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OPTIMAL CONTROL FOR FINAL APPROACH OF RENDEZVOUS WITH NON-COOPERATIVE TARGET

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Abstract: This paper considers the final approach of rendezvous problem for servicing satellite to capture a tumbling satellite with external forces/torques as control inputs. The main purpose is to design an optimal control law to ensure a servicing satellite with the least fuel consumption. It is formulated as a constrained optimal control problem, where specific requirements of this final approach problem are all incorporated in the problem formulation. Then, by utilizing the specific features of the problem, this final approach of rendezvous problem is transformed into an equivalent standard optimal control problem subject to continuous state inequality constraints. A computational method is developed, based on the control parametrization in conjunction with a time scaling transform and the constraint transcription method, to design an optimal controller for this new constrained optimal control problem. Numerical results are presented for illustration.

Key words: *final approach, non-cooperative target, optimal control*

Mathematics Subject Classification: *49N90, 90C90, 93C95*

1 Introduction

Satellite on-orbit autonomous servicing is a challenging problem. It has attracted significant interests from many scientists and engineers in the past and current. To date there have been on-orbit demonstrations such as Robot Technology Experiment (ROTEX) [5], a German experiment, is the first autonomous space robotic system flown by NASA and Engineering Test Satellite VII (ETS-VII) [17] from JAXA demonstrated the space manipulator to capture a cooperative satellite whose attitude is stabilized during the demonstration via tele-operation from the ground control station. NASA did an autonomous rendezvous mission through the DART mission, where the mission failed due to more than expected fuel usage during rendezvous maneuvering [2]. DARPA is currently developing a more advanced technology demonstration mission Orbital Express Program [24, 29, 28] and it has completed first autonomous free flight and capture.

In order to perform on-orbit autonomous servicing, the chaser should firstly rendezvous and capture the satellite to be serviced in orbit. In a general satellite capture problem, we suppose that there is a target satellite and a chaser satellite flying in space. The target satellite (target) is non-cooperative satellite which moves with spinning or tumbling motions in an orbit (see Figure 1), while the task of the chaser satellite (chaser) is to rendezvous with the target in space in a desired way and finally capture it. In Fig. 1, the trajectory 1

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is safe and feasible, and the trajectory 2 shows the chaser will knock into target, hence it is infeasible.

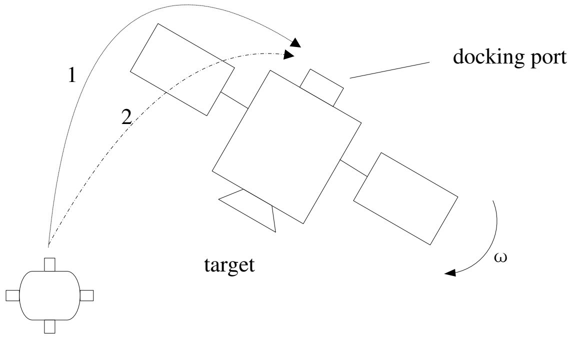

Figure 1: Rendezvous with non-cooperative target

Most of the current and past on-orbit servicing missions focus only on the capture of a cooperative satellite which is supposed to move smoothly in its orbit without rapid attitude changing. In reality, a malfunctioning satellite may spin or tumble in orbit. Such a satellite is considered as a non-cooperative satellite. Capture of a non-cooperative satellite is a tremendous challenge. Very few research work on the problem of capturing a tumbling satellite have been done. Most of the proposed methods require a manipulator onboard the chaser satellite (e.g. [14, 30]). Even with a very capable manipulator, the chaser still has to align with the tumbling satellite before any subsequent robotic operations can proceed. Sakawa studied the problem of controlling a single freely flying object to fly from one position and orientation to another in an optimal manner [21]. Matsumoto, et al. studied fly-by and optimal orbits for maneuvering to a rotating satellite [15]. Nakasuka and Fujiwara proposed a method for matching angular velocities between the chaser and target by changing the target's moments of inertia [18]. Fitz-Coy and Liu proposed a two phase navigation solution for rendezvous with a tumbling satellite in 2D space [3]. Artificial intelligence method was applied to autonomous rendezvous and docking [1, 9, 12, 19].

This paper focuses on the final approach of rendezvous problem for servicing satellite to capture a tumbling satellite with external forces/torques as control inputs. The main purpose is to design the optimal control law to ensure the final approach problem with the least possible fuel consumption. The final approach of rendezvous problem is formulated as a constrained optimal control problem, where specific requirements of the problem are all incorporated in problem formulation. Then, by taking into consideration of the specific features of the problem, this optimal final approach problem is transformed into an equivalent standard optimal control problem involving terminal state constraint. There are some methods available in the literature which can be used to solve this constrained optimal control problem. Example of these methods are [4, 16, 20, 25] and the references cited therein. Relevant software packages are $MISER3.2$ [6], $SCOS$ [7, 8, 10], $RIOTS\_95$ [22, 23]. In this paper, the constraint transcription method [25] is used to transform the constraints into canonical form. In this way, we obtain a sequence of optimal control problems with canonical constraints. We then develop an efficient computational method based on the control parametrization technique [26, 27] in conjunction with a time scaling transform [11]. This computational method can make use of the optimal control software package, MISER 3.2.

2 Problem Formulation

The optimal control law was designed based on the relative motion equation which was described in the target's body fixed frame. Assuming that the target's motion (position, orientation, and velocities) in space are known and except for the control thrusts of the chaser, no other external forces are considered.

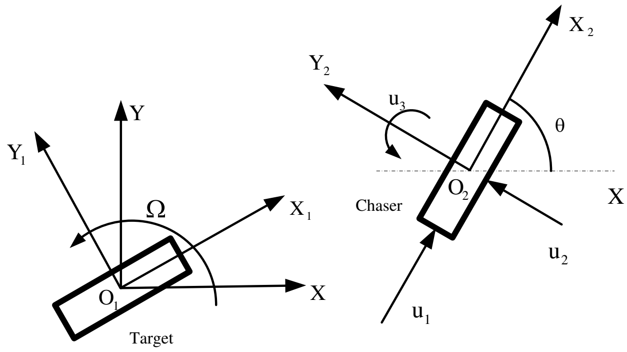

Figure 2: Chaser and target's motion in a plane

We consider the planar motion of the final approach as shown in Fig.2, the mass center of the target O_1 is assumed to be moving along a straight line at a constant speed. A translating reference frame XY is fixed to point O_1. Consider also a body-fixed frame X_1Y_1, attached to the target also at O_1. The two frames are assumed to be initially coincident. The target with its body fixed frame X_1Y_1 is rotating at a constant angular velocity Ω about the axis through O_1 perpendicular to the plane. A body fixed frame X_2Y_2 is attached to the chaser at its mass center O_2, whose coordinates in the XY frame are (x, y). The orientation of the chaser is denoted by θ, which is defined as the angle between the X_2 and X axes. There are two external forces u_1 and u_2, respectively in X_2 and Y_2 directions, and one external torque u_3, in the direction perpendicular to the plane, working as control inputs to the chaser. The motion of the chaser with respect to the XY frame can then be described as

$$\begin{cases} \dot{x} = v_x \\ \dot{y} = v_y \\ \dot{\theta} = \omega \\ \dot{v}_x = (u_1 \cos\theta - u_2 \sin\theta)/m \\ \dot{v}_y = (u_1 \sin\theta + u_2 \cos\theta)/m \\ \dot{\omega} = u_3/I_z \end{cases} \tag{2.1}$$

where m is the mass of the chaser, I_z is the polar moment of inertia of the chaser about the point O_2, (x, y) are the coordinates of O_2 in the XY frame, dot means time derivative and v_x , v_y and ω represent the translational and rotating velocities of the chaser observed in the XY frame.

The position vector with respect to the XY frame, $[x\ y]^T$, can be expressed in the X_1Y_1 frame as $[x_r\ y_r]^T$ by using the orthogonal transformation $[x_r\ y_r]^T = \mathbf{A}\,[x\ y]^T$ where

$$\mathbf{A} = \begin{bmatrix} \cos\Omega t & \sin\Omega t \\ -\sin\Omega t & \cos\Omega t \end{bmatrix}$$

in which t represents time. System (2.1) can then be rewritten as the relative motion equation in the target's body-fixed frame X_1Y_1

$$\begin{cases} \dot{x}_r = v_{xr} \\ \dot{y}_r = v_{yr} \\ \dot{\theta}_r = \omega_r \\ \dot{v}_{xr} = \Omega^2 x_r + 2\Omega v_{yr} + u_1 \cos\theta_r - u_2 \sin\theta_r \\ \dot{v}_{yr} = \Omega^2 y_r - 2\Omega v_{xr} + u_1 \sin\theta_r + u_2 \cos\theta_r \\ \dot{\omega}_r = u_3 \end{cases} \tag{2.2}$$

with initial and terminal conditions

$$\begin{array}{lll} x_r(0) = x_{r0}, & y_r(0) = y_{r0}, & \theta_r(0) = \theta, \\ v_{xr}(0) = v_{xr0}, & v_{yr}(0) = v_{yr0}, & \omega_r(0) = \omega_{r0}. \end{array} \tag{2.3}$$

$$\begin{array}{lll} x_r(t_f) = x_{rf}, & y_r(t_f) = y_{rf}, & \theta_r(t_f) = \theta_f, \\ v_{xr}(t_f) = 0, & v_{yr}(t_f) = 0, & \omega_r(t_f) = 0. \end{array} \tag{2.4}$$

where subscript r indicates the relative motion of the chaser with respect to the target, observed from the target's body-fixed frame X_1Y_1. For the final approach of the rendezvous problem, here, $t_f > 0$ is a given rendezvous time, $v_{xr}(t_f)$, $v_{yr}(t_f)$ and $\omega_r(t_f)$, which are the relative velocities of the chaser with respect to the target, must be equal to zero. Note that $\theta_r = \theta - \Omega t$ and $\omega_r = \omega - \Omega$ and the normalized control input u_1, u_2 and u_3, satisfying $u_{i\min} \le u_i(t) \le u_{i\max}$, $i = 1, 2, 3$, is defined as

$$u_1 = \hat{u}_1/m, \quad u_2 = \hat{u}_2/m, \quad u_3 = \hat{u}_3/I_z$$

where $\hat{u}_1$, $\hat{u}_2$ and $\hat{u}_3$ are physical control inputs.

In practice, the propeller only produces continuous forces. Thus, u_1, u_2 and u_3 are to be generated by their respective virtual input signals, which are

$$\dot{u}_1(t) = w_1(t), \quad \dot{u}_2(t) = w_2(t), \quad \dot{u}_3(t) = w_3(t) \tag{2.5}$$

where w_1, w_2 and w_3 are the respective rates of change of u_1, u_2 and u_3. The initial conditions for (2.5) are

$$u_1(0) = \zeta_1, \quad u_2(0) = \zeta_2, \quad u_3(0) = \zeta_3 \tag{2.6}$$

where ζ_1, ζ_2 and ζ_3 are parameters to be determined.

Different optimal controls can be designed for different optimality criteria, such as time optimal, fuel-consumption optimal, etc. Here, we discuss the least possible fuel consumption optimal control problem with specified initial and final conditions in a fixed time interval. For System (2.2), in time interval $[0,\ t_f]$, the terminal constraints specified in (2.4) can be appended into the cost function, the optimal control law should minimize the cost function by using the penalty function idea as shown below

$$\begin{aligned} J = {} & k_1 \left(x_r(t_f) - x_{rf}\right)^2 + k_2 \left(y_r(t_f) - y_{rf}\right)^2 \\ & + k_3 \left(\theta_r(t_f) - \theta_f\right)^2 + k_4 v_{xr}^2(t_f) + k_5 v_{yr}^2(t_f) \\ & + k_6 \omega_r^2(t_f) + \int_0^{t_f} \sqrt{u_1^2(t) + u_2^2(t) + u_3^2(t)}dt \end{aligned} \tag{2.7}$$

where k_1, k_2, k_3, k_4, k_5 and k_6 are penalty parameters. We can adjust these parameters to achieve the required accuracy of satisfying the terminal constraints. The second term represents the fuel consumption during the time interval.

We write the continuous state inequality constraints as

$$\begin{aligned} g_i(t) &= u_i(t) - u_{i\,\max} \le 0, \;\; i = 1, 2, 3 \\ g_j(t) &= u_{j-3\,\min} - u_{j-3}(t) \le 0, \;\; j = 4, 5, 6 \end{aligned} \tag{2.8}$$

We may now state the corresponding optimal final approach control problem as:

Problem (P): Subject to the system described by (2.2) and (2.5) with initial and terminal conditions (2.3) and (2.4), find control functions w_1, w_2, w_3 and decision variables ζ_1, ζ_2 ζ_3 such that the cost function (2.7) is minimized subject to the continuous state inequality constraints (2.8), where the penalty parameters k_1, k_2, k_3, k_4, k_5 and k_6 are to be appropriately adjusted.

3 A Computational Method

We now develop an efficient computational method for solving Problem (P) as follows.

Let the time interval $[0,\ t_f]$ be partitioned into n_p subintervals with $n_p + 1$ partition points denoted by τ_0^p, τ_1^p, ..., $\tau_{n_p}^p$ such that

$$\tau_0^p = 0, \tau_{n_p}^p = t_f \text{ and } \tau_{k-1}^p < \tau_k^p, \text{for } k = 1, ..., n_p \tag{3.1}$$

where n_p satisfies $n_{p+1} > n_p$. We now approximate the control functions in the form of piecewise constant functions as

$$\begin{aligned} w_1^p(t) &= \sum\nolimits_{k=1}^{n_p} \sigma_{1,k}^p \chi_{[\tau_{k-1}^p, \tau_k^p)}(t) \\ w_2^p(t) &= \sum\nolimits_{k=1}^{n_p} \sigma_{2,k}^p \chi_{[\tau_{k-1}^p, \tau_k^p)}(t) \\ w_3^p(t) &= \sum\nolimits_{k=1}^{n_p} \sigma_{3,k}^p \chi_{[\tau_{k-1}^p, \tau_k^p)}(t) \end{aligned} \tag{3.2}$$

where $\chi_{[\tau_{k-1}^p, \tau_k^p)}(t)$ denotes the indicator function of $[\tau_{k-1}^p,\ \tau_k^p)$ defined by

$$\chi_{[\tau_{k-1}^p, \tau_k^p)}(t) = \begin{cases} 1, & t \in \left[\tau_{k-1}^p, \tau_k^p\right) \\ 0, & \text{elsewhere} \end{cases}$$

$\sigma_{1,k}^p$, $\sigma_{2,k}^p$, $\sigma_{3,k}^p$, $k = 1,\ 2,\ \ldots,\ n_p$ are control parameters, and τ_k^p, $k = 0,\ 1,\ \ldots,\ n_p$ are switching time points such that (3.1) are satisfied.

Let Γ^p be the set of all vectors τ^p satisfying (3.1). σ_1^p=($\sigma_{1,1}^p$, ..., σ_{1,n_p}^p), σ_2^p=($\sigma_{2,1}^p$, ..., σ_{2,n_p}^p), σ_3^p=($\sigma_{3,1}^p$, ..., σ_{3,n_p}^p) and τ^p=(τ_1^p, ..., $\tau_{n_p}^p$). We consider Problem (P) with its control functions w_1^p, w_2^p and w_3^p expressed, respectively, by (3.2) to be referred to as Problem (P_p). For each $p \geqslant 1$, Problem (P_p) is an optimal parameter selection problem. The gradient formulae of the cost function (2.7) with respect to the control parameter vectors σ_1^p, σ_2^p and σ_3^p as well as the initial condition parameters ζ_1, ζ_2 and ζ_3 can be easily obtained. See Theorem 5.2.1 of [25]. In fact, the gradient formulae of the cost function with respect to the switching vector τ^p can also be derived by using an argument similar to that given for Theorem 5.3.1 of [25]. However, these gradient formulae are not effective for numerical calculation. For details, see comments given in [11, 13]. In this paper, we will employ the idea of a time scaling transform [11] to map the variable switching time points into pre-assigned fixed knots.

We introduce a transformation which will map $t \in [0,\ t_f]$ into $s \in [0,\ 1]$:

$$dt(s)/ds = \vartheta^p(s) \tag{3.3}$$

with initial and terminal condition $t(0) = 0$ and $t(1) = t_f$, where ϑ^p is given by

$$\vartheta^p(s) = \sum\nolimits_{i=1}^{n_p} \delta_i^p \chi_{[\varsigma_{i-1}^p, \varsigma_i^p)}(s)$$

where $\delta_i^p \geq 0$, $i = 0, 1, \ldots, n_p$, represent the duration between consecutive control switches and $\varsigma_i^p \geq 0$, $i = 0, 1, \ldots, n_p$, are preassigned time points in a new time horizon [0, 1]. The function $\vartheta^p(s)$, with possible discontinuity points at $s = \varsigma_k^p = k/n_p$, $k = 0, 1, \ldots, n_p$, is called a time scaling control. Let δ_k^p, $k = 1, \ldots, n_p$, be referred to collectively as δ^p, and Ω^p be the set of all such δ^p. Define $\widetilde{\omega}_1^p(s) = w_1^p(t(s))$, $\widetilde{\omega}_2^p(s) = w_2^p(t(s))$ and $\widetilde{\omega}_3^p(s) = w_3^p(t(s))$. Then, $\widetilde{\omega}_1^p(s)$, $\widetilde{\omega}_2^p(s)$and $\widetilde{\omega}_3^p(s)$ are determined uniquely by σ_1^p, σ_2^p, σ_3^p and δ^p.

Define $\widetilde{u}_1(s) = u_1(t(s))$, $\widetilde{u}_2(s) = u_2(t(s))$, $\widetilde{u}_3(s) = u_3(t(s))$, $\widetilde{x}_r(s) = x_r(t(s))$, $\widetilde{y}_r(s) = y_r(t(s))$, $\widetilde{v}_{xr}(s) = v_{xr}(t(s))$, $\widetilde{v}_{yr}(s) = v_{yr}(t(s))$, $\widetilde{\theta}_r = \theta_r(t(s))$ and $\widetilde{\omega}_r(s) = \omega_r(t(s))$. Then, Problem (P_p), after the time scaling transform (3.3) and the application of the constraint transcription method to the constraints (2.8), becomes:

Problem $(P_{p,\varepsilon,\lambda})$: Given the dynamical system:

$$\begin{cases} \dot{\widetilde{x}}_r(s) = \vartheta^p(s)\widetilde{v}_{xr}(s) \\ \dot{\widetilde{y}}_r(s) = \vartheta^p(s)\widetilde{v}_{yr}(s) \\ \dot{\widetilde{\theta}}_r(s) = \vartheta^p(s)\widetilde{\omega}_r(s) \\ \dot{\widetilde{v}}_{xr}(s) = \vartheta^p(s)(\Omega^2\widetilde{x}_r(s) + 2\Omega\widetilde{v}_{yr}(s) + \widetilde{u}_1(s)\cos\widetilde{\theta}_r(s) - \widetilde{u}_2(s)\sin\widetilde{\theta}_r(s)) \\ \dot{\widetilde{v}}_{yr}(s) = \vartheta^p(s)(\Omega^2\widetilde{y}_r(s) - 2\Omega\widetilde{v}_{xr}(s) + \widetilde{u}_1(s)\sin\widetilde{\theta}_r(s) + \widetilde{u}_2(s)\cos\widetilde{\theta}_r(s)) \\ \dot{\widetilde{\omega}}_r(s) = \vartheta^p(s)\widetilde{u}_3(s) \\ \dot{\widetilde{u}}_1(s) = \vartheta^p(s)\widetilde{\omega}_1^p(s) \\ \dot{\widetilde{u}}_2(s) = \vartheta^p(s)\widetilde{\omega}_2^p(s) \\ \dot{\widetilde{u}}_3(s) = \vartheta^p(s)\widetilde{\omega}_3^p(s) \\ \dot{t}(s) = \vartheta^p(s) \end{cases} \tag{3.4}$$

with initial conditions:

$$\begin{array}{lllll} \widetilde{x}_r(0) = x_{r0}, & \widetilde{y}_r(0) = y_{r0}, & \widetilde{\theta}_r(0) = \theta, & \widetilde{v}_{xr}(0) = v_{xr0}, & \widetilde{v}_{yr}(0) = v_{yr0}, \\ \widetilde{\omega}_r(0) = \omega_{r0}, & \widetilde{u}_1(0) = \zeta_1, & \widetilde{u}_2(0) = \zeta_2, & \widetilde{u}_3(0) = \zeta_3, & t(0) = 0. \end{array}$$

where ϑ^p is determined uniquely by δ^p, find control parameter vectors σ_1^p, σ_2^p and σ_3^p, initial condition parameter constants ζ_1, ζ_2 and ζ_3, and time scaling parameter vector $\delta^p \in \Omega^p$ such that the cost function:

$$\begin{aligned} J = {} & k_1\left(\widetilde{x}_r(1) - x_{rf}\right)^2 + k_2\left(\widetilde{y}_r(1) - y_{rf}\right)^2 \\ & + k_3(\widetilde{\theta}_r(1) - \theta_f)^2 + k_4\widetilde{v}_{xr}^2(1) + k_5\widetilde{v}_{yr}^2(1) + k_6\widetilde{\omega}_r^2(1) \\ & + \int_0^1 \vartheta^p(s)\sqrt{\widetilde{u}_1^2(s) + \widetilde{u}_2^2(s) + \widetilde{u}_3^2(s)}ds \end{aligned} \tag{3.5}$$

is minimized subject to the constraint:

$$\sum\nolimits_{k=1}^{n_p} \delta_k^p / n_p = t_f$$

and the canonical inequality constraints:

$$\begin{aligned} & F_{i,\varepsilon,\lambda}^p(\sigma_1^p, \sigma_2^p, \sigma_3^p, \xi_1, \xi_2, \xi_2, \delta^p) = -\lambda \\ & \quad + \textstyle\int_0^1 L_\varepsilon\left(g_i(s, \sigma_1^p, \sigma_2^p, \sigma_3^p, \xi_1, \xi_2, \xi_3, \delta^p)\right) ds \leq 0 \end{aligned} \tag{3.6}$$

$i = 1, 2, \ldots, 6$, where $\lambda \geq 0$ is adjustable constant value, g_i, $i = 1, 2, \ldots, 6$, are defined by (2.8), and

$$L_\varepsilon(g(s)) = \begin{cases} 0 & if \quad g_i(s) < -\varepsilon \\ (g_i(s) + \varepsilon)^2 / 4\varepsilon & if \quad -\varepsilon \leq g_i(s) < \varepsilon \\ g_i(s) & if \quad g_i(s) > \varepsilon \end{cases}$$

where $\varepsilon > 0$ is an adjustable constant. The following theorem shows the relationship between the continuous state inequality constraints (2.8) and their approximate canonical inequality constraints.

Theorem 3.1. *$\forall\ \varepsilon > 0$, let $(\sigma_1^{p,\varepsilon,\lambda}, \sigma_2^{p,\varepsilon,\lambda}, \sigma_3^{p,\varepsilon,\lambda})$ be the control parameter vectors, $(\xi_1^{p,\varepsilon,\lambda}, \xi_2^{p,\varepsilon,\lambda}, \xi_2^{p,\varepsilon,\lambda})$ the initial condition parameters, and $\delta^{p,\varepsilon,\lambda}$ the time scaling control parameter vector. Then, there exists a $\lambda(\varepsilon)$, such that for all λ satisfying $0 < \lambda < \lambda(\varepsilon)$, if $\Pi = (\sigma_1^{p,\varepsilon,\lambda}, \sigma_2^{p,\varepsilon,\lambda}, \sigma_3^{p,\varepsilon,\lambda}, \xi_1^{p,\varepsilon,\lambda}, \xi_2^{p,\varepsilon,\lambda}, \xi_2^{p,\varepsilon,\lambda}, \delta^{p,\varepsilon,\lambda})$ satisfies the constraints*

$$F_{i,\varepsilon,\lambda}^p(\Pi) = \int_0^1 L_\varepsilon(g_i(s, \Pi))ds - \lambda \leq 0$$

then it satisfies the continuous constraints (2.8).

Proof. The proof is similar to that given for Theorem 8.5.1 of [25]. □

During the computation, we assign initial values of ε and λ. Then, check whether the continuous state inequality constraints (2.8) are satisfied or not. If they are not satisfied, decrease the value of λ. By *Theorem* 3.1 we see that for each ε the reduction of λ needs only be carried out a finite number of steps for the fulfillment of the continuous state inequalities constraints (2.8). Once the continuous state inequalities constraints (2.8) are satisfied, decrease the value of ε, and then reducing the values of λ until the continuous state inequalities constraints (2.8) are satisfied. The process is repeated until ε is smaller than or equal to a given tolerance.

4 Numerical Simulations

In this section, simulation results are given. The angular velocity of the target is assumed to be $\Omega = 0.1 rad/s$. The initial condition is taken as

$$\begin{aligned} &x_r(0) = 10, \quad y_r(0) = 10, \quad \theta_r(0) = \pi/2, \\ &v_{xr}(0) = 1, \quad v_{yr}(0) = 1, \quad \omega_r(0) = 1. \end{aligned}$$

and the desired final state is

$$\begin{aligned} &x_r(t_f) = 1, \quad y_r(t_f) = 0, \quad \theta_r(t_f) = 0, \\ &v_{xr}(t_f) = 0, \quad v_{yr}(t_f) = 0, \quad \omega_r(t_f) = 0. \end{aligned}$$

The normalized control input are assumed to be

$$|u_i(t)| \leq 1, \quad i = 1, 2, 3.$$

Let the terminal time, t_f, be 10s, $[0, t_f]$ is divided into 20 segments for numerical simulation.

The least fuel consumption of the final approach of rendezvous problem with the given parameter values are solved using the computational method presented in Section III for which the optimal control software MISER3.2 is used. The computer used is Pentium (R) 1.73GHz with 512M memory. The results obtained are

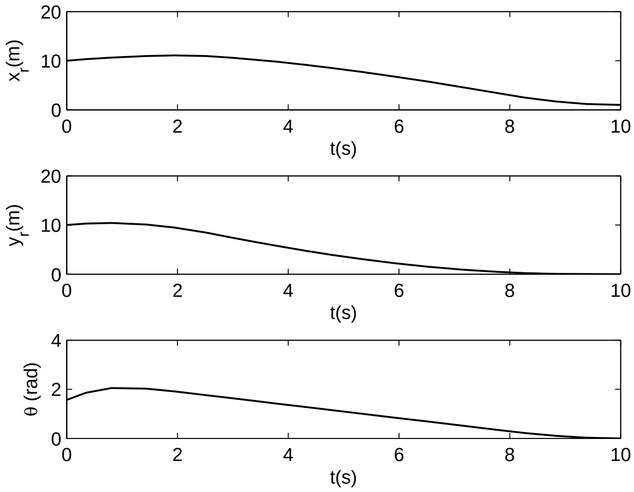

Figure 3: Optimal trajectories I

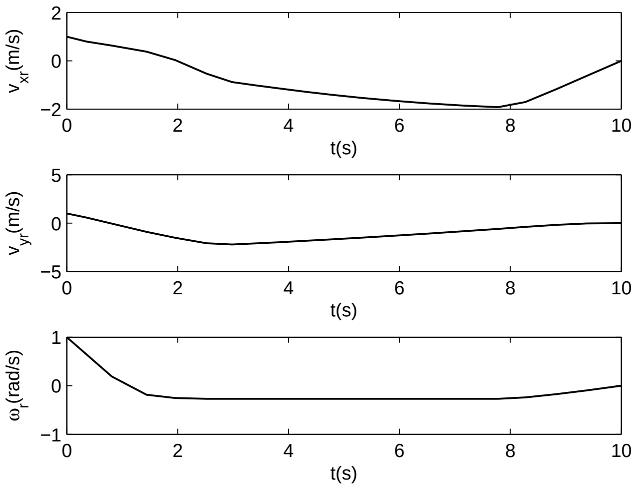

Figure 4: Optimal trajectories II

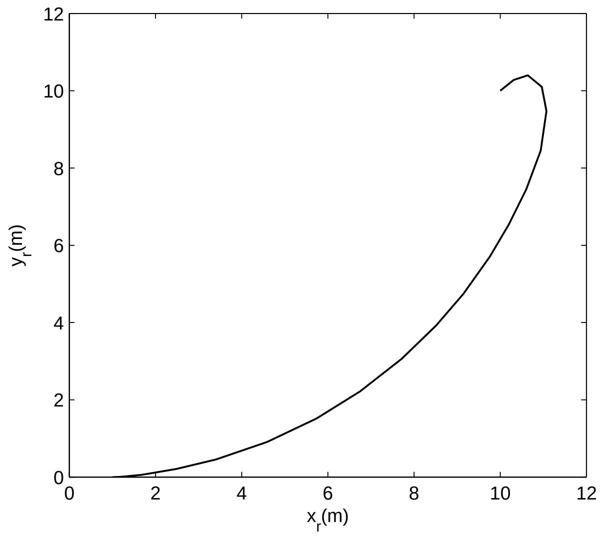

Figure 5: Optimal trajectory of the chaser in X_1-Y_1 plane

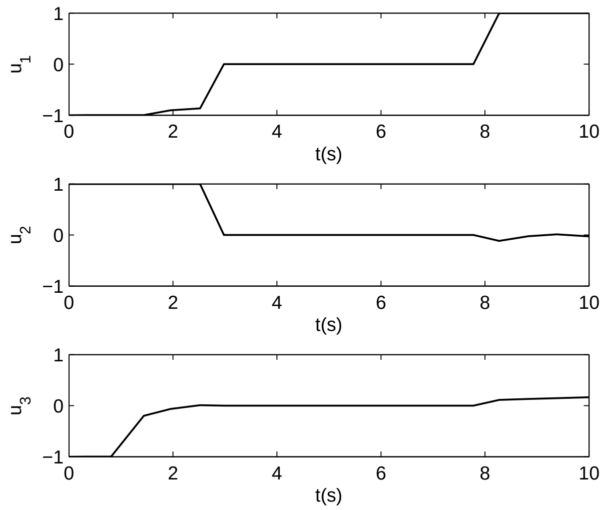

Figure 6: Optimal control forces

The corresponding value of the cost function J in equation (3.5) is 6.15. Fig. 3 and Fig. 4 describe the state trajectories and Fig. 5 shows the chaser's corresponding optimal moving trajectory in the target's body fixed frame X_1Y_1. Fig. 3 and Fig. 4 show the optimal trajectory starting from the initial state and ending at the final state. Fig. 6 depicts the normalized control signals. By using the thrust acceleration components show in the Fig. 6, the spacecraft of chaser can achieve no relative motion with the target, and at the same time it rendezvous to desire terminal point showed in Fig. 5. The results indicate that the control parametrization method is simple and efficient in the sense that convergence to the final solution was obtained with a not large number of iteration.

5 Conclusion

This paper deals with the final approach of rendezvous problem for a servicing spacecraft to approach a non-cooperative target with least fuel consumption. By utilizing specific features associated with the least fuel final approach problem, we obtained an equivalent standard optimal control problem. An efficient computational method was developed based on the control parametrization method in conjunction with a time scaling transform, where the constraint transcription method is used to approximate the continuous state inequality constraints. Numerical simulation showed that the proposed computational method is highly effective and efficient.

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