Yokohama Publishers
ISSN 1880-5221 ONLINE JOURNAL

Pacific Journal of Optimization
Vol. 5, No. 1, pp. 111–126, January 2009

OIL SPILL BOOM MODELING, NUMERICAL APPROXIMATION AND CONTINGENCY PLAN OPTIMIZATION*

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Abstract: Using a non-linear membrane theory we establish a mechanical model for oil spill floating boom. The model is based on the minimization of the internal strain energy minus the external energy of the applied forces. The discretisation of the boom uses a four-nodes quadrilateral finite-element. The non-linear variational problem is solved using the Newton-Raphson method. The vertical angle of the boom skirt is computed and is used to evaluate the oil containment efficiency. A boom plan tactical optimization problem is formulated. Several real-life operational constraints are given for the boom plan definition. Numerical examples illustrate the capability of the numerical model.

Key words: *membrane theory, finite-element method, fluid-solid interaction, tactical optimization*

Mathematics Subject Classification: *74K15, 74G05, 74-02, 76B75*

1 Introduction

Oil pollution is a major problem within maritime and coastal environments. For a survey of the Asian-Pacific region, a $CEDRE$ bulletin [2] during 1997, mentions the ship wreck of the tanker $Nakhodka$, in the West of the Honshu island Japan $6,240T$ spilled, forty accidents in the Hong-Kong harbor during the years 1995-1996 from $20L$ to $50T$, and six hundreds sabotages of Colombia's pipe-lines during the decay 1986-1996 up to $140,000T$.

Before any optimization of response technologies, the most adapted one must be found first, in accordance with the pollution location, the oil properties and the operational conditions. In this paper, we study the floating oil barriers, named booms. Near the shoreline, the objective of the technology is to contain the oil on the sea surface, or to deviate the oil to a coastal point.

In this paper, we are interested in the mechanical modeling of oil spill boom. The proposed model is based on the minimization of the total mechanical energy of the boom structure. We describe the continuous problem and the pressure term involved. A discrete solution is obtained using the finite-element method.

A boom plan tactical optimization problem is given so that minimizing oil leakage under the boom. We give the operational point of view which must be included in any boom plan optimization procedure. Note that many others optimization problems are not treated here,

*This research is supported by the French Research Agency ANR, and $ADEME$, under grant N^0 $BAR3D/0694C0067$, N^0 $ANR-06-ECOT-003-01$.

such as oil skimming, waste treatment, oil slick survey, waste storage, total response time for clean up operations [20], work force strategy [18](200,000 voluntary persons during the *Nakhodka* clean up operation).

This work is motivated by some recent numerical results [12] and by recent numerical and experimental study of the oil boom hydrodynamic [4]. Indeed, in the historical reference [6], the authors have provided the membrane finite-element considered in this paper. The established existence and regularity of the solution of the mechanical membrane problem is not treated here.

Mathematical studies concerning oil spill problems are numerous, and focus on different aspects such as stochastic approach in harbor [7], probabilistic approach of boom containment [3], and oil boom hydrodynamic [23]. Boom hydrodynamic studies [5] [22] are most numerous than boom structural studies [1].

External loads on boom come from three sources, the sea current, the waves [9], and the wind, given in decreasing order of importance. In this paper, we will study the first loading factor, the sea current. It is studied as an inefficiency factor for an oil volume contained by booms [3] [17].

The paper is organized as follows. Boom theory is given in section 2 using membrane theory. Numerical and mechanical approximations are described in section 3. Optimization problems are set in section 4. Numerical examples and a real-life boom are considered in section 5.

2 Boom Theory

Let us first give the boom domain definition.

Definition 2.1. A boom domain imbedded in the three dimensional euclidian space $\mathbb{R}^3$ is defined as a set ω of N boom sections including each four devices

$$\omega = \sum_{i=1}^{N} \sum_{j=1}^{4} \omega_{i,j}$$

where $\omega_{i,1}$ is the boom section numbered i composed of a float, a skirt, including at its bottom a chain and at its top a leach, $\omega_{i,2}$ is a mini skirt $i = 1...N-1$ linking adjacent boom sections, $\omega_{i,3}$ is a mooring device between a boom section end and a buoyancy coffer, $\omega_{i,4}$ is a mooring line device between a buoyancy coffer and an anchor or a dead mass moored on the sea bed. The superscript RL added to $\omega_{i,3-4}^{RL}$ means the right or the left end of a boom section. The superscript $+-$ added to $\omega_{i,3-4}^{RL,+-}$ means the end far away $(+)$ or closed $(-)$ to a boom section.

The sea level at a given time is denoted l, the water depth is h, the immersed boom height is Z, the common line between the float and the skirt of a boom section is denoted $l_{i,1}$, the floating line of a boom section is denoted $f_{i,1}$. We have

$$\omega_{i,1}|_{z=l} = f_{i,1}$$

We show the boom domain on the figure 1. Note that the buoyancy coffer positions will respect

$$\omega_{i,3}^{RL,+} = \omega_{i,4}^{RL,-}, \forall RL, \forall i$$

It means that the head of the mooring line (coffer-anchor) closed to the boom coincides with the end away from the boom of the mooring device (boom-coffer). It corresponds to the buoyancy coffer position.

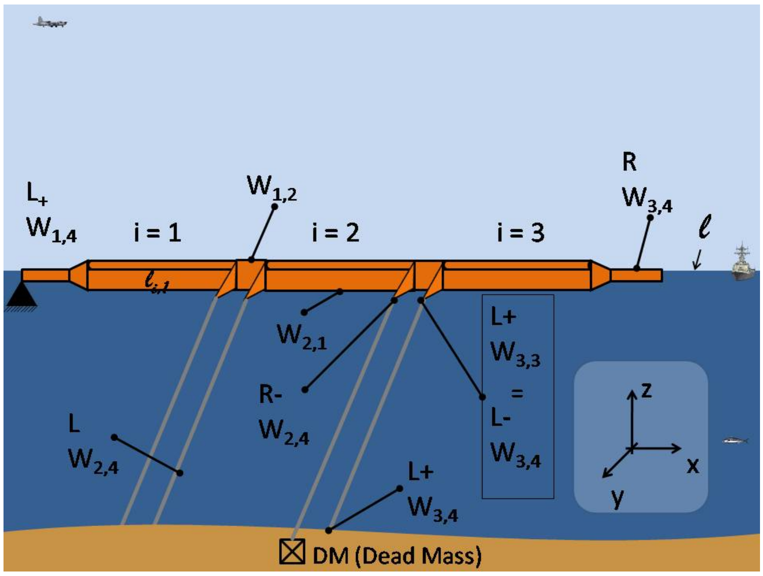

Figure 1: Boom domain decomposition

The boom domain is placed in the euclidian space with the following convention. The first axis x follows the boom domain. It is given by $l_{i,1}$ for straight boom plan before boom deformation. The second axis y follows the sea current to the downstream direction, when the current is normal to the boom. The third axis z follows the sea water depth from the sea bed.

Definition 2.2. The set of the boom admissible displacements V_a is defined by

$$V_a = \left\{ u : \omega \longrightarrow \mathbb{R}^3, u|_{\omega_{i,4}^{RL,+}} = 0_(x, y, z), u|_{\omega_{i,4}^{RL,-}} = 0_z \right\}$$

where, the first condition corresponds to a fixed anchor, a dead mass or a point on the shore, and the second condition corresponds to the null vertical displacement of a buoyancy coffer on the sea surface.

For the boom mooring on a mast, a tanker hull magnet, or a vertical stressed line along a quay, we use the following condition instead of the first one.

$$u|_{\omega_{i,4}^{RL,+}} = 0_(x, y)$$

We will define the functional to be minimized in the boom mechanical problem. The boom domain after displacement is denoted $\omega + u$ and u is the boom displacement.

Definition 2.3. The total mechanical energy related to an admissible displacement u of a boom is given by the functional $e : V_a \longrightarrow \mathbb{R}$ defined by

$$e(u) = e_i(u) - e_e(u)$$

The internal mechanical stress energy e_i is given by

$$e_i(u) = \frac{1}{2}\int_{\omega} tr(\sigma(u)x(u))d\omega$$

where tr indicates the trace operator, σ the Piola-Kirchhoff stress tensor of second kind, x the Green strain tensor.

The external load potential energy is given by

$$e_e(u) = \int_{\omega+u} (-p\overrightarrow{n}.\overrightarrow{u} + \overrightarrow{d}.\overrightarrow{u}) \ \ d(\omega+u) - \int_{\omega+u} \rho g\overrightarrow{z}.\overrightarrow{u} \ \ d(\omega+u)$$

where $\overrightarrow{d}$ indicates the tangential drag friction force, p the normal current drag pressure and the normal inflating pressure, $\overrightarrow{n}$ the unit external normal to $\omega + u$, ρ the membrane surface density, g the gravity acceleration, and $\overrightarrow{z}$ the vertical unit vector oriented from the sea bed to the sea surface.

The external energy functional e_e is composed of two summations. The first one comes from the three fluids (oil, water, air) actions on the boom (pressure and drag), while the second one comes from the gravity forces (body force). We consider no projected load, no temperature change, and no electric or magnetic body force on the boom. Note that the summation defining e_e is performed on the deformed boom geometry, after displacement. The material surface density ρ is the local summation of the material density through the membrane thickness. It is considered within the deformed configuration of the boom.

Note that a deformed boom section is generally composed of a totaly immersed skirt, and an inflated float which is partially immersed and partially in contact with air. Consequently the pressure term taken into account in the model is composed of two parts. It will be described in the section 3.

We show the vertical cross-section of a boom on the figure 2. Note that the sea level differs up-stream and down-stream on both sides of the boom (head loss). It is a consequence of the hydrodynamic action of the sea current. The sea current velocity is denoted V. Figure 2 shows the skirt vertical angulation θ of a boom cross-section. The vertical angle θ will play an important role in the sequel. The angle $\theta = 0$ indicates a vertical skirt, normal to the sea surface. The horizontal angle between the sea current V and the boom normal is denoted α. It indicates with $\alpha = \frac{\pi}{2}$ a boom parallel with the sea current.

The following proposition gives the equation to be solved in the membrane displacement problem.

Proposition 2.4. *Neglecting tangential hydrodynamic external force* $\overrightarrow{d}$ *and inertial effect of the boom, at a given time* t *(sea level and sea current given), the displacement* u *of the boom is solution of the following problem*

$$Find \quad u \in V_a \quad such \quad that \quad \frac{d \ \ e(u)}{du}.v = 0 \qquad \forall v \in V_a$$

Proof. We use the virtual work principle written in Lagrangian formulation with the hypothesis of a membrane structure having large displacements. We neglect the inertial effect of the boom (dynamic effect) and the friction of the sea water on the boom. □

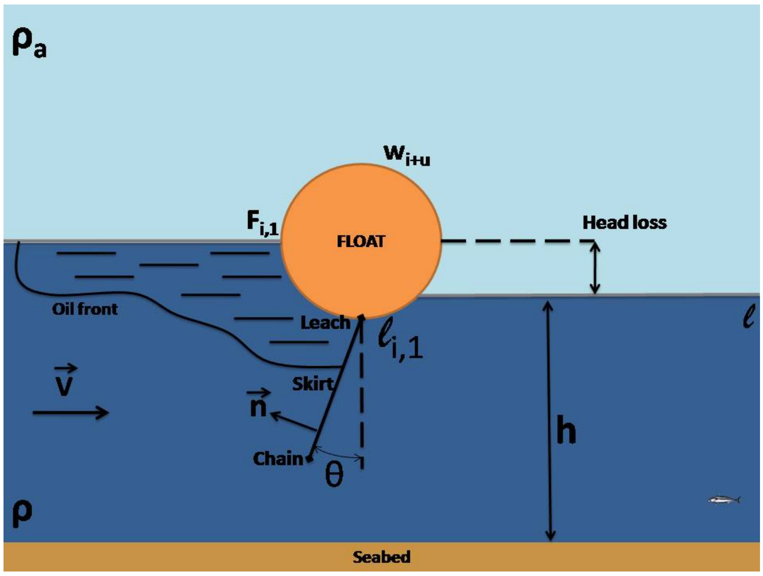

Figure 2: Boom vertical cross-section

Two remarks introduce now two important operational boom plan concepts.

Remark 2.5. The vertical coordinate of a mooring line is such that

$$z|_{\omega+u} \geq l - h$$

meaning that a mooring line must remain in the sea domain without intrusion in the sea bed. The sleeping part of a mooring line $\omega_{i,4}$ is the subset of $\omega_{i,4}$ defined by

$$S = \{m \in \omega_{i,4}, z|_{\omega+u} = l - h\}$$

The sleeping length S_l is the length of S. At any time t, S_l must be sufficient so that the friction force between the mooring line and the sea bed guarantees that

$$\overrightarrow{u}|_{\omega_{i,4}^{RL,+}} = 0$$

The constraint $z|_{\omega+u} \geq l - h$ concerns potentially any parts $\omega_{i,j}$ of a boom. It is a consequence of an eventual tide. Any part of a boom can potentially ground at down tide (the sea surface level l changes during time).

Remark 2.6. The avoiding part, during time t, of the head $\omega_{i,4}^{RL,-}$ of a mooring line is its locations set in the (x, y) dimensions (buoyancy coffer positions on the sea surface). This set is given by

$$A = \left\{m_t = \omega_{i,4}^{RL,-}, t \geq 0\right\}$$

for the buoyancy coffer i. The avoiding radius A_r is the size of A.

We show the sleeping length and the avoiding radius of a boom mooring line on the figure 3.

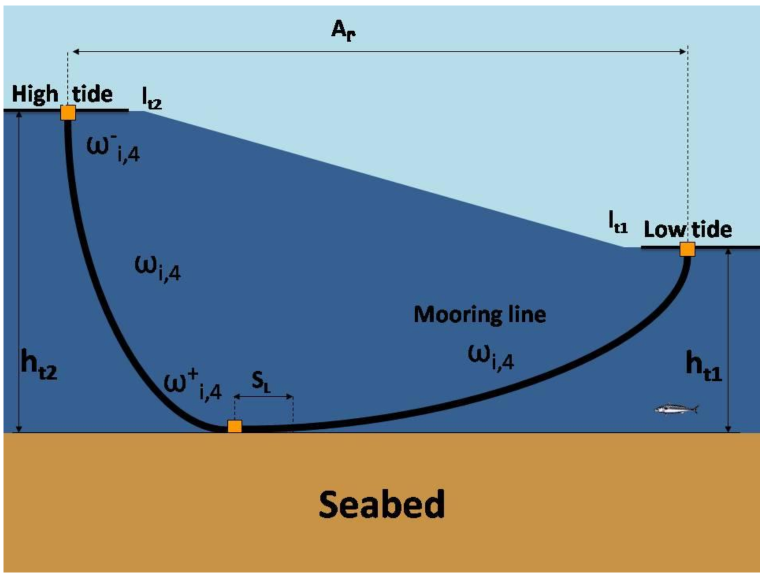

Figure 3: Sleeping length and avoiding radius of a mooring line

During time t, A_r must be minimized. It guarantees short boom displacements. It permits to avoid an eventual local boom grounding on emerged rocks, or a limitation on the maritime circulation in the vicinity of the boom.

Note that during time $\omega_{i,4}^{RL,-}$ has also a vertical displacement component. It is a consequence of the sea level change during tide.

3 Mechanical and Numerical Approximations

3.1 Mechanical Approximations

We recall that ω is the boom domain, u is the boom displacement, $\omega+u$ is the boom domain after displacement, and $l_{i,1}$ is the intersection line i between the float and the skirt.

The pressure force on a boom can be described by the following relation. It considers approximations made during experimental tests on a rigid boom in an hydrodynamic channel. The following relation results after simplifications and calculations.

$$\int_{\omega+u} p\overrightarrow{n}\ d(\omega+u) =$$

$$+ \frac{1}{2}\rho \ C_{\theta_0} \ (V_m \ f(\alpha))^2 \int_{\omega+u} \overrightarrow{n} \ d(\omega+u)$$

$$+ \ p_t \int_{\omega+u} \overrightarrow{n} \ d(\omega+u)$$

$$+ \ \sum_{i=1}^{N} \int_{l_{i,1}} P_1 \overrightarrow{z} \ d(l_{i,1})$$

The first hypothesis defines a drag hydrodynamic coefficient C on the skirt and the float by using a dimensional approach. This coefficient is defined experimentally on the basis of an a priori vertical skirt angulation θ_0 and a partially immersed float. More precisely, the order of the immersed height of the float is $10^{-1}m$, while the immersed height order of the skirt is $1m$.

The second hypothesis contains two approximations: the normal sea current along a boom section is assumed to take an averaged value V_m; on the other hand, along a boom section the sea current angle α with respect to the boom normal is taken into account by using a weighting function $f(\alpha)$.

Along a boom section, the function $f(\alpha)$ will be asymmetric. The function $f(\alpha)$ is defined by supposing that a boom section at the level of the sea surface $f_{i,1}$ (boom floating line) is a parabolic curve P_β, where the depth β of the parabolic curve is proportional to V_m^2 the square of the current velocity. The depth β is given a priori and depends on the boom deformation. By denoting $\overrightarrow{n_{P_\beta}}$ the normal to the parabolic curve P_β, the function $f(\alpha)$ is defined by

$$f(\alpha) = cos\langle\overrightarrow{n_{P_\beta}}, V_m\rangle \tag{3.1}$$

The third hypothesis concerns the internal pneumatic pressure (float inflation) p_t. It is considered constant along a boom section and remains independent of the float deformation. The displacement of the boom introduces a bending of the float. We neglect with this hypothesis the float volume variation and consequently the float internal pressure change.

The hypothesis 4 concerns the immersed part of the boom. The hydrostatic curvilinear force P_1 corresponds to the local vertical Archimedes force. Note that, this vertical force is applied on the intersection between the float and the skirt $l_{i,1}$. This reaction curvilinear force on $l_{i,1}$ balances the boom body force. However, this Archimedes force P_1 will be defined on the boom geometry before displacement. The value of P_1 will remain independent of the deformed boom geometry. The force P_1 is computed on the basis of the initial geometry of the float.

3.2 Numerical Approximations

The discrete boom displacement problem is given in the following lemma.

Lemma 3.1. *Considering the continuous membrane displacement u and virtual displacement v*

$$u = u_h \in V_{a,h}$$

$$v = v_h \in V_{a,h}$$

where $V_{a,h}$ is the finite dimensional space of the bilinear functions defined on a finite-element mesh ω_h of a boom (quadrilateral element), the membrane equilibrium problem written in a finite dimensional space takes the following form.

$$\frac{d \ e(u_h)}{du_h}.v_h = \langle F_h, V_h\rangle \tag{3.2}$$

Proof. The derivative operation $\frac{d}{du}e(u).v$ defines a linear continuous form with respect to v. The displacement v_h belongs to a finite dimensional space $V_{a,h}$. Consequently, it exists vectors $F_h \in \mathbb{R}^n$ and $V_h \in \mathbb{R}^n$ such that the result holds. F_h and V_h are the nodal mesh vectors representing the nodal out-of-balance forces and the nodal virtual displacements. The total number of nodal degree of freedom is n, three times the number of nodes. □

The discrete displacement space $V_{a,h}$ is a set of functions with some part imposed to be 0 (fixed degrees of freedom). This null displacement part concerns subsets of $\omega_{i,4}$. As a consequence, the corresponding components of the vector V_h are null. Let us denote by $V_{h,BC}$ these fixed components of V_h. We have by denoting $V_{h,F}$ the free components of V_h

$$\langle F_h, V_h \rangle = \langle \left\{ \begin{array}{c} F_{h,F} \\ F_{h,BC} \end{array} \right\}, \left\{ \begin{array}{c} V_{h,F} \\ V_{h,BC} = 0 \end{array} \right\} \rangle$$

where $F_{h,BC}$ can be interpreted as the reaction forces on the fixed part of the boom.

The solution of the discrete displacement problem is presented in the following lemma.

Lemma 3.2. *The solution of the boom equilibrium equation*

$$F_{h,F} = 0 \tag{3.3}$$

by the Newton-Raphson method uses the Hessian matrix K_h associated to the functional $e(u_h)$. It is defined by

$$\frac{d^2}{du^2}(e(u_h).v_h, w_h) = \langle K_h.V_h, W_h \rangle \tag{3.4}$$

where V_h and W_h are the displacement vectors of the finite element displacement functions v_h and w_h.

Proof. The Newton-Raphson method applied to $F_{h,F} = 0$ uses the tangent matrix $[\frac{\partial}{\partial U_h} F_{h,F}]$. Derivative of equation 3.2. along w_h gives

$$\frac{d^2}{du^2}(e(u_h).v_h, w_h) = \langle [\frac{\partial F_h}{\partial U_h}].V_h, W_h \rangle$$

where the tangent matrix $[\frac{\partial F_h}{\partial U_h}]$ is denoted K_h. □

Remark 3.3. To improve the convergence of the Newton-Raphson method, we must use at each iteration an upper limitation on the norm of the nodal displacement correction. When the displacement correction has a large component with respect to the spatial dimension, a scaling is applied to the displacement correction. Using SI units we used generally a maximal correction of a nodal displacement component $U_{lim} = 5cm$. Depending of the avoiding radius A_r of the mooring line heads of a boom plan, the iteration number is at least A_r/U_{lim}.

4 Boom Optimization

4.1 Tactical Optimization

The tactical optimization of a boom plan is a complex problem. Consequently, in this section, we give the beginning of a formulation of an optimization problem. It takes the operational point of view used for boom contingency plans. The solution method of the proposed optimization problem is not treated here.

The strategic optimization of an emergency plan including oil boom is also not treated here. In that field, we can cite the strategic optimization of a global oil-spill emergency plan [16] ($Exxon - Valdez$), and the optimal policy measure in harbor [7].

First, we define the cost function associated to a boom plan. Two approaches, focussing on oil contingency efficiency are proposed. The first one is based on the normal sea current velocity $\langle \overrightarrow{V}, \overrightarrow{n} \rangle$ in the vicinity of the boom. It is used in the FORBAR project. The second approach is based on the skirt vertical angulation θ. It is used in the SIMBAR project.

Definition 4.1. The oil contingency local inefficiency at a boom position can be given by the indicator function f_{pol} based on the Lee's Criteria [10]

$$f_{pol} = 1_{\left|\langle \overrightarrow{V}, \overrightarrow{n} \rangle\right| \geq V_s} \tag{4.1}$$

where, V_s is a critical sea current velocity depending of the oil density, sea water density and oil surface tension. The indicator function f_{pol} can also be based on the Simbar's criteria [11]

$$f_{pol} = 1_{|\theta| \geq \theta_s} \tag{4.2}$$

where θ_s is a critical value of the skirt angulation. It is defined by experimental observations, and numerical computations [21]. The arguments of θ_s are the sea current velocity and the oil properties (density, viscosity).

From the operational point of view, the empirical values used in the emergency contingency plans (*in-situ* observations) are $0.35m/s$ for V_s and 10^{deg} for θ_s.

The leakage criteria given by 4.1 using the velocity of the fluid suggests the construction of an oil leakage debt law. The construction of this debt law is not treated here. The leakage criteria given by 4.2 takes into account the deformed geometry of the boom. That suggests to construct a leakage model based on a fluid/structure approach.

Using the local cost function defined by 4.1 or by 4.2 we give the formulation of a boom plan tactical optimization problem. This problem concerns the geometry of the contingency plan, in an estuary or an harbor for examples. The problem of the boom design optimization, for example the boom geometry (boom skirt height) or the boom constitutive materials are not treated here. This strategic optimization problem is more complex to formulate because an optimal boom design must be a compromise between different kinds of coastal sites.

For a given boom design, and a given site to be protected, we define a tactical optimization problem in the following way.

$$Min \sum_{i=1}^{N} \int_{\omega_{i,1}+u_h} f_{pol} \;\; d(\omega + u_h) \tag{4.3}$$

$$||\sigma(u_h)|| \leq \sigma_{max} \tag{4.4}$$

$$||F_{h,BC}|| \leq F_{max} \tag{4.5}$$

where σ_{max} is the maximal stress supported by the boom plan materials (fabric, chain, mooring line), and F_{max} is the maximal force supported by the boom plan boundaries (friction of a dead-mass on the sea floor, tension on a fixed boundary).

From the operational point of view, the standard values used in coastal emergency contingency plans are listed. The dead masses weight $6T$. It permits to define F_{max} using the Archimedes force of a dead-mass, and its friction coefficient on the sea-bed. The fabric stress limit is $650daN/5cm$ and the chain stress limit is $32T$. It permits to define σ_{max} on each parts of the boom plan.

The enumeration of the decision variables of the tactical optimization problem is given bellow.

- angle between the sea current and the boom normal α
- number of boom sections of the plan N
- length of the boom sections $||\omega_{i,1}||$
- length of the mooring lines $||\omega_{i,4}||$
- position of the dead-masses on the sea bed $\omega_{i,4}^{RL,+}$

The enumeration of the parameters to be taken into account is given bellow. A parameter should be given, or its influence analyzed case-by-case.

- time t
- boom design (boom kind)
- boom stress limit σ_{max}, boundary force limit F_{max}
- sea current V
- water depth h
- morphology of the site (sea bed and coastal geophysical data)
- boom-coffer device design $\omega_{i,3}$

In particular, the time t should be considered to handle the presence of tide. During the SIMBAR project, the Elorn estuary protection (Brittany, France), is computed at six different times, every two hours of a reference day. It permits to analyze a full tide cycle [13].

The operational supplementary constraints to be considered for a coastal boom plan are listed bellow.

- α such that $cos(\alpha) \leq \frac{0.35}{||V||}$
- $6Z \leq h$
- $3h \leq ||\omega_{i,4}||$
- $||\omega_{i,3}^{R,+} - \omega_{i,3}^{L,+}|| \leq \frac{||\omega_{i,1}||}{1.07}$

The first constraint permits to tackle the critical current velocity V_s normal to the boom. Note that $||V|| \longrightarrow +\infty$ gives $\alpha \longrightarrow \frac{\pi}{2}$. In this case the boom and the current directions coincide. The boom is feathered within a flag behavior. The second constraint permits to limit the velocity variation of the fluid flowing under the boom in presence of small water depth. The third constraint permits to create the sleeping length S_l of a mooring line. The constraint 4 is a negative pre-stress of the cord of the boom section $\omega_{i,1}$. It permits to decrease the boom stress σ [14], which is balanced by a higher radius of curvature of $l_{i,1} + u$.

Remark 4.2. In the case of boom plan in harbor, the cost function 4.3 must be considered either tide up, or tide down of the installation period. It depends on the oil pollution location in the harbor. In any case of boom plan, the constraints 4.4 and 4.5 may be active during the maximal current velocity $||V||$ of the installation period. It can be attained for either tide kind.

Remark 4.3. The velocity current amplitude $||V||$ can be spatially non-uniform in the vicinity of the boom plan. Consequently, the decision variable α can be non-uniform along the boom plan. The angle between the boom normal and the current is higher when the current amplitude is higher. In the case of a boom plan in a river, the current amplitude can be lower in the vicinity of the river shore. Consequently the boom plan is installed so that the oil is deviated from the high current with a high angle α. The oil il also concentrated in the low current with a low angle α, where the skimming system can be installed.

Remark 4.4. Other optimization problems can be set by using the operational point of view. Here is a non exhaustive enumeration of cost functions.

- boom installation time (useful for oil terminal located in river)
- number, volume and strategic location of equipment points [8]
- computational time of the boom numerical model
- time delay for boom stock renovation
- number of boom installation exercises per year
- coordination of counter pollution means (strategic oil spill response problem)[18]
- number of emergency plans up to date
- life cycle duration of a boom material, washing efficiency

4.2 Comments

The optimization problem set here is a complex problem as a consequence of handling a coastal natural environment. The great number of variables leads to choice as method a flexible simplex method (flexible polyhedra), or a genetic algorithm for problem solution. Note that a general integer program [20] has been used to formulate a tactical decision problem for clean up operations.

Note that several data of the problem such as sea current, water depth and coastal morphology, can be approximative. It makes difficult the interpretation of the results. Note that sea current action modeling can used a stochastic approach [19].

Note equally that supplementary mooring lines $\omega_{i,4}$ can be added to limit locally the avoiding radius A_r of a boom plan. Generally it is used in presence of tide and small water depth, for avoiding boom grounding on rocks by low tide.

The volume of the buoyancy coffers is not taken into account in our numerical model. Note that this volume is $2m^3$ in coastal boom plan.

We end this section by underlining an influence having the decision variables of the proposed optimization problem. The tactical optimization problem concerns a boom plan having a variable number N of sections. This number must be increased when the variable α increases. It represents the angle between the boom section normal and the sea current. The concept of a minimal coastal length to be protected is not taken into account here.

The figure 4 shows four graphs for a boom plan adapted to estuaries and rivers. On the figure 4, the N edges represent the boom sections. The graph nodes include DM mooring devices, and SZ sacrificed zones (oil recuperation zones).

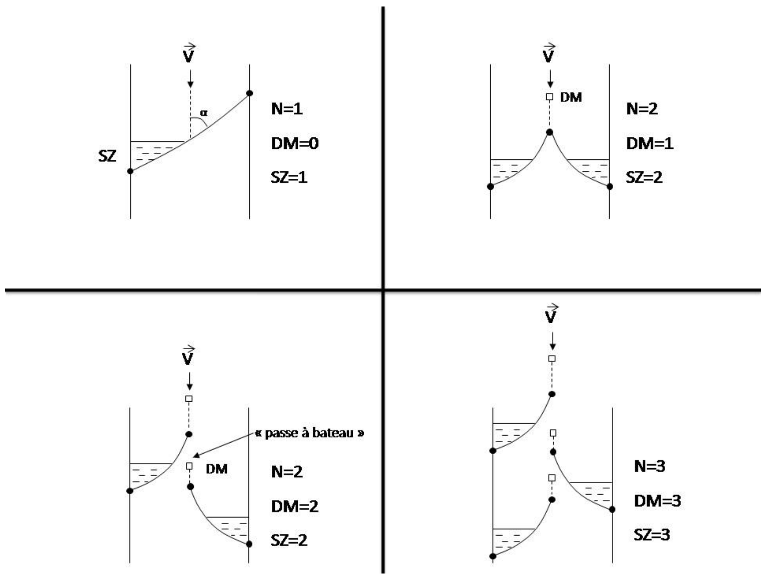

Figure 4: Graphs of boom plans in estuaries or rivers

The figure 4 shows two non-connected boom graphs. It generalizes the definition given in section 1 for connected boom graph. A non-connected boom graph permits to maintain a maritime traffic in estuary or river.

The figure 4 shows that the boom efficiency can be fitted by using the number of boom sections, the number of dead-masses, and the sacrifice of coastal zones.

5 Numerical Results

In this section we present a numerical comparison between two kinds of boundary conditions, applied to a $30m$ long boom section. It shows a first step for the identification of a boom anchorage optimal design. After, we present the computation of a straight boom plan having $N = 5$ sections of $150m$ long each.

5.1 Boom Section 30 m Long

Two computations of a boom section of $30m$ long are presented. The objective is to show the influence of the geometry of the boom-coffer fixation device $\omega_{i,3}$ on the deformed boom geometry and stress. Two geometries of this fixation device are presented. The first geometry is a rectangle. In this case, the boundaries away from the boom $\omega_{i,3}^{+}$ are fixed on two rigid vertical masts. The second geometry is a triangle. In this case the boundary points away from the boom $\omega_{i,3}^{+}$ are fixed to two points. The height with respect to the sea surface of these points is a parameter. In the case presented, this height corresponds to the middle of the boom float. The boom design considered in these two computations are, the skirt height $0.75m$, the float diameter $0.55m$, the sea current velocity $0.3m/s$ (normal to the initial boom geometry), the inflated float pressure p_t $150mbar$. The boom float is pinched at its ends to allow their free rotations along a vertical axis. The boom self weight is $12kg/m$ corresponding solely to the chain mass. The mass $1kg/m$ of the fabric (float and skirt parts) is here neglected. The hydrodynamic pressure of the sea current acts solely on the skirt. The Archimedes force P_1 on the intersection line between the float and skirt is uniform along the boom section.

The figure 5 shows the two deformed mesh geometries, with the two kinds of boom-coffer fixation devices.

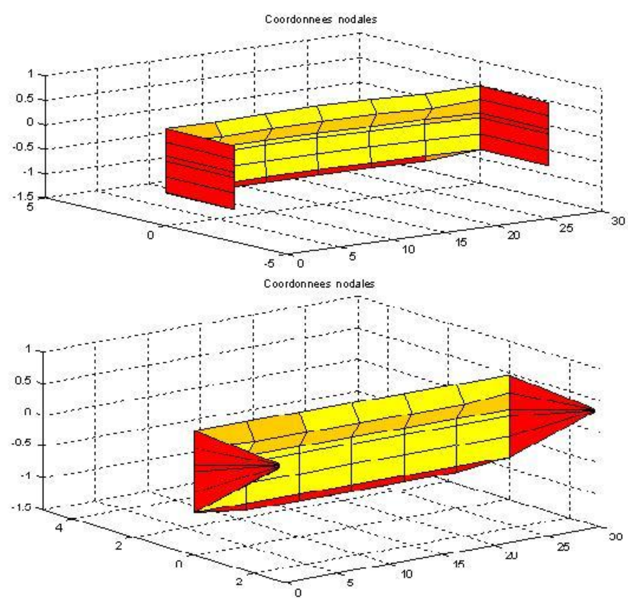

Figure 5: Rectangular and triangular fixation devices

The rectangular fixation on a mast gives for the membrane stress in the middle of the skirt bottom $576N/m$, the vertical skirt angle -5.35^{deg} and the chain stress $3.13\ 10^3N$. The triangular fixation on a point gives for the skirt bottom stress $584N/m$, the skirt angle -3.71^{deg} and the chain stress $2.88\ 10^3N$. The triangular fixation design minimizes the skirt angle, and the boom stress.

5.2 Straight Boom Plan 758 m Long

We present the computation of a straight boom plan normal to an $0.3m/s$ uniform sea current. The water depth is $10m$. The boom plan contains $N = 5$ boom sections. Each boom section has the design of the boom section of $30m$ long presented previously. The boom plan is $758m$ long. The weight of the mooring lines is taken into account. The hydrodynamic pressure on the mooring lines is neglected. The mooring lines displacements on the boom plan ends belong to the vertical plane (y, z) normal to the boom plan direction. The mooring line bottoms $\omega_{i,4}^{+}$ are fixed on the sea bed. The mooring line tops $\omega_{i,4}^{-}$ have a null vertical displacement along z, indicating that the buoyancy coffers stay at the level of the sea surface l. A chain is considered at the bottom of each mini-skirt of $2m$ long $\omega_{i,2}$. Consequently, the chain stress can circulate continuously between adjacent boom sections $\omega_{i,1}$ and $\omega_{i+1,1}$. The Archimedes force P_1 on the intersection line between the float and skirt is uniform along the boom. The CPU time is $258mn$. The Newton-Raphson method needs 280 iterations to converge. The figure 6 shows the three views of the deformed finite-element mesh.

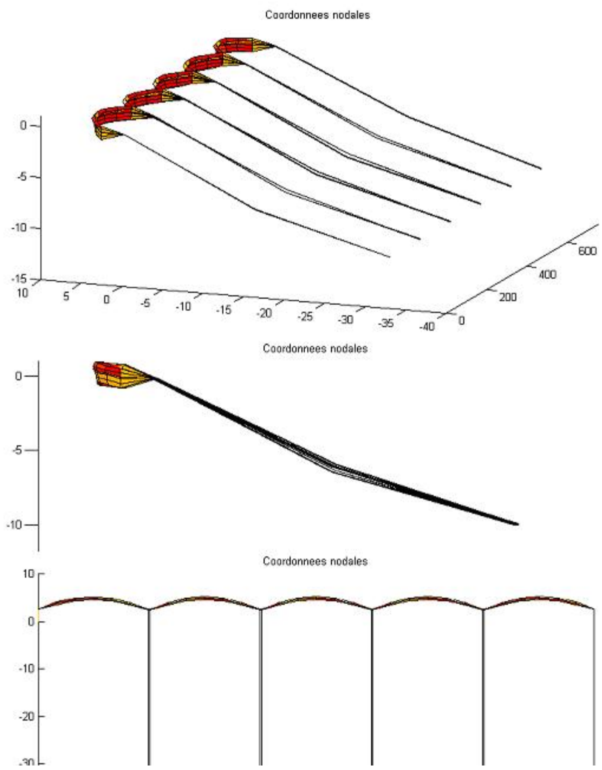

Figure 6: Straight boom plan with 5 sections

The vertical skirt angles θ at the middle of each section are respectively using ^{deg} as unit -9.38 -9.83 -9.82 -9.85 -9.4. These angles are less than the empiric threshold value θ_s [15]. Considering a light oil pollutant the cost function $\int_{\omega+u_h} f_{pol}$ is null for this boom plan under these conditions. Considering a Heavy Fuel Oil pollutant (HFO) the cost function associated to this boom plan may be higher [21].

The stress σ in the chain at the middle of each section are respectively using 10^5 N/m as unit 6.61 6.63 6.63 6.63 6.63. These stress values are less than the standard boom stress limit σ_{max}.

Acknowledgement

The author acknowledge the French framework $PRECODD$ dedicated to the Environmental Technologies and Sustainable Development. The network “Réseau de Recherche et d'Innovations Technologiques” $RITMER$ on “Pollutions Marines Accidentelles et leurs Conséquences Ecologiques” is gratefully acknowledged.

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Manuscript received 5 September 2008
revised 6 October 2008
accepted for publication 22 November 2008

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