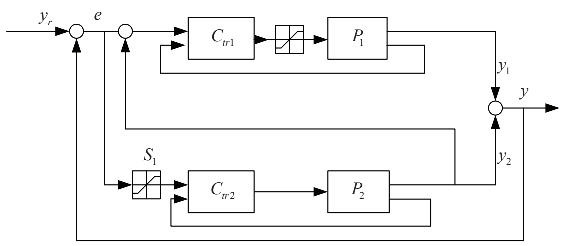

Figure 1: Block diagram of the dual-stage HDD

3 Control System Design

The procedure can be summarized in two steps for control system design. First, design the primary actuator controller by applying DSPTOS method. Then, a parametric Lyapunov equation approach to low gain feedback controller is designed for secondary actuator.

A. Design of controller for primary-actuator

The role of the primary actuator is to provide large travel range beyond that of the secondary actuator rapidly. The PTOS has been widely used as a primary actuator controller for dual-stage HDDs since Workman proposed it in [10], such as [5], [8], [2], [3], [9]. However, it has a constant velocity gain and seeking performance is restricted when the seeking length is large. To improve long stroke seeks and to further reduce access time, we use DSPTOS which was first proposed in [18].

First we review the algorithms of PTOS, and then we will talk about the details of DSPTOS:

$$u_1 = sat\,(k_2\,(f\,(e_1) - \dot{y}_1)) \tag{3.1}$$

where

$$f\,(e_1) = \begin{cases} \frac{k_1}{k_2} e_1 & for\,|e_1| \leq y_l \\ sgn\,(e_1) \left(\sqrt{2\bar{u}_1 b_1 \alpha\,|e_1|} - \frac{\bar{u}_1}{k_2} \right) & for\,|e_1| > y_l \end{cases}$$

$$e_1 = y_r - y_1$$

and $sat(\cdot)$ is with the saturation level of $\bar{u}_1$, α $(0 < \alpha \leq 1)$ is the acceleration discount factor, k_1 and k_2 are constant gains, and y_l represents the size of the linear region. We have the following constraints:

$$\alpha = \frac{2k_1}{b_1 k_2^2} \tag{3.2}$$

$$y_l = \frac{\bar{u}_1}{k_1} \tag{3.3}$$

which ensure that $f(e_1)$ and $f'(e_1)$ are continuous, then the control input remains continuous as well.

Here we define

$$C(e_1) = \begin{cases} k_1 & |e_1| \leq y_l \\ 2\sqrt{\frac{k_1 \bar{u}_1}{|e_1|}} - \frac{\bar{u}_1}{|e_1|} & |e_1| > y_l \end{cases}, \tag{3.4}$$

then we can get $u_1(t) = C(e_1) e_1 + k_2 \dot{e}_1$ if we assume $u_1 \leq \bar{u}_1$. As we know, y_r is constant and $\dot{y}_r = 0$, $\ddot{y}_r = 0$ when $t > 0$. The primary closed loop error dynamics can then be expressed as

$$\ddot{e}_1 + b_1 k_2 \dot{e}_1 + k_1 C(e_1) e_1 = 0. \tag{3.5}$$

We can get the closed-loop damping coefficient from (3.5) as

$$\varsigma_p = \sqrt{\frac{k_1}{2\alpha C(e_1)}}. \tag{3.6}$$

Otherwise, in DSPTOS, the condition of α is modified to make the servo system have a predetermined constant closed-loop damping coefficient ς_p around the target reference. From (3.6) and (3.2), we can get

$$\alpha = \max\left\{\frac{k_1}{2\varsigma_p^2 C(e_1)}, 1\right\}, \tag{3.7}$$

$$k_2 = \sqrt{\frac{2k_1}{\alpha b_1}}. \tag{3.8}$$

The DSPTOS has a large deceleration when the error is large. As the error becomes small, the closed-loop damping coefficient is maintained. The large deceleration helps with the rapid rising in the early stage, and then the large damping restricts the overshoot in the followed stage. These features are the reasons why DSPTOS makes settling fast, especially in long strokes larger than y_l.

B. Design of controller for secondary-actuator

The goal of the controller design for the secondary actuator Σ_2 in (2.1) is to provide a larger damping ratio and a higher precision. The main obstacle for secondary controller design is that the displacement range of secondary actuator is limited and the input signal for the actuator is constrained to prevent damage. That is the reason why low gain feedback is applied in this part. One of the key features of low gain feedback is that, for a given stabilizable linear system with all its open loop poles in the closed left-half plane and with its initial state in an arbitrarily large bounded set, the peak magnitude of the low gain feedback control goes to zero as the low gain parameter approaches zero. That is to say, for such a system, actuator saturation can be avoided by decreasing the low gain parameter as long as the initial state lies in a bounded set.

To explain the algorithms clearly, we introduces the following three lemmas which are all cited from [4]:

Lemma 3.1. *Consider the linear system*

$$\begin{cases} \dot{x}_2 = A_2 x_2 + B_2 u_2, x_2(0) = 0 \\ y_2 = C_2 x_2 \end{cases} \tag{3.9}$$

we assume that (A_2, B_2) *is stabilizable,* (A_2, C_2) *is detectable. For a positive scalar* γ*, define a cost function*

$$J(u_2) = \int_0^\infty e^{\gamma t} u_2^T(t) R u_2(t) \, dt.$$

Then, $J(u_2)$ *is minimized with*

$$u_2^*(t) = -R^{-1} B_2^T P x_2(t)$$

where P *is the unique positive-definite solution of the following ARE*

$$A_2^T P + P A_2 - P B_2 R^{-1} B_2^T P = -\gamma P. \tag{3.10}$$

Furthermore, the closed-loop system (3.9) is globally exponentially stable with

$$\lim_{t \to \infty} e^{\frac{\gamma}{2} t} x_2(t) = 0.$$

Lemma 3.2. *Let* (A_2, B_2) *be controllable and let* $\gamma > 0$ *be such that*

$$\gamma > -2 \min\{Re(\lambda(A_2))\} \tag{3.11}$$

where $Re(\lambda(A_2))$ *denotes the set of the real parts of the eigenvalues of* A_2*. Then, the ARE (3.10) has a unique positive-definite solution given by* $P(\gamma) = W^{-1}(\gamma)$*, where* $W(\gamma)$ *is the unique positive-definite solution to the following Lyapunov matrix equation*

$$W \left(A_2 + \frac{\gamma}{2} I \right)^T + \left(A_2 + \frac{\gamma}{2} I \right) W = B_2 R^{-1} B_2^T, \tag{3.12}$$

i.e. $W(\gamma)$ *is analytically given by*

$$W(\gamma) = \int_0^\infty e^{-\left(A_2 + \frac{\gamma}{2} I\right) t} B_2 R^{-1} B_2^T e^{-\left(A_2 + \frac{\gamma}{2} I\right)^T t} dt. \tag{3.13}$$

As is well known that such a system can be semi-globally stabilized if and only if (A_2, B_2) is stabilizable, (A_2, C_2) is detectable, and all eigenvalues of A_2 are in the closed left-half plane. Clearly, (A_{2-}, B_{2-}) where A_{2-} contains all eigenvalues of A_2 that have negative real parts does not affect the stabilizability property of the system. Without loss of generality, we will assume that (A_2, B_2) is controllable with all the eigenvalues of A_2 on the imaginary axis.

Lemma 3.3. *Let* (A_2, B_2) *be controllable and all eigenvalues of* A_2 *be on the imaginary axis. Then, the family of state feedback laws*

$$u_2 = -R^{-1} B_2^T P(\gamma) x_2, \gamma > 0$$

semi-globally stabilizes Σ_2 *with guaranteed convergence rate* $e^{-\frac{\gamma}{2} t}$*, where* $P(\gamma)$ *is the unique positive-definite solution to the ARE (3.10). That is, for any given arbitrarily large bounded set* $\chi \subset R^n$*, there exists a* γ^* *such that for any* $\gamma \in (0, \gamma^*]$*, the closed-loop system is asymptotically stable with* χ *contained in the domain of attraction. Furthermore, the convergence to the origin is no slower than* $e^{-\frac{\gamma}{2} t}$*.*

To apply the low gain feedback controller, we choose a proper γ and design a linear feedback control law

$$u_2(t) = -R^{-1}B_2^T P(\gamma)\, x_2(t) \tag{3.14}$$

which enables the secondary actuator control system to be semi-globally stable and the saturation of u_2 to be absent. Then the corresponding closed-loop system is described as $C_2\left(sI - A_2 + B_2R^{-1}B_2^T P(\gamma)\right)^{-1} B_2$.

C. Algorithm description

As we can see, the role of the primary actuator is to provide large travel range rapidly. DSPTOS method is applied because of its two features. First, the maximum acceleration helps the rapid rising when the tracking error is large; second, the large damping around the target restricts the overshoot. The main obstacle for secondary controller design is to avoid input saturation. That is the reason why parametric Lyapunov low gain feedback method is applied. The combination of these two methods will provide fast settling time and high precision for both long span and short span.

The algorithm is summarized as follows:

1. select proper k_1 and ς_p;
2. calculate $C(e_1)$, α and k_2 according to (3.4), (3.7) and (3.8);
3. u_1 can be obtained by (3.1);
4. choose γ in accordance with (3.11) and a proper R;
5. calculate $W(\gamma)$ and $W^{-1}(\gamma)$ according to (3.13);
6. u_2 can be obtained by (3.14).

4 Design Example

To verify the effectiveness of the proposed algorithm, this section presents a design example using the procedure described in section 3. This paper is concerned with the practical dual stage HDD system which was initially proposed in [9].

The dual-stage HDD model parameters are given by

$$b_1 = 1.7 * 10^8, a_1 = -10^9, a_2 = -3.1 * 10^4, b_2 = 4.3 * 10^8, \overline{u}_1 = 3, \overline{u}_2 = 1.25.$$

For the primary actuator, we can see from section 3 that there are only two parameters to be tuned, k_1 and ς_p. Here we select $k_1 = 0.75$ and $\varsigma_p = 0.85$, then we can obtain the other parameters. For the secondary control design, as $Re(\lambda(A_2)) = -15500$, we choose $\gamma = 31100$, $R = I$ and then we can get $W(\gamma)$ by solving (3.13), $P(\gamma)$ is obtained by $P(\gamma) = W^{-1}(\gamma)$.

The measured time responses and the inputs of both actuators for the position y_1, y_2, and y for $2\mu m$, $20\mu m$ and $200\mu m$ track seek are shown in Fig. 2- 4. The tests of these seeking lengths are conducted by applying the methods proposed in this paper referred to as "proposed" and by using the PTOS algorithms for primary controller design and the low gain feedback method for secondary controller design referred to as "conventional". The results of the dual-stage servo system will also be compared with those of the servo system with a single-stage primary actuator which are done on the same primary actuator by keeping the secondary inactive throughout the whole implementation process.

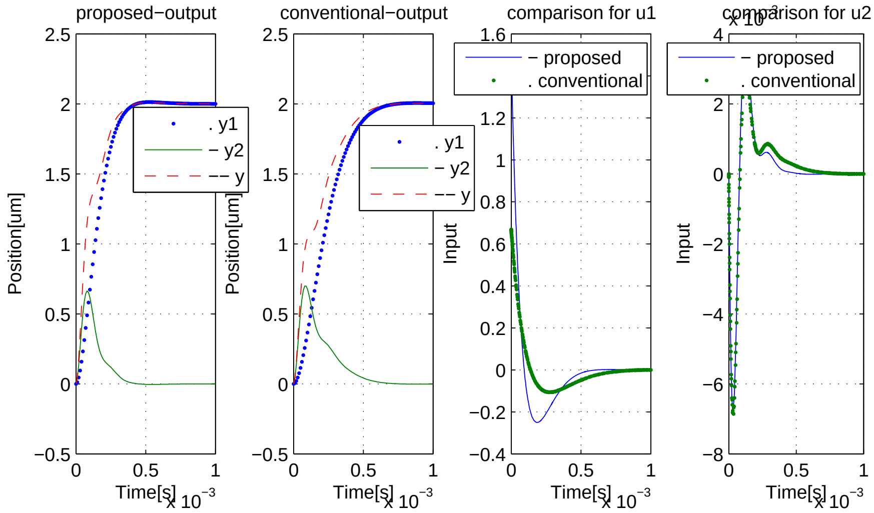

Figure 2: Seeking time and inputs for yr=2: “proposed” vs “conventional”

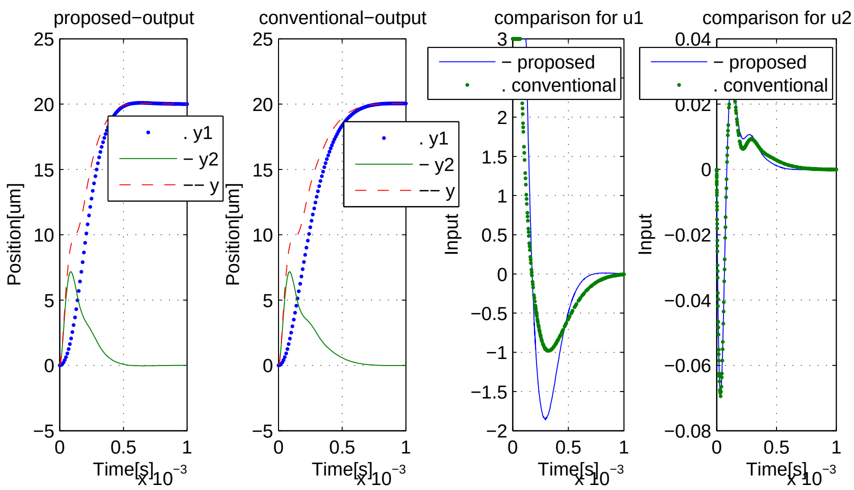

Figure 3: Seeking time and inputs for yr=20: “proposed” vs “conventional”

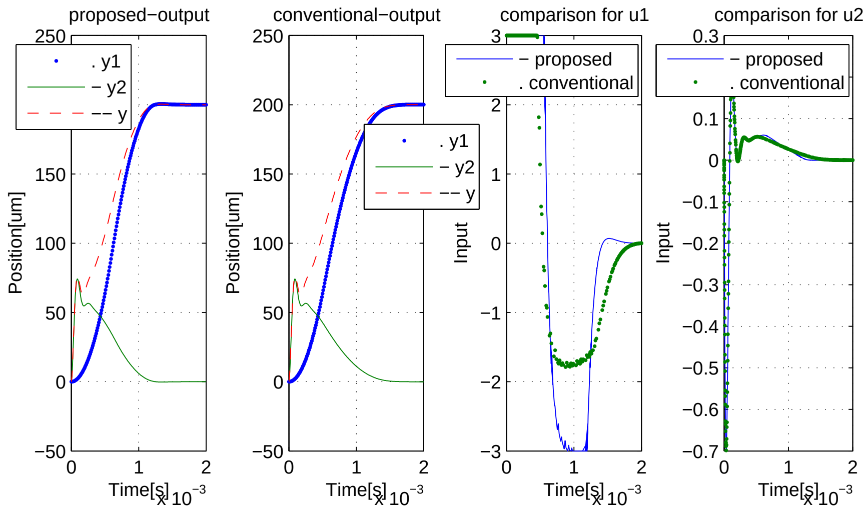

Figure 4: Seeking time and inputs for yr=200: "proposed" vs "conventional"

Table 1: Experiment result summary

Seeking Length (um)	Seeking Time (ms)		
	DSPTOS Dual	DSPTOS Single	PTOS Dual
2	0.39	0.40	0.65
20	0.48	0.50	0.66
200	1.15	1.18	1.41

The seeking times are summarized in Table. 1 for easy comparison. The overshoot is 1%. It is shown that the proposed control can reduce the seeking time by more than 2.5% compared with the single-stage control. We can also see that the seeking time under the proposed control is significantly reduced by 37% compared with the "conventional" control for medium- and short-span seeking. When the seeking length is large, the proposed control scheme could reduce the seeking time by 22%.

5 Conclusion

This paper has proposed a method with decoupling master-slave structure for track-seeking controllers design for dual-stage servo systems. Distinct from the original control which uses the PTOS method, the primary actuator control loop is designed to further reduce the settling time by applying DSPTOS method. Then, a parametric Lyapunov equation approach to low gain feedback controller is designed to the secondary actuator. The simulation results show good track-seeking performances. Compared with the "conventional" control method and the single-stage control method, the simulation results demonstrate that the proposed control strategy can further reduce the settling time and yield better performances.

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Guang-Ren Duan
Center for Control Theory and Guidance Technology, P.O. BOX 416
Harbin Institute of Technology, Harbin 150001, P.R. China
E-mail address: g.r.duan@hit.edu.cn

Shi Li
Center for Control Theory and Guidance Technology, P.O. BOX 416
Harbin Institute of Technology, Harbin 150001, P.R. China
E-mail address: lishi0530@163.com

Yan-Ming Fu
Center for Control Theory and Guidance Technology, P.O. BOX 416
Harbin Institute of Technology, Harbin 150001, P.R. China
E-mail address: fuyanming@hit.edu.cn