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JOINT SOURCE AND RELAY OPTIMIZATION FOR A MIMO COMMUNICATION SYSTEM

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In honor of Professor Liansheng Zhang on the occasion of his 70th birthday.

Abstract: In this paper, we consider the joint source and relay optimization problem for a Multi-Input-Multi-Output (MIMO) communication system employing a non-regenerative MIMO relay. Given a fixed total transmission power budget for the source and the relay, we formulate the MIMO transmitter and relay design problem using the Minimum Mean Square Error (MMSE) criterion. Since the original formulation is nonconvex (thus difficult to solve), we present equivalent reformulations which are amenable to solutions by modern convex optimization techniques. In particular, we show that when the channel matrices are diagonal, the optimal MMSE joint source and relay design problem can be solved iteratively as a sequence of Second Order Cone Programs (SOCP). The latter can be solved using highly efficient interior point methods. Computer simulation through SeDuMi (Self-Dual-Minimization) software shows that this new approach (optimal joint source-relay power control strategy) is not only efficient, but also effective, leading to substantially improved mean square error performance than the non-optimized uniform power control strategy.

Key words: *joint source-relay optimization, linear matrix inequality, OFDM, SOCP*

Mathematics Subject Classification: *15A42, 90C22*

1 Introduction

Introducing relaying nodes in a wireless communication network is a powerful method to improve network performance. As is well known, strong shadowing, shielding and the interference from the neighboring nodes are the major detriments to the long-term channel quality of wireless communication links. When high carrier frequencies are used, these detriments can be much more pronounced. An effective solution to mitigate these undesirable channel effects is to let some nodes in the network act as cooperative relays, resulting in a tetherless multiple antenna array which can substantially improve the quality of network services while maintaining a fixed level transmission power from the source. In practice, there are two types of relays. One is non-regenerative relays which serve as simple amplify-and-forward stations, whereas the other is regenerative relays which perform detection/decoding, storage, regeneration and aid in routing.

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In recent years, a substantial amount of research has focused on wireless networks with relays. Important bounds on the capacity of the relay networks have been established [3]. For a fixed wireless TDMA (Time Division Multiple Access) system with non-regenerative relays, the authors of [1] considered the scheme of letting all idled users (or nodes) serve as relays provided that they transmit on different frequency bands. A simple analytic formula for an optimum power allocation strategy is established in [1] which is reminiscent of the well known water-filling principle in information theory [3]. The work of [4] considered a non-regenerative MIMO relay system whereby the source covariance matrix and the relay matrix are jointly optimized to maximize the source-destination capacity.

In this paper, we also consider a MIMO communication system using a non-regenerative relay, just like [4], but using the minimum mean squared error (MMSE) criterion. It turns out that the direct formulation of this joint source and relay optimization problem is non-convex, making it difficulty to solve in practice [2]. Hence, we develop an alternative but equivalent formulation using the Linear Matrix Inequality (LMI) technique. Unfortunately, this formulation remains nonconvex due to a cross term in the objective function. When channel matrices are diagonal as is the case when all nodes employ the OFDM (Orthogonal Frequency Division Multiplexing) modulation scheme [6], we can further simplify our formulation to a Second Order Cone program (SOCP). The latter can be solved by the highly efficient interior point method. Computer simulation through SeDuMi software shows that this new approach is not only efficient, but also effective, leading to substantially improved mean square error performance for a non-regenerative MIMO relay communication system.

Our notational conventions are as follows: The n-dimensional Euclidean space is denoted by $\Re^n$ and the nonnegative orthant of dimension n is denoted by $\Re^n_+$. Vectors and matrices will be represented by bold lowercase and uppercase letters respectively, and the superscript ^{H} will denote the Hermitian transpose. For a random vector $\mathbf{x}$, $\mathcal{E}(\mathbf{x})$ will denote its mean and $\mathcal{E}(\mathbf{x}\mathbf{x}^H)$ will denote its correlation matrix. Moreover, for any symmetric matrix $\mathbf{X}$, the notation $\mathbf{X} \succeq \mathbf{0}$ (or $\mathbf{X} \succ \mathbf{0}$) signifies that $\mathbf{X}$ is positive semidefinite (or positive definite respectively), and the notation $\mathrm{Tr}\,(\mathbf{X})$ denotes the trace of $\mathbf{X}$.

2 Problem Formulation

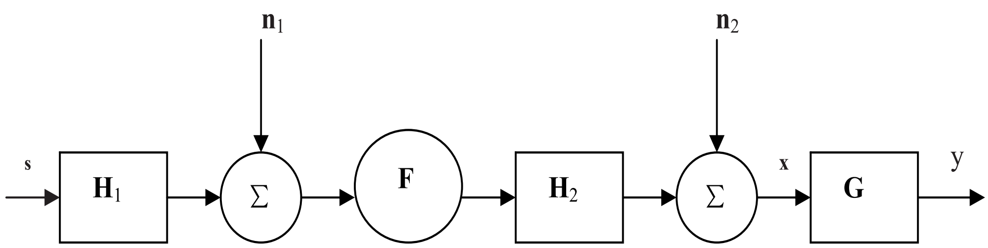

Figure 1: A non-regenerative MIMO relay system

Consider a non-regenerative MIMO relay system (see Fig. 1) whose input signal is $\mathbf{s}$. Let $\mathbf{H}_1$ denote the channel matrix between the source and the relay, $\mathbf{H}_2$ the channel matrix between the relay and the destination. The channel noise vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ are assumed to be additive Gaussian and uncorrelated with each other and with the input signal $\mathbf{s}$. Let the relay transmit matrix and the receiver equalizer matrix be denoted by $\mathbf{F}$ and $\mathbf{G}$ respectively.

Then, the equalizer output signal $\mathbf{y}$ takes the form

$$\mathbf{y} = \mathbf{G}(\mathbf{H}_2\mathbf{F}\mathbf{H}_1\mathbf{s} + \mathbf{H}_2\mathbf{F}\mathbf{n}_1 + \mathbf{n}_2).$$

Let $\mathbf{e}$ denote the error vector, then

$$\mathbf{e} = \mathbf{y} - \mathbf{s} = \mathbf{G}\left(\mathbf{H}_2\mathbf{F}\mathbf{H}_1\mathbf{s} + \mathbf{H}_2\mathbf{F}\mathbf{n}_1 + \mathbf{n}_2\right) - \mathbf{s}.$$

The mean squared error (**MSE**) at the receiver can be written as

$$\begin{aligned}
\mathbf{MSE} &= \mathrm{Tr}\left(\mathcal{E}\left(\mathbf{e}\mathbf{e}^H\right)\right)\\
&= \mathrm{Tr}\left(\mathcal{E}((\mathbf{G}\left(\mathbf{H}_2\mathbf{F}\mathbf{H}_1\mathbf{s} + \mathbf{H}_2\mathbf{F}\mathbf{n}_1 + \mathbf{n}_2\right) - \mathbf{s})(\mathbf{G}\left(\mathbf{H}_2\mathbf{F}\mathbf{H}_1\mathbf{s} + \mathbf{H}_2\mathbf{F}\mathbf{n}_1 + \mathbf{n}_2\right) - \mathbf{s})^H)\right)\\
&= \mathrm{Tr}\left(\mathcal{E}((\mathbf{G}\mathbf{H_2}\mathbf{F}\mathbf{H}_1 - \mathbf{I})\mathbf{s}\mathbf{s}^H(\mathbf{G}\mathbf{H_2}\mathbf{F}\mathbf{H}_1 - \mathbf{I})^H + (\mathbf{G}\mathbf{H_2}\mathbf{F})\mathbf{n}_1\mathbf{n}_1^H(\mathbf{G}\mathbf{H_2}\mathbf{F})^H + \mathbf{G}\mathbf{n}_2\mathbf{n}_2^H\mathbf{G}^H\right)\\
&= \mathrm{Tr}\left((\mathbf{G}\mathbf{H_2}\mathbf{F}\mathbf{H}_1 - \mathbf{I})\mathbf{Q}(\mathbf{G}\mathbf{H_2}\mathbf{F}\mathbf{H}_1 - \mathbf{I})^H + \rho_1^2\mathbf{G}\mathbf{H}_2\mathbf{F}(\mathbf{G}\mathbf{H}_2\mathbf{F})^H + \rho_2^2\mathbf{G}\mathbf{G}^H\right)\\
&= \mathrm{Tr}\ \Big(\mathbf{G}\left(\mathbf{H_2}\mathbf{F}\mathbf{H_1}\mathbf{Q}\left(\mathbf{H_2}\mathbf{F}\mathbf{H_1}\right)^H + \rho_1^2\mathbf{H}_2\mathbf{F}\left(\mathbf{H}_2\mathbf{F}\right)^H + \rho_2^2\mathbf{I}\right)\mathbf{G}^H\\
&\quad -\mathbf{G}\mathbf{H_2}\mathbf{F}\mathbf{H}_1\mathbf{Q} - \mathbf{Q}\left(\mathbf{G}\mathbf{H}_2\mathbf{F}\mathbf{H}_1\right)^H + \mathbf{Q}\Big),
\end{aligned}$$

where we have used the fact that the signal $\mathbf{s}$, the noise $\mathbf{n}_1$ and the noise $\mathbf{n}_2$ are mutually uncorrelated

$$\mathcal{E}(\mathbf{s}\mathbf{n}_i^H) = \mathbf{0},\ \mathcal{E}(\mathbf{n}_i\mathbf{n}_j^H) = \mathbf{0},\ i, j = 1, 2,\ i \neq j,$$

and that the signal and the noise autocorrelation matrices are given by

$$\mathcal{E}(\mathbf{s}\mathbf{s}^H) = \mathbf{Q},\ \mathcal{E}(\mathbf{n}_i\mathbf{n}_i^H) = \rho_i^2\mathbf{I},\ i = 1, 2.$$

Let us define the matrix

$$\mathbf{W} = \left(\mathbf{H_2}\mathbf{F}\mathbf{H}_1\mathbf{Q}\left(\mathbf{H}_2\mathbf{F}\mathbf{H}_1\right)^H + \rho_1^2\mathbf{H}_2\mathbf{F}\left(\mathbf{H}_2\mathbf{F}\right)^H + \rho_2^2\mathbf{I}\right)^{-1}. \tag{2.1}$$

Then the total **MSE** can be simplified as

$$\mathbf{MSE} = \mathrm{Tr}\ \left(\mathcal{E}\left(\mathbf{e}\mathbf{e}^H\right)\right) = \mathrm{Tr}\left(\mathbf{G}\mathbf{W}^{-1}\mathbf{G}^H - \mathbf{G}\mathbf{H}_2\mathbf{F}\mathbf{H}_1\mathbf{Q} - \mathbf{Q}\left(\mathbf{G}\mathbf{H}_2\mathbf{F}\mathbf{H}_1\right)^H + \mathbf{Q}\right). \tag{2.2}$$

We wish to minimize the mean squared error by choosing $\mathbf{F}$, $\mathbf{G}$ and $\mathbf{Q}$ appropriately. As a fifth order polynomial in the variables $\mathbf{F}$, $\mathbf{G}$ and $\mathbf{Q}$, the mean squared error **MSE** is not convex. To circumvent this difficulty, we first eliminate $\mathbf{G}$ by minimizing **MSE** while fixing $\mathbf{F}$. In particular, we differentiate **MSE** with respect to $\mathbf{G}$ and set it to zero. This leads to the following LMMSE (Linear Minimum Mean Square Error) equalizer

$$\mathbf{G} = \mathbf{Q}\mathbf{H}_1^H\mathbf{F}^H\mathbf{H}_2^H\mathbf{W}. \tag{2.3}$$

Substituting (2.3) into (2.2), the **MSE** can be further simplified as:

$$\begin{aligned}
\mathbf{MSE} &= \mathrm{Tr}\left(\mathcal{E}(\mathbf{e}\mathbf{e}^H)\right)\\
&= \mathrm{Tr}\left(\mathbf{Q}\mathbf{H}_1^H\mathbf{F}^H\mathbf{H}_2^H\mathbf{W}\mathbf{W}^{-1}\left(\mathbf{Q}\mathbf{H}_1^H\mathbf{F}^H\mathbf{H}_2^H\mathbf{W}\right)^H - \mathbf{Q}\mathbf{H}_1^H\mathbf{F}^H\mathbf{H}_2^H\mathbf{W}\mathbf{H}_2\mathbf{F}\mathbf{H}_1\mathbf{Q}\right.\\
&\quad \left. - \mathbf{Q}\left(\mathbf{H}_2\mathbf{F}\mathbf{H}_1\right)^H\left(\mathbf{Q}\mathbf{H}_1^H\mathbf{F}^H\mathbf{H}_2^H\mathbf{W}\right)^H + \mathbf{Q}\right)\\
&= \mathrm{Tr}\left(\mathbf{Q} - \mathbf{Q}^H\mathbf{H}_1^H\mathbf{F}^H\mathbf{H}_2^H\mathbf{W}\mathbf{H}_2\mathbf{F}\mathbf{H}_1\mathbf{Q}\right).
\end{aligned} \tag{2.4}$$

In practical applications, there are transmission power constraints on the source and the relay:

$$\text{Tr}\ (\mathbf{Q}) \leq p_1, \tag{2.5}$$

$$\text{Tr}\ \left(\mathbf{F}\mathbf{H}_1\mathbf{Q}\mathbf{H}_1^H\mathbf{F}^H + \rho_1^2\mathbf{F}\mathbf{F}^H\right) \leq p_2, \tag{2.6}$$

where $p_1 > 0$ and $p_2 > 0$ are user-specified bounds on the transmitting power for the source and relay, respectively. Our goal is to find the transmitting matrix $\mathbf{F}$ and the relay matrix $\mathbf{G}$ which satisfy the constraints given by (2.5) and (2.6) so that the total **MSE** $\text{Tr}\,(\mathcal{E}(\mathbf{e}\mathbf{e}^H))$ is minimized. In other words, we aim to solve

$$\begin{aligned} \mathbf{min}_{\mathbf{F},\ \mathbf{Q}} \quad & \text{Tr}\ \left(\mathbf{Q} - \mathbf{Q}^H\mathbf{H}_1^H\mathbf{F}^H\mathbf{H}_2^H\mathbf{W}\mathbf{H}_2\mathbf{F}\mathbf{H}_1\mathbf{Q}\right) \\ \mathbf{s.t.} \quad & \mathbf{W} = \left(\mathbf{H_2}\mathbf{F}\mathbf{H}_1\mathbf{Q}\left(\mathbf{H}_2\mathbf{F}\mathbf{H}_1\right)^H + \rho_1^2\mathbf{H}_2\mathbf{F}\left(\mathbf{H}_2\mathbf{F}\right)^H + \rho_2^2\mathbf{I}\right)^{-1}, \\ & \text{Tr}\ (\mathbf{Q}) \leq p_1,\ \text{Tr}\ \left(\mathbf{F}\mathbf{H}_1\mathbf{Q}\mathbf{H}_1^H\mathbf{F}^H + \rho_1^2\mathbf{F}\mathbf{F}^H\right) \leq p_2, \\ & \mathbf{Q} \succeq \mathbf{0}. \end{aligned} \tag{2.7}$$

Because of the cross item in the objective function and in the matrix inverse of the first constraint, the above direct formulation of optimal joint transmitter-relay problem is non-convex and hence difficult to solve due to the usual difficulties with the local solution and the selection of the step size and starting point. In what follows, we will simplify the nonconvex optimization problem (2.7).

It is straightforward to check the following matrix identity:

$$\mathbf{Q}-\mathbf{Q}\mathbf{H}_1^H\mathbf{F}^H\mathbf{H}_2^H\mathbf{W}\mathbf{H}_2\mathbf{F}\mathbf{H}_1\mathbf{Q} = \left(\mathbf{Q}^{-1} + \mathbf{H}_1^H\mathbf{F}^H\mathbf{H}_2^H\left(\rho_1^2\mathbf{H}_2\mathbf{F}\left(\mathbf{H}_2\mathbf{F}\right)^H + \rho_2^2\mathbf{I}\right)^{-1}\mathbf{H}_2\mathbf{F}\mathbf{H}_1\right)^{-1}.$$

Thus, we can rewrite the above joint source and relay optimization problem (2.7) as

$$\begin{aligned} \mathbf{min}_{\mathbf{F},\ \mathbf{Q}} \quad & \text{Tr}\ \left(\left(\mathbf{Q}^{-1} + \mathbf{H}_1^H\mathbf{F}^H\mathbf{H}_2^H\left(\rho_1^2\mathbf{H}_2\mathbf{F}\left(\mathbf{H}_2\mathbf{F}\right)^H + \rho_2^2\mathbf{I}\right)^{-1}\mathbf{H}_2\mathbf{F}\mathbf{H}_1\right)^{-1}\right) \\ \mathbf{s.t.} \quad & \text{Tr}\ (\mathbf{Q}) \leq p_1,\ \text{Tr}\ \left(\mathbf{F}\mathbf{H}_1\mathbf{Q}\mathbf{H}_1^H\mathbf{F}^H + \rho_1^2\mathbf{F}\mathbf{F}^H\right) \leq p_2, \\ & \mathbf{Q} \succeq \mathbf{0}. \end{aligned} \tag{2.8}$$

Introducing a new matrix variable $\mathbf{U} = \mathbf{Q}^{-1}$ and a matrix variable $\mathbf{V}$ satisfying

$$\mathbf{V} \succeq \mathbf{F}\mathbf{H}_1\mathbf{Q}\mathbf{H}_1^H\mathbf{F}^H = \mathbf{F}\mathbf{H}_1\mathbf{U}^{-1}\mathbf{H}_1^H\mathbf{F}^H,$$

we can reformulate the above optimization problem as

$$\begin{aligned} \mathbf{min}_{\mathbf{F},\ \mathbf{U},\mathbf{V}} \quad & \text{Tr}\ \left(\left(\mathbf{U} + \mathbf{H}_1^H\mathbf{F}^H\mathbf{H}_2^H\left(\rho_1^2\mathbf{H}_2\mathbf{F}\left(\mathbf{H}_2\mathbf{F}\right)^H + \rho_2^2\mathbf{I}\right)^{-1}\mathbf{H}_2\mathbf{F}\mathbf{H}_1\right)^{-1}\right) \\ \mathbf{s.t.} \quad & \text{Tr}\ (\mathbf{Q}) \leq p_1,\ \mathbf{U}\mathbf{Q} = \mathbf{I},\ \text{Tr}\ \left(\mathbf{V} + \rho_1^2\mathbf{F}\mathbf{F}^H\right) \leq p_2, \\ & \begin{bmatrix} \mathbf{U} & \mathbf{H}_1^H\mathbf{F}^H \\ \mathbf{F}\mathbf{H}_1 & \mathbf{V} \end{bmatrix} \succeq \mathbf{0},\quad \mathbf{U} \succeq \mathbf{0}, \end{aligned}$$

where we have used the equivalence (Schur complement) [2]

$$\mathbf{V} \succeq \mathbf{F}\mathbf{H}_1\mathbf{U}^{-1}\mathbf{H}_1^H\mathbf{F}^H \iff \begin{bmatrix} \mathbf{U} & \mathbf{H}_1^H\mathbf{F}^H \\ \mathbf{F}\mathbf{H}_1 & \mathbf{V} \end{bmatrix} \succeq \mathbf{0}.$$

By monotonicity, we can replace the equality constraint $\mathbf{UQ} = \mathbf{I}$ in the above optimization problem by

$$\mathbf{UQ} \succeq \mathbf{I}$$

which is further equivalent to

$$\begin{bmatrix} \mathbf{Q} & \mathbf{I} \\ \mathbf{I} & \mathbf{U} \end{bmatrix} \succeq \mathbf{0}.$$

Thus, the optimal joint MMSE source and relay optimization problem (2.7) can be stated as

$$\begin{array}{ll} \mathbf{min}_{\mathbf{F},\mathbf{Q},\mathbf{U},\mathbf{V}} & \mathrm{Tr}\left(\left(\mathbf{U} + \mathbf{H}_1^H\mathbf{F}^H\mathbf{H}_2^H\left(\rho_1^2\mathbf{H}_2\mathbf{F}\left(\mathbf{H}_2\mathbf{F}\right)^H + \rho_2^2\mathbf{I}\right)^{-1}\mathbf{H}_2\mathbf{F}\mathbf{H}_1\right)^{-1}\right) \\ \mathbf{s.t.} & \mathrm{Tr}\ (\mathbf{Q}) \le p_1,\ \mathrm{Tr}\ \left(\mathbf{V} + \rho_1^2\mathbf{F}\mathbf{F}^H\right) \le p_2, \\ & \begin{bmatrix} \mathbf{U} & \mathbf{H}_1^H\mathbf{F}^H \\ \mathbf{F}\mathbf{H}_1 & \mathbf{V} \end{bmatrix} \succeq \mathbf{0},\ \begin{bmatrix} \mathbf{Q} & \mathbf{I} \\ \mathbf{I} & \mathbf{U} \end{bmatrix} \succeq \mathbf{0}. \end{array} \tag{2.9}$$

It is clear that all the constraints in the optimization problem (2.9) are linear matrix inequalities, and therefore convex. Unfortunately, the objective function of (2.9) is nonconvex due to the cross term $\mathbf{H}_1^H\mathbf{F}^H\mathbf{H}_2^H\left(\rho_1^2\mathbf{H}_2\mathbf{F}\left(\mathbf{H}_2\mathbf{F}\right)^H + \rho_2^2\mathbf{I}\right)^{-1}\mathbf{H}_2\mathbf{F}\mathbf{H}_1$. This lack of convexity makes it difficult to compute global optimal solution. In the next section, we consider a simplified design which turns out to be convex.

3 Simplifying Design for an OFDM Non-Regenerative MIMO Relay System

In this section, we consider OFDM modulations so that the channel matrices $\mathbf{H_1}$, $\mathbf{H}_2$ are diagonal. We will substantially decrease the optimization variables in a way similar to [6]. More specifically, employing the Inverse Fast Fourier Transform (IFFT) at the transmitter and FFT at the front end of the relay (along with a cyclic prefix), we can effectively diagonalize the channel matrix $\mathbf{H}_1$. The channel matrix $\mathbf{H}_2$ can be diagonalized in a similar way. Hence, without lose of generality, we can assume the channel matrices $\mathbf{H}_1$ and $\mathbf{H}_2$ are diagonal.

Theorem 3.1. *Suppose the channel matrices $\mathbf{H}_1$ and $\mathbf{H}_2$ are both diagonal, then the optimization problem* (2.9) *can achieve its minimum when the source covariance matrix $\mathbf{F}$, $\mathbf{Q}$, $\mathbf{U}$, $\mathbf{V}$ are all diagonal.*

Proof. Assume $\mathbf{H}_1$, $\mathbf{H}_2$ are diagonal. Let $\mathbf{F}^*$, $\mathbf{Q}^*$ be an optimal solution of the joint source-relay optimization problem (2.7) which is non-diagonal. Define two new matrices which equal the diagonal part of $\mathbf{F}^*$, $\mathbf{Q}^*$ respectively:

$$\bar{\mathbf{F}} = \mathrm{Diag}(\mathbf{F}^*), \quad \bar{\mathbf{Q}} = \mathrm{Diag}(\mathbf{Q}^*)$$

It can be checked that $\bar{\mathbf{F}}$ and $\bar{\mathbf{Q}}$ are feasible for (2.7) and have a smaller objective value than that of $\mathbf{F}^*$, $\mathbf{Q}^*$. This then further implies that the optimal solution for (2.9) are all diagonal. Since the proof is the same as that of [8, 7], and we omit the details. □

According to Theorem 3.1, we only need to consider diagonal designs in the optimization problem (2.9), or equivalently (2.8). Let $\mathbf{u}$, $\mathbf{q}$, $\mathbf{f}$ and $\mathbf{v}$ denote the vectors of diagonal entries of $\mathbf{U}$, $\mathbf{Q}$, $\mathbf{F}$ and $\mathbf{V}$ respectively. Let $\mathbf{h}_1$ and $\mathbf{h}_2$ denote the vector of the diagonal entries of channel matrices $\mathbf{H}_1$ and $\mathbf{H}_2$ respectively. Then we can simplify (2.8) as

$$\begin{array}{ll} \mathbf{min}_{\mathbf{f},\,\mathbf{q}} & \displaystyle\sum_{i=1}^{L}\left(\mathbf{q}_i^{-1}+\frac{|\mathbf{h}_1(i)|^2|\mathbf{h}_2(i)|^2\mathbf{f}_i^2}{\rho_1^2|\mathbf{h}_2(i)|^2\mathbf{f}_i^2+\rho_2^2}\right)^{-1} \\ \mathbf{s.t.} & \displaystyle\sum_{i=1}^{L}\mathbf{q}_i \le p_1,\ \sum_{i=1}^{L}\mathbf{f}_i^2\left(\rho_1^2+|\mathbf{h}_1(i)|^2\mathbf{q}_i\right)\le p_2, \\ & \mathbf{q}_i \ge 0, \quad i=1,2,...,L. \end{array}$$

Notice that $\mathbf{u}_i = \mathbf{q}_i^{-1}$ and define

$$\mathbf{g}_i = \frac{|\mathbf{h}_1(i)|^2|\mathbf{h}_2(i)|^2\mathbf{f}_i^2}{\rho_1^2|\mathbf{h}_2(i)|^2\mathbf{f}_i^2+\rho_2^2}, \qquad i=1,2,..,L.$$

Then we have

$$\mathbf{f}_i^2 = \frac{\rho_2^2\mathbf{g}_i}{|\mathbf{h}_1(i)\mathbf{h}_2(i)|^2-\rho_1^2|\mathbf{h}_2(i)|^2\mathbf{g}_i}, \qquad i=1,2,..,L.$$

Hence, by eliminating the variables $\mathbf{f}_i$, we can rewrite the above optimization in the form

$$\begin{array}{lll} \mathbf{min}_{\mathbf{g},\mathbf{q},\mathbf{u}} & \displaystyle\sum_{i=1}^{L}(\mathbf{u}_i+\mathbf{g}_i)^{-1} & \\ \mathbf{s.t.} & \displaystyle\sum_{i=1}^{L}\mathbf{q}_i \le p_1, & \displaystyle\sum_{i=1}^{L}\frac{\rho_2^2\mathbf{g}_i\left(\rho_1^2+|\mathbf{h}_1(i)|^2\mathbf{q}_i\right)}{|\mathbf{h}_1(i)\mathbf{h}_2(i)|^2-\rho_1^2|\mathbf{h}_2(i)|^2\mathbf{g}_i}\le p_2, \\ & \multicolumn{2}{l}{\mathbf{q}_i\ge 0, \quad \mathbf{q}_i\mathbf{u}_i\ge 1, \quad 0\le\mathbf{g}_i\le\dfrac{|\mathbf{h}_1(i)|^2}{\rho_1^2}, \quad i=1,2,...,L.} \end{array} \tag{3.1}$$

Define a new variable $\mathbf{r}_i$ such that $\mathbf{r}_i = \mathbf{g}_i^{-1}$. Then, we have

$$\begin{aligned} \frac{\rho_2^2\mathbf{g}_i\left(\rho_1^2+|\mathbf{h}_1(i)|^2\mathbf{q}_i\right)}{|\mathbf{h}_1(i)\mathbf{h}_2(i)|^2-\rho_1^2|\mathbf{h}_2(i)|^2\mathbf{g}_i} &= \frac{\rho_2^2\left(\rho_1^2+|\mathbf{h}_1(i)|^2\mathbf{q}_i\right)}{|\mathbf{h}_1(i)\mathbf{h}_2(i)|^2\mathbf{r}_i-\rho_1^2|\mathbf{h}_2(i)|^2} \\ &= \frac{\rho_2^2\rho_1^2}{|\mathbf{h}_1(i)\mathbf{h}_2(i)|^2\mathbf{r}_i-\rho_1^2|\mathbf{h}_2(i)|^2} \\ &\quad +\frac{\rho_2^2|\mathbf{h}_1(i)|^2}{\left(|\mathbf{h}_1(i)\mathbf{h}_2(i)|^2\mathbf{r}_i-\rho_1^2|\mathbf{h}_2(i)|^2\right)\mathbf{u}_i}. \end{aligned}$$

Therefore, we can further simplify the above problem to

$$\begin{array}{ll} \mathbf{min}_{\mathbf{g},\mathbf{q},\mathbf{r},\mathbf{v}} & \displaystyle\sum_{i=1}^{L}(\mathbf{u}_i+\mathbf{g}_i)^{-1} \\ \mathbf{s.t.} & \displaystyle\sum_{i=1}^{L}\mathbf{q}_i\le p_1,\ \mathbf{q}_i\ge 0,\ \mathbf{q}_i\mathbf{u}_i\ge 1,\ \mathbf{g}_i\mathbf{r}_i\ge 1,\ 0\le\mathbf{g}_i\le\frac{|\mathbf{h}_1(i)|^2}{\rho_1^2},\ i=1,2,...,L, \\ & \displaystyle\sum_{i=1}^{L}\frac{\rho_2^2\rho_1^2}{|\mathbf{h}_1(i)\mathbf{h}_2(i)|^2\mathbf{r}_i-\rho_1^2|\mathbf{h}_2(i)|^2}+\sum_{i=1}^{L}\frac{\rho_2^2|\mathbf{h}_1(i)|^2}{\left(|\mathbf{h}_1(i)\mathbf{h}_2(i)|^2\mathbf{r}_i-\rho_1^2|\mathbf{h}_2(i)|^2\right)\mathbf{u}_i}\le p_2. \end{array}$$

We need to introduce another four variables

$$\mathbf{v}_i=(\mathbf{u}_i+\mathbf{g}_i)^{-1}, \quad \mathbf{s}_i=\left(|\mathbf{h}_1(i)\mathbf{h}_2(i)|^2\mathbf{r}_i-\rho_1^2|\mathbf{h}_2(i)|^2\right)^{1/2}\mathbf{u}_i^{1/2},$$
$$\mathbf{t}_i=\left(|\mathbf{h}_1(i)\mathbf{h}_2(i)|^2\mathbf{r}_i-\rho_1^2|\mathbf{h}_2(i)|^2\right)^{-1}.$$

With these new variables and using monotonicity, we can further simplify the above optimization problem to

$$\begin{aligned}
\mathbf{min} \quad & \sum_{i=1}^{L} \mathbf{v}_i \\
\mathbf{s.t.} \quad & \sum_{i=1}^{L} \mathbf{q}_i \le p_1,\ \sum_{i=1}^{L} \rho_2^2 \rho_1^2 \mathbf{t}_i + \sum_{i=1}^{L} \rho_2^2 |\mathbf{h}_1(i)|^2 \mathbf{s}_i^{-2} \le p_2, \\
& \mathbf{q}_i \ge 0,\ \mathbf{q}_i \mathbf{u}_i \ge 1,\ \mathbf{v}_i(\mathbf{u}_i + \mathbf{g}_i) \ge 1,\ \ \mathbf{g}_i \mathbf{r}_i \ge 1,\ 0 \le \mathbf{g}_i \le \frac{|\mathbf{h}_1(i)|^2}{\rho_1^2},\ i = 1, 2, ..., L, \\
& |\mathbf{s}_i| \le \left(|\mathbf{h}_1(i)\mathbf{h}_2(i)|^2 \mathbf{r}_i - \rho_1^2 |\mathbf{h}_2(i)|^2\right)^{1/2} \mathbf{u}_i^{1/2},\ \mathbf{t}_i \ge \left(|\mathbf{h}_1(i)\mathbf{h}_2(i)|^2 \mathbf{r}_i - \rho_1^2 |\mathbf{h}_2(i)|^2\right)^{-1}.
\end{aligned}$$

Finally, let us define $\mathbf{w}_i = \mathbf{s}_i^{-1}$ and rearrange the constraints, then the above optimization problem can be written in the following equivalent form:

$$\begin{aligned}
\mathbf{min} \quad & \sum_{i=1}^{L} \mathbf{v}_i \\
\mathbf{s.t.} \quad & \sum_{i=1}^{L} \mathbf{q}_i \le p_1,\ \sum_{i=1}^{L} \rho_2^2 \rho_1^2 \mathbf{t}_i + \sum_{i=1}^{L} \rho_2^2 |\mathbf{h}_1(i)|^2 \mathbf{w}_i^2 \le p_2, \\
& \mathbf{q}_i \mathbf{v}_i \ge 1,\ \mathbf{w}_i \mathbf{s}_i \ge 1,\ \mathbf{v}_i(\mathbf{u}_i + \mathbf{g}_i) \ge 1,\ \ \mathbf{g}_i \mathbf{r}_i \ge 1, \\
& (|\mathbf{h}_1(i)\mathbf{h}_2(i)|^2 \mathbf{r}_i - \rho_1^2 |\mathbf{h}_2(i)|^2)\mathbf{u}_i \ge |\mathbf{s}_i|^2,\ \mathbf{t}_i(|\mathbf{h}_1(i)\mathbf{h}_2(i)|^2 \mathbf{r}_i - \rho_1^2 |\mathbf{h}_2(i)|^2) \ge 1, \\
& \mathbf{q}_i \ge 0,\ 0 \le \mathbf{g}_i \le \frac{|\mathbf{h}_1(i)|^2}{\rho_1^2},\ i = 1, 2, ..., L.
\end{aligned} \tag{3.2}$$

Since the objective function of (3.2) is linear, and all the constraints are either linear, convex quadratic cone or rotated convex quadratic cone, we see that the original nonconvex optimization problem (2.9) is equivalent to a SOCP (which is convex).

In other words, in the case of OFDM modulations (diagonal channel matrices), the original nonconvex joint source-relay optimum design problem has been transformed into an equivalent SOCP formulation. We can use recently developed interior point methods to efficiently solve it in polynomial time with complexity $O(L^{3.5} \log \frac{1}{\epsilon})$, where $\epsilon > 0$ is the relative accuracy of the computed solution [5].

4 Computer Simulations

In this section, we will demonstrate the effectiveness of our SOCP formulation (3.2) through two simulation examples. In both examples, the SOCP programs are solved using the interior point optimization code SeDuMi [9] developed in MATLAB. Two simulation schemes are compared in the simulations: one is a uniform-power control scheme, while the other is an optimal joint source-relay power control scheme.

Orthogonal Frequency Division Multiplexing (OFDM) is one of the important multicarrier modulating schemes. It breaks down a frequency selective channel into frequency flat subchannels for which communication is easy and well-understood. In the uniform-power control scheme, the power of every subchannel in the non-regenerative MIMO relay communication system is kept the same, while in the joint source-relay power control scheme, the power of every subchannel is obtained by the MMSE design (3.2). For a fair comparison, the power budget of both the transmitter and the relay are kept the same in the two power control schemes.

In the first example, we suppose the channel matrices $\mathbf{H}_1$ and $\mathbf{H}_2$, both having sizes 4×4, are as follows:

$$\mathbf{H}_1 = \begin{bmatrix} -0.0428 - 0.0242j & -0.0313 + 0.0313j & 0.2003 + 0.0524j & 0.0035 - 0.0064j \\ 0.0035 - 0.0064j & -0.0428 - 0.0242j & -0.0312 + 0.0312j & 0.2003 + 0.0524j \\ 0.2003 + 0.0524j & 0.0035 - 0.0064j & -0.0428 - 0.0242j & -0.0312 + 0.0312j \\ -0.0313 + 0.0313j & 0.2003 + 0.0524j & 0.0035 - 0.0064j & -0.0428 - 0.0242j \end{bmatrix},$$

$$\mathbf{H}_2 = \begin{bmatrix} 0.2088 + 0.0147j & 0.0156 + 0.0062j & -0.0483 - 0.0343j & -0.0018 + 0.0018j \\ -0.0018 + 0.0018j & 0.2088 + 0.0146j & 0.0156 + 0.0062j & -0.0482 - 0.0343j \\ -0.0483 - 0.0343j & -0.0018 + 0.0018j & 0.2088 + 0.0147j & 0.0156 + 0.0062j \\ 0.0156 + 0.0062j & -0.0483 - 0.0343j & -0.0018 + 0.0018j & 0.2088 + 0.0147j \end{bmatrix}.$$

Also, suppose that the noise in the channels is white, both having a variance of 1. The power budgets of the source and the relay are assumed the same, that is, $p_1 = p_2$. For SNR= 1dB, the MSE of the optimal design obtained from solving (3.2) with SeDuMi software is 3.1773, while the MSE of the nonoptimized uniform-power distribution control scheme is 7.3678. Thus, compared with uniform-power distribution control scheme, the MMSE optimized solution provides a 56.88% improvement in the MSE performance. This result shows when the value of SNR is low, optimal joint source-relay power control scheme for a non-regenerative MIMO relay has a substantially superior mean square error performance than uniform-power distribution control scheme.

In the second example, we suppose the impulse response of the channel between the source and relay is

$$\begin{bmatrix} 0.1507 + 0.1048j & 0.0695 + 0.0497j & 0.0394 - 0.0485j \end{bmatrix},$$

where $j = \sqrt{-1}$, and the impulse response of the channel between the relay and destination is

$$\begin{bmatrix} 0.0098 - 0.0687j & -0.0348 + 0.0161j & 0.1239 + 0.0293j \end{bmatrix}.$$

The noise in the channels is supposed to be white, both having a variance of 0.25. The envelopes of both channels are supposed to suffer from rayleigh fading. The power limits of the source input and the relay input are both assumed the same, that is, $p_1 = p_2$. Suppose the values of SNR vary from 1dB $\sim$ 30dB, and for each SNR point we compute the MSE averages of both under the same two schemes as those in the first example over 100 independent channel realizations. Figure 2 shows the simulation result. From the result, it is easily concluded that with the increasing values of SNR, the values of MSE are reduced much more quickly under the optimal joint source-relay power control than that of the uniform-power distribution scheme.

5 Concluding Remarks

The work reported in this paper has demonstrated the potential of applying convex optimization in solving the joint optimization problems for MIMO communication systems. While the initial formulation of the joint source and relay MIMO optimization problem turns out to be nonconvex (thus difficult to solve), we have successfully transformed the problem into an equivalent SOCP when the OFDM modulations are used. The resulting SOCP can be solved with a low computational complexity due to the Cartesian product form the conic constraints. Computer simulation through SeDuMi software shows that our new SOCP design approach has much more superior mean square error performance than the uniform-power distribution control scheme. Furthermore, when the value of SNR increases, the MSE performance improvement over the uniform-power distribution scheme is more pronounced.

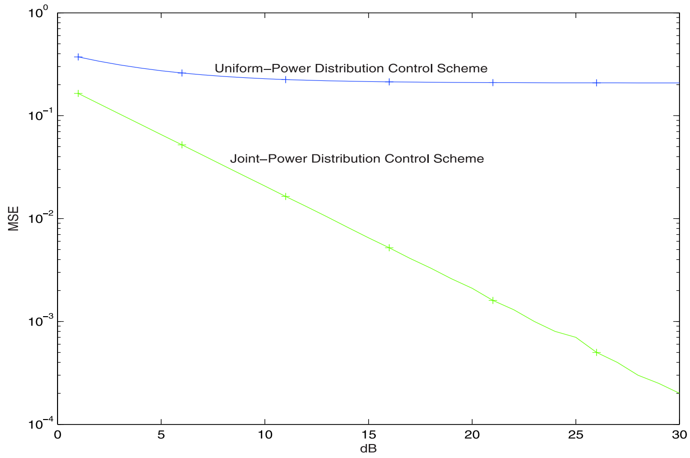

Figure 2: SNR versus MSE

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