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VALUING FUEL-SWITCHING UNITS USING LEAST-SQUARES MONTE CARLO SIMULATION APPROACH*

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Abstract: In this paper, we use Least-Squares Monte Carlo (LSMC) simulation approach to value a fuel-switching unit, which can convert different fuels, such as natural gas, fuel oil, to electricity. The fuel-switching potential between gas and oil is studied based on simulated electricity price, natural gas price and fuel oil price. The simulation results show that fuel-switching capability may increase asset value by 10%, and that asset valuation by LSMC is much more efficient than the lattice model. Moreover we show that the asset value of a generation unit is significantly related to the ability to make a fuel choice and the correlation coefficients between gas price and oil price.

Key words: *electricity generation asset valuation, least-squares Monte Carlo (LSMC) simulation, fuel-switching option (FSO), price uncertainty*

Mathematics Subject Classification: *91B28*

Nomenclature

j : index for fuel ($j = 1, 2$, where "1" represents natural gas and "2" is fuel oil);

t : index for time in hours ($t = 0, \cdots, T$, where T is the number of hours in the planning horizon);

u_t : zero-one generation unit commitment decision variable in time period t;

v_t : zero-one fuel decision variable in time period t (where "1" means the present fuel is natural gas and "0" says the present fuel is fuel oil);

x_t : state variable indicating the unit status in time period t;

t_j^{on} : the minimum number of periods the unit must remain on after it has been turned on using fuel j ($j = 1, 2$);

t_j^{off} : the minimum number of periods the unit must remain off after it has been turned off using fuel j ($j = 1, 2$);

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t_j^{cold} : the minimum number of periods required to cool the unit from shutdown when fuel j $(j = 1, 2)$ is used;

Φ : the state space for the fuel-switching unit
$\{\Phi \equiv (-t_1^{\text{cold}}, \cdots, -t_1^{\text{off}}, \cdots, t_1^{\text{on}} \cup -t_2^{\text{cold}}, \cdots, -t_2^{\text{off}}, \cdots, t_2^{\text{on}})\}$;

i : index for the unit status $(i \in \Phi)$;

q_t : decision variable indicating the amount of power the unit is generating in time period t;

$q_j^{\min}$: minimum rated capacity of the unit using fuel j $(j = 1, 2)$;

$q_j^{\max}$: maximum rated capacity of the unit using fuel j $(j = 1, 2)$;

P_t^E : electricity price (\$/MWh) in time period t;

$P_t^{F_j}$: fuel j $(j = 1, 2)$ price (\$/MMBtu) in time period t;

$H(q)$: heat (MMBtu) required to generate q (MW) of power;

$\pi_t(x_t, v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2})$: profit for operating the unit at output level q_t in time period t when the electricity, fuel 1 and fuel 2 prices are P_t^E, $P_t^{F_1}$ and $P_t^{F_2}$ respectively;

$S_t(u_t, u_{t-1}, v_t)$: startup/shutdown cost when the unit is turned on/off in time period t;

J_t : the asset value of the generation unit in time period t;

$h_t^{1,1}$: the regression function when the unit is on in both time periods t and $t + 1$;

$h_t^{1,0}$: the regression function when the unit is on in time period t and off in time period $t + 1$;

$h_t^{0,1}$: the regression function when the unit is off in time period t and is on in time period $t + 1$;

$h_t^{0,0}$: the regression function when the unit is off in both time periods t and $t + 1$;

$h_t^{0,0,n}$: the regression function when the unit is cold in both time periods t and $t+1$ without switching fuel;

$h_t^{0,0,s}$: the regression function when the unit is cold in both time periods t and $t + 1$ with fuel switched.

1 Introduction

The earliest introduction of market concepts to electric power systems took place in Chile in the late 1970s. The UK government privatized the UK Electricity Supply Industry in 1990. Then the British process was used as a model for the deregulation of several countries, such as Norway, Australia, and New Zealand. The deregulation of the US electric power market in the last few years has replaced the vertical utility by a series of independently operating units with a more horizontal relationship [1]. After the restructuring, the electric utility industry throughout the US has been facing pressure to increase its efficiency, to reduce operational costs, and to lower purchase cost of power equipment [2]. In order to adapt well in the new business environment, generation asset investors must consider market uncertainty in appraising the asset value.

In addition to market uncertainty, the investors must also consider an asset's operating flexibility that enables the unit to respond to changing exogenous economic conditions and provide solutions that enhance profitability. The importance of such *operating options* becomes critical especially when the market environment is highly volatile. For example, when facing exogenous stochastic prices a generator with operating options, such as fuel switching can protect itself against adverse price movements, with the capability of switching into an alternative fuel that may be less affected by the adverse price realizations [3], [4]. There is a rich literature about optimal switching application in economics activities and asset valuation under uncertainty [15], [16], [17], and [18] etc.

In the present competitive environment, a generator will invest only when an adequate return on the investment is expected. Since electricity is non-storable in nature, the generation asset value may be replicated on future spark spread options [5], [6], and [19]. In addition to pure spark spread options, physical constraints such as minimum uptime and downtime constraints may also affect the value. Tseng and Barz [7] proposed a Monte Carlo (MC) simulation to formulate the power plant valuation problem as a multi stage stochastic problem. In addition, discrete-time price trees for correlated price processes for both electricity and fuel, such as geometric mean reverting processes are employed to value a power asset. The computational efficiency of the valuation problem may be improved by using stochastic dynamic programming via a price tree [8]. Although the method produces a satisfactory result for a two-factor case (referring to two uncertainties such as electricity price and gas price), it may not be applicable to a three-factor case if an additional uncertainty must be considered (e.g., for a unit capable of switching fuels). The tree approach is especially prohibitive when the time horizon is long because of the 'curse of dimensionality'. Therefore, valuing a generation asset with fuel-switching option (considering three price uncertainties) is a challenging task. In this paper, we explore the Least-Squares Monte Carlo (LSMC) approach and use it to value the fuel-switching option.

This paper is organized as follows. In Section 2, we provide an overview of the fuel-switching unit including physical constraints and fuel-switching options. We then model the generation asset valuation problem using LSMC in Section 3. We present numerical results in Section 4 and conclude this paper in Section 5.

2 Problem Description

2.1 Overview of Fuel-switching Units

A thermal generation unit with fuel-switching options can use two different fuels to generate electricity, and can be quickly switched from one fuel to the other. The operation of the unit involves three commodities with different market prices. In this paper, the following conditions are implicitly assumed: (1) fuel switching does not affect the normal operation of a unit; (2) fuel switching can be finished within a reasonable time; (3) fuel-switching cost can be viewed as a constant; (4) a fuel-switching plant cannot use two fuels simultaneously. Under these conditions, the fuel-switching options can be implemented easily in power plants and provide the following advantages:

- Fuel-switching capabilities may help stabilizing the operation of a unit under limited resources available to a country or region;

- Fuel-switching options may help saving the operation cost of a unit in the long term;
- Fuel-switching options may help solving some pollution issues, such as reducing emissions of greenhouse gases.

In the power industry, players may prefer natural gas to fuel oil because natural gas is much 'cleaner' than fuel oil under environmental restrictions on emission of greenhouse gases. Therefore, fuel switching has been considered as a method of emission abatement. On the other hand, although natural gas prices are much more expensive than fuel oil prices based on cost per BTU and so power producers may have to react to the soaring price of natural gas by switching to cheaper, more environmentally harmful fuel sources.

To reduce power producers' exposure to price volatility or possible supply disruptions in the present deregulated environment, industrial users are expected to increasingly seek the flexibility of switching fuels using hybrid technologies, especially in the U.S., duel fuel units consist of 14% of the total generation capacity [14].

In the current energy market, oil and gas prices are correlated and oil-to-gas competition begins to set a potential opportunity for mutual fuel switching (gas-to-oil or oil-to-gas). When technical performance for hourly fuel switching is proven and readily available, a fuel switching unit may become a viable solution to this problem.

2.2 Fuel-switching Options

Generally speaking, fuel-switching options for a thermal unit refers to the ability to burn alternate fuels, such as natural gas or fuel oil. Switching occurs when one fuel out-of-the-money is replaced by another in-the-money (or less out-of-the-money) from the view of economy.

Since a fuel-switching unit can operate using different fuels, the cost characteristics of the unit depend on the fuel used. Different fuel may have a different amount of MMBtu required to produce a MW [11], [12]. This property differs from unit to unit and fuel to fuel. The value of a fuel-switching unit is affected by the operational constraints such as the minimum up/down time constraints.

In reality, fuel switching is a transitional process, which may take from minutes to hours dependent upon the unit. In this paper, we assume that the fuel switching can be done instantaneously under other conditions to be addressed next.

There are normally two ways to switch fuels: on-line switching and off-line switching. In on-line switching, the switch takes place when the unit has already been on-line for at least its minimum uptime periods (i.e., state t_1^{on}). During the switching, it transits to the shutdown status using another fuel. For off-line switching, the unit must be off-line while the fuel switching takes place. Therefore, the state transits (from t_1^{cold} to t_2^{cold} or from t_2^{cold} to t_1^{cold}). Fig. 1 illustrates the state transitions of both on-line and off-line switching. In this paper, we focus on the off-line switching model.

In this paper we concentrate on the economic rather than technical issues in fuel switching: how to employ fuel-switching options to minimize production costs, reduce market risks and make more profit in the short term. In other words, we ignore the impacts from technical improvement, environmental restrictions, resource availability etc.

2.3 Profit Function of a Fuel-switching Unit

In this paper, the input-output characteristic of a generating unit is captured by $H(q)$ (MMBtu), which is a function describing the fuel required to generate q (MW) of power

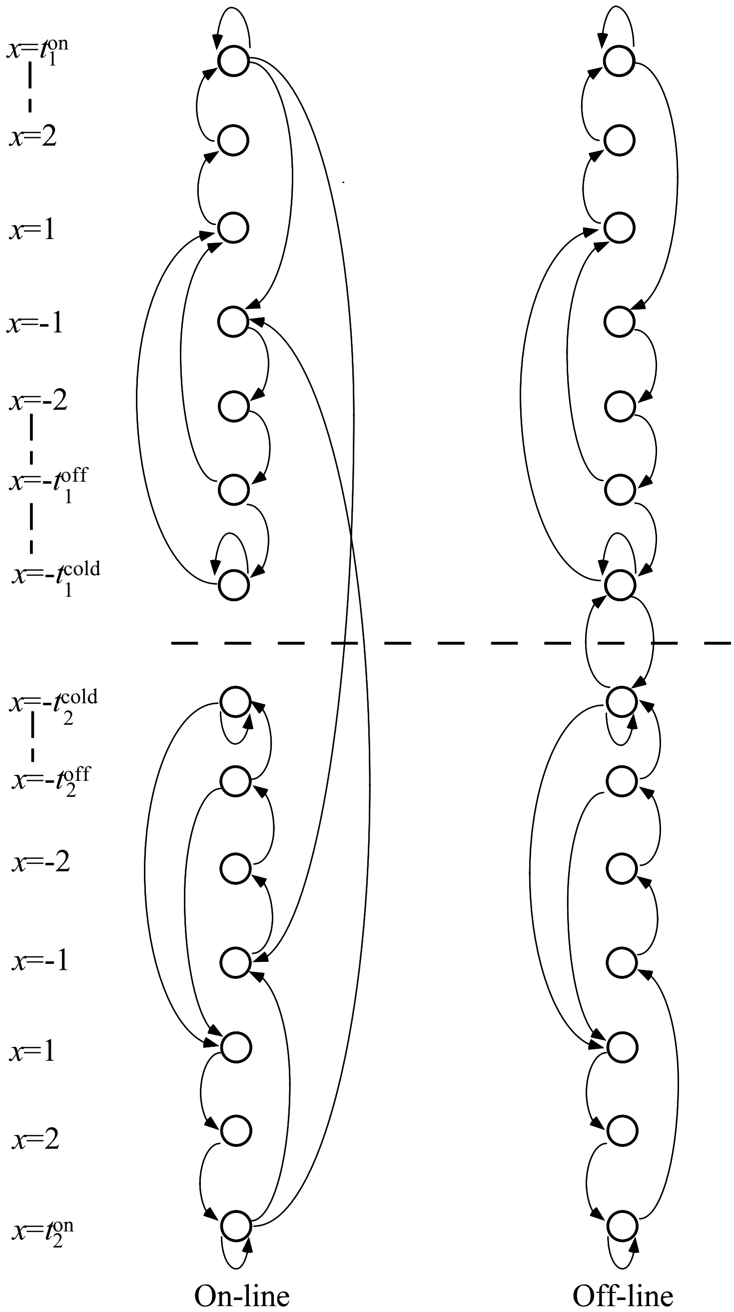

Figure 1: A state transition diagram between two fuels (on-line and off-line).

[13]. Now this function is dependent on the fuel type used, denoted by $H_j(q)$ for fuel j. The profit at time t may be represented as follows:

$$\pi_t(x_t, v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2})$$

$$= \begin{cases} P_t^E q_t - H_1(q_t) P_t^{F_1} v_t - H_2(q_t) P_t^{F_2}(1 - v_t), & \text{if } x_t > 0, \\ 0, & \text{otherwise.} \end{cases} \tag{2.1}$$

where q_t is the generation level at time t and we assume that q_t is dispatched *instantaneously* and *optimally* after the prices $(P_t^E, P_t^{F_1}, P_t^{F_2})$ are revealed.

2.4 Operational Constraints of a Fuel-switching Unit

The operational constraints are formulated as follows.

- Minimum up/downtime constraints:

$$u_t = \begin{cases} 1, & \text{if } 1 \le x_t < t_1^{\text{on}} v_t + t_2^{\text{on}}(1 - v_t), \\ 0, & \text{if } -t_1^{\text{off}} v_t - t_2^{\text{off}}(1 - v_t) < x_t \le -1 \\ 0 \text{ or } 1, & \text{otherwise.} \end{cases} \tag{2.2}$$

- Switch constraints:

$$v_t = \begin{cases} 1 - v_{t-1} \text{ or } v_{t-1}, & \text{if } x_t = t_1^{\text{cold}} \text{ or } t_2^{\text{cold}} \\ v_{t-1}, & \text{otherwise.} \end{cases} \tag{2.3}$$

  switch happens only when the unit is *cold* at time t.

- State transition constraints:

$$x_t = \begin{cases} \min(t^{\text{on}}, \max(x_{t-1}, 0) + 1), & \text{if } u_t = 1, \\ \max(-t^{\text{cold}}, \min(x_{t-1}, 0) - 1), & \text{if } u_t = 0 \end{cases} \tag{2.4}$$

  where

$$t^{\text{on}} = t_1^{\text{on}} v_t + t_2^{\text{on}}(1 - v_t) \tag{2.5}$$

$$t^{\text{off}} = t_1^{\text{off}} v_t + t_2^{\text{off}}(1 - v_t) \tag{2.6}$$

$$t^{\text{cold}} = t_1^{\text{cold}} v_t + t_2^{\text{cold}}(1 - v_t) \tag{2.7}$$

- Unit capacity constraints:

$$u_t(q_1^{\min} v_t + q_2^{\min}(1 - v_t)) \le q_t \le u_t(q_1^{\max} v_t + q_2^{\max}(1 - v_t)) \tag{2.8}$$

- Startup/shutdown costs:

$$S_t(u_t, u_{t-1}, v_t) = \begin{cases} S_1^u v_t + S_2^u(1 - v_t), & \text{if } u_t = 1 \text{ and } u_{t-1} = 0, \\ S_1^d v_t + S_2^d(1 - v_t), & \text{if } u_t = 0 \text{ and } u_{t-1} = 1, \\ 0, & \text{otherwise.} \end{cases} \tag{2.9}$$

  where S_1^u and S_2^u represent constant startup costs for fuel 1 and fuel 2; S_1^d and S_2^d are constant shutdown costs for fuel 1 and fuel 2.

The operational constraints including the switch constraints have been fully captured in the state transition diagram in Fig. 1.

3 The Least-Squares Monte Carlo Approach

The method to be introduced in this section can be viewed as an extension of the method proposed in [9] for valuing American-style options. We extend their approach to a more complicated situation involving multi-stage decision making and fuel-switching options.

Let $J_t(x_t, u_t, v_t, q_t^*; P_t^E, P_t^{F_1}, P_t^{F_2})$ be the so-called value-to-go function indicating the total value of the unit for the remaining period at state x_t at time t, and assume a finite time horizon $[0, T]$. The asset valuing problem can be formulated as the following recursive relation:

$$J_t(x_t, u_t, v_t, q_t^*; P_t^E, P_t^{F_1}, P_t^{F_2}) = \pi_t(x_t, v_t, q_t^*; P_t^E, P_t^{F_1}, P_t^{F_2}) + \max_{u_t, v_t}\{E_t[J_{t+1}(x_{t+1}, u_{t+1}, v_{t+1}, q_{t+1}; P_{t+1}^E, P_{t+1}^{F_1}, P_{t+1}^{F_2})] - S_t(u_t, u_{t-1}, v_t)\} \quad (3.1)$$

where q_t^* represents the optimal generation level in time t within the capacity range $[q^{\min}, q^{\max}]$; E_t denotes the expectation operator given the price information available at time t.

3.1 Solution Procedure

From (3.1), it can be seen at time t to make an optimal commitment decision, one must know $E_t[J_{t+1}(x_{t+1}, u_{t+1}, v_{t+1}, q_{t+1}; P_{t+1}^E, P_{t+1}^{F_1}, P_{t+1}^{F_2})]$, which implicitly is a function of current price information, P_t^E, $P_t^{F_1}$, and $P_t^{F_2}$. The main idea of the LSMC method is to approximate such a function by regression. In terms of the states and the switching option, different functions are to be approximated using regression. They are defined below.

If $x_t = x_{t+1} = t^{\text{on}}$,

$$h_t^{1,1}(v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2}) = E_t[J_{t+1}(t^{\text{on}}, u_{t+1}, v_{t+1}, q_{t+1}; P_{t+1}^E, P_{t+1}^{F_1}, P_{t+1}^{F_2})] \quad (3.2a)$$

If $x_t = t^{\text{on}}$ and $x_{t+1} = -1$,

$$h_t^{1,0}(v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2}) = E_t[J_{t+1}(-1, u_{t+1}, v_{t+1}, q_{t+1}; P_{t+1}^E, P_{t+1}^{F_1}, P_{t+1}^{F_2})] \quad (3.2b)$$

If $-t^{\text{cold}} \le x_t \le -t^{\text{off}}$ and $x_{t=1} = 1$,

$$h_t^{0,1}(v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2}) = E_t[J_{t+1}(1, u_{t+1}, v_{t+1}, q_{t+1}; P_{t+1}^E, P_{t+1}^{F_1}, P_{t+1}^{F_2})] \quad (3.2c)$$

If $-t^{\text{cold}} < x_t \le -t^{\text{off}}$ and $x_{t+1} < 0$,

$$h_t^{0,0}(v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2}) = E_t[J_{t+1}(x_t - 1, u_{t+1}, v_{t+1}, q_{t+1}; P_{t+1}^E, P_{t+1}^{F_1}, P_{t+1}^{F_2})] \quad (3.2d)$$

If $x_t = x_{t+1} = -t^{\text{cold}}$,

$$h_t^{0,0,n}(v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2}) = E_t[J_{t+1}(-t^{\text{cold}}, u_{t+1}, v_{t+1}, q_{t+1}; P_{t+1}^E, P_{t+1}^{F_1}, P_{t+1}^{F_2})] \quad (3.2e)$$

If $x_t = -t^{\text{cold}}$ and $x_{t+1} = -(t_1^{\text{cold}}(1 - v_t) + t_2^{\text{cold}} v_t)$,

$$h_t^{0,0,s}(v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2}) = E_t[J_{t+1}(x_{t+1}, u_{t+1}, v_{t+1}, q_{t+1}; P_{t+1}^E, P_{t+1}^{F_1}, P_{t+1}^{F_2})] \quad (3.2f)$$

If the above regression functions $h_t(v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2})$ are available at time t, one could know the expected unit value for the next time period when the uncertain prices $(P_t^E, P_t^{F_1}, P_t^{F_2})$ are revealed. Then, one could also know how to make optimal decisions at t. Especially when the unit stay at the *cold* state and $h_t^{0,0,n} < h_t^{0,0,s}$, fuel switching may happen at this time period. Since analytical forms of $h_t(\cdot)$ are nonexistent in general, we will use numerical methods based on Monte Carlo simulation to approximate $h_t(\cdot)$. Through simulating a set of random variables, the expected value yields the least square error. Therefore, to approximate the above expected function, we generate N data samples of prices based on the mean reverting uncertainty model (3.4). Thus, the expected value of $J_{t+1}(x_{t+1}, u_{t+1}, v_{t+1}, q_{t+1}; P_{t+1}^E, P_{t+1}^{F_1}, P_{t+1}^{F_2})$ can be approximated by the function that best regressions J_{t+1} on the data of price $(P_t^E, P_t^{F_1}, P_t^{F_2})$ and possible decision values of (u_t, v_t, q_t). That means any realization of (u_t, v_t, q_t) at time t, one would know how to optimally make decisions for the next time period based on the above regression functions $h_t(v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2})$.

The expected value of $J_{t+1}(\cdot)$ can be approximated though *forward-moving* simulation and *backward-moving* dynamic programming iterations from $T-1$ to $T-2, \cdots, 0$. Thus, the asset value can be obtained by $J_0(x_0, u_0, v_0, q_0; P_0^E, P_0^{F_1}, P_0^{F_2})$, where $(x_0, u_0, v_0, q_0; P_0^E, P_0^{F_1}, P_0^{F_2})$ are determined by the initial conditions of the unit. As to J_T, it is determined by the boundary conditions as follows.

3.2 Boundary Conditions

At time T, there is no commitment decision to make, because the power plant value is only conditioned on the state x_T as follows:

$$J_T(x_T, u_t, v_T, q_T^*; P_T^E, P_T^{F_1}, P_T^{F_2}) = \begin{cases} P_T^E q_T^* - H_1(q_T^*) P_T^{F_1} v_T - H_2(q_T^*) P_T^{F_2}(1 - v_T), & \text{if } x_T > 0, \\ 0, & \text{otherwise.} \end{cases} \tag{3.3}$$

where q_T^* represents the optimal generation level in time T within the capacity range $[q_j^{\min}, q_j^{\max}]$ for fuel j, which is determined by v_T.

At time $T-1$, for the remaining two time periods, one can use the LSMC to estimate $J_{T-1}(x_T, u_{T-1}, v_{T-1}, q_{T-1}; P_{T-1}^E, P_{T-1}^{F_1}, P_{T-1}^{F_2})$ for the data samples. The decision maker can choose whether to switch fuel immediately or not and revisit the exercise decision at the next time period when the unit is at *cold* state. The value of $J_{T-1}(\cdot)$ is maximized path-wise, and hence the value of fuel-switching option is greater than or equal to 0 unconditionally. This procedure can be repeated for time T-2,$\cdots$, 0. The last iteration, starting with the initial conditions at time 0, provides the optimal planning and asset estimation for the whole time horizon.

3.3 Price Processes

Mean reversion refers to how likely it is for the short-term interest rate to be pulled back, over time, toward its mean value: mean reverting processes are widely applied to finance and energy commodities. For example, the oil price tends to revert back to a 'normal' long-term equilibrium level, although it has significant short-term oscillations. In this paper, we assume that price of electricity P_t^E, price of fuel 1 $P_t^{F_1}$, and price of fuel 2 $P_t^{F_2}$ are all functions of y_1, y_2, and y_3, respectively, which are governed by the following (mean-reverting) stochastic differential equations [7], [8].

$$dy_l = -\mu_l[y_l(t) - m_{t,l}(t)]dt + \sigma_l dB_l, \tag{3.4}$$

where l=1, 2, and 3 represents electricity, fuel 1, and fuel 2 respectively; μ_l is a drift function; σ_l is a constant volatility and B_l is a Wiener process with correlation ρ_{lm} (l,m=1, 2, 3).

There exists a one-to-one transformation between y_1 and P_t^E, between y_2 and $P_t^{F_1}$, and between y_3 and $P_t^{F_2}$. Therefore, $(P_t^E, P_t^{F_1}, P_t^{F_2})$ can be obtained through the corresponding (y_1, y_2, y_3), and vice versa.

For the purpose of carrying out the simulation process, the time horizon of fuel-switching options is divided into T subintervals of length Δt (=1 hour). The discrete version of the process for y_l is

$$\Delta y_l = -\mu_l(y_l - m_{t,l})\Delta t + \sigma_l \epsilon_l \sqrt{\Delta t} \tag{3.5}$$

where Δy_l is the change in y_l in time Δt; ϵ_l is a random sample from a standardized normal distribution. The coefficient of correlation between ϵ_l and ϵ_m is ρ_{lm} for $1 \leq l, m \leq 3$. One simulation trial involves obtaining T samples of the $\epsilon_l (1 \leq l \leq 3)$ from a multivariate standard normal distribution. These are substituted into equation (3.5) to produce simulated paths for each y_l and enable a sample value for the real option to be calculated.

In (3.5), to generate three correlated normal random variables, ϵ_1, ϵ_2, and ϵ_3, first we generate three mutually independent standard normal random variables, Z_1, Z_2, and Z_3. We then consider the following linear transformation:

$$\epsilon_l = \sum_{k=1}^{l} \alpha_{lk} Z_k \qquad (l = 1, 2, 3) \tag{3.6}$$

To meet the variances and covariances of $\epsilon_l (l = 1, 2, 3)$, the following two constraints are imposed.

$$\sum_k \alpha_{lk}^2 = 1 \tag{3.7}$$

and

$$\sum_k \alpha_{lk} \alpha_{mk} = \rho_{lm} \tag{3.8}$$

The first sample ϵ_1 is set equal to Z_1. Then the above equations can be solved, ϵ_2 is calculated from Z_1 and Z_2, ϵ_3 is calculated from Z_1, Z_2 and Z_3.

From (3.6), we have

$$\begin{aligned} \epsilon_1 &= \alpha_{11} Z_1, \\ \epsilon_2 &= \alpha_{21} Z_1 + \alpha_{22} Z_2, \\ \epsilon_3 &= \alpha_{31} Z_1 + \alpha_{32} Z_2 + \alpha_{33} Z_3. \end{aligned} \tag{3.9}$$

In matrix form, it is

$$\epsilon = \alpha Z \tag{3.10}$$

where $\epsilon = (\epsilon_1, \epsilon_2, \epsilon_3)^T$, $Z = (Z_1, Z_2, Z_3)^T$ and

$$\alpha = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix}$$

Then, according to (3.7) and (3.8), we obtain

$$\alpha = \begin{pmatrix} 1 & 0 & 0 \\ \rho_{12} & \sqrt{1-\rho_{12}^2} & 0 \\ \rho_{13} & \frac{\rho_{23}-\rho_{12}\rho_{13}}{\sqrt{1-\rho_{12}^2}} & \sqrt{1 - \frac{\rho_{13}^2 - 2\rho_{12}\rho_{13}\rho_{23} + \rho_{23}^2}{1-\rho_{12}^2}} \end{pmatrix} \tag{3.11}$$

Define new uncertainty variables z_l, $l = 1, 2, 3$ that satisfy the following linear relation.

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \alpha^{-1} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \tag{3.12}$$

It can be shown that z_l, $l = 1, 2, 3$ have been 'decoupled'. That is, given a realization of z_l, $l = 1, 2, 3$ at time t, their increments dz_l, $l = 1, 2, 3$ are uncorrelated. We generate sample path data (over time) for z_l, $l = 1, 2, 3$ first, which are then converted to y_l, $l = 1, 2, 3$, and then P_t^E, $P_t^{F_1}$, and $P_t^{F_2}$ to evaluate the payoffs. This procedure preserves the correlation among the price data that is critical to the value of the generation assets.

3.4 Algorithm Development

Now, we can use the above regression functions and boundary conditions to value the fuel-switching unit through backward dynamic programming based on the pre-generated price database. The detailed algorithm is as follows:

Data: Initial conditions $(x_0, u_0, v_0, q, 0, P_0^E, P_0^{F_1}, P_0^{F_2})$ are given, and data set size $N > 0$ is also given.

Step 0: Set $t \leftarrow T-1$, $k \leftarrow 1$, $j \leftarrow 1$, $i \equiv x_t \leftarrow t_j^{\text{on}}$, $J_T(x_T, u_T, v_T, q_T^*; P_T^E, P_T^{F_1}, P_T^{F_2})$ get from (3.3).

Step 1: Obtain a set of sample prices $(P_t^{E(k)}, P_t^{F_1(k)}, P_t^{F_2(k)})$.

Step 2: Regress $J_{t+1}^{(k)}$ on $(P_t^{E(k)}, P_t^{F_1(k)}, P_t^{F_2(k)})$ to obtain $h_t(\cdot)$.

Step 3: If $x_t = t_j^{\text{on}}$,
$J_t^{(i,j,k)} \leftarrow \pi_t(x_t, v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2}) + \max(h_t^{1,1}, h_t^{1,0} - S_t)$;
else if $0 < x_t < t_j^{\text{on}}$,
$J_t^{(i,j,k)} \leftarrow \pi_t(x_t, v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2}) + J_{t+1}^{(i+1,j,k)}$;
else if $x_t = 0$,
$J_t^{(i,j,k)} \leftarrow -\infty$;
else if $-t_j^{\text{off}} < x_t < 0$,
$J_t^{(i,j,k)} \leftarrow J_{t+1}^{(i-1,j,k)}$;
else if $-t_j^{\text{cold}} < x_t \le -t_j^{\text{off}}$,
$J_t^{(i,j,k)} \leftarrow \max(h_t^{0,0}, h_t^{0,1} - S_t)$;
else if $x_t = -t_j^{\text{cold}}$,
$J_t^{(i,j,k)} \leftarrow \max(h_t^{0,0,n}, h_t^{0,0,s}, h_t^{0,1} - S_t)$.

Step 4: If $i \ge -t_j^{\text{cold}}$, $i \leftarrow i-1$, go to Step 3.

Step 5: If $j < 2$, $j \leftarrow j+1$, $i \leftarrow t_j^{\text{on}}$, go to Step 3.

Step 6: If $t > 0$, $t \leftarrow t-1$, $j \leftarrow 1$, $i \leftarrow t_j^{\text{on}}$, go to Step 1.

Step 7: Stop.

Note that the fuel switching will happen when $x_t = -t^{\text{cold}}$, and $h_t^{(0,0,n)} < h_t^{(0,0,s)}$. In other words, when the unit is off-line and under the *cold* status, the operator will determine to switch fuel to make more money based on the condition of $h_t^{(0,0,n)} < h_t^{(0,0,s)}$. Now, according to the initial unit status (x_0, u_0, v_0, q_0), and initial prices $(P_0^E, P_0^{F_1}, P_0^{F_2})$, the expected value of the fuel-switching unit is

$$J_0(x_0, u_0, v_0, q_0; P_0^E, P_0^{F_1}, P_0^{F_2}) \approx \sum_{k=1}^{N} J_0^{(k)}/N \tag{3.13}$$

In the above algorithm, the most difficult part is to obtain the regression function $h_t(v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2})$. What remains to show is an appropriate functional form for $h_t(\cdot)$. Our experience shows that the following polynomial form works well for the regression.

$$\begin{aligned} h_t(v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2}) \approx\ & a_1 + a_2 q_t + a_3 q_t^2 + a_4 q_t^3 + a_5 q^* q_t + a_6 P_t^E + a_7 P_t^F \\ & + a_8 q^* P_t^F + a_9 (P_t^E)^2 / P_t^F \end{aligned} \tag{3.14}$$

where

$$\begin{aligned} q^* =\ & \max(q_1^{\min}, \min(q_1^{\max}, (P_t^E/P_t^{F_1} - c_{1,1})/2c_{2,1}))v_t \\ & + \max(q_2^{\min}, \min(q_2^{\max}, (P_t^E/P_t^{F_2} - c_{1,2})/2c_{2,2}))(1 - v_t) \end{aligned} \tag{3.15}$$

q^* is determined by the optimal dispatch rule and the present fuel, $P_t^F = P_t^{F_1} v_t + P_t^{F_2}(1 - v_t)$, and a_1 to a_9 are the parameters to be fitted in the regression. Here we assume that the heat rate function of the unit follows the quadratic function given in (4.1a) and $c_{1,j}$ and $c_{2,j}$ are the coefficients for different fuel j. In the numerical results for the example described in the next section we find a R^2 value of around 0.83, which means the regression finds a good fit between the polynomial form above and the data.

Although $h_t(v_t, q_t; P_t^E, P_t^{F_1}, P_t^{F_2})$ is a highly nonlinear function in (v_t, q_t) and $(P_t^E, P_t^{F_1}, P_t^{F_2})$, and it may not be smooth everywhere because it involves q^*, $h_t(\cdot)$ is still a linear function of the regression parameters $a_1, \cdots, a_9$, which can be figured out efficiently by solving a system of linear equations (a 9×9 linear system in this case.)

4 Numerical Results

To value a fuel-switching capable unit, we compare the following two examples: the same unit with and without fuel-switching options.

4.1 Baseline: a Non-switching Case

Without considering the fuel-switching option, the valuation problem only involves two uncertainties and can be solved using a two-factor lattice method [8]. This serves as a baseline case, by which we can calibrate the performance of the LSMC method.

Consider a natural gas-fired generating unit with the following input-output characteristics.

$$H_1(q_t) = c_{0,1} + c_{1,1} q_t + c_{2,1} q_t^2 \tag{4.1a}$$

$$C_1(q_t) = H_1(q_t) \times P_t^{F_1} \tag{4.1b}$$

where the cost function $C_1(\cdot)$ is assumed to be a quadratic function of q_t, $H_1(\cdot)$ is the heat rate function, $P_t^{F_1}$ is the fuel price at time period t. Assume $P_0^{F_1}$ is \$2.2/MMBtu, P_0^E is \$20/MW, and $q_1^{\min} = 225$MW, $q_1^{\max} = 700$MW, $c_{0,1} = 540$, $c_{1,1} = 9.223$, and $c_{2,1} = 0.00234$. We also assume that $t_1^{\text{on}} = 5$, and $t_1^{\text{off}} = t_1^{\text{cold}} = 10$ to fully capture the influence of the physical constraints. Let the startup cost be \$2300 and shutdown cost be \$1000. Hourly

Table 1: Mean reverting process coefficients of electricity, gas and oil

Coefficients	σ	m_u	m_t	P_0
Electricity	0.27	0.072	2.878	20
Gas	0.24762	0.010570	1.01945	2.2
Oil	0.10209	0.003704	0.48054	0.58

Table 2: Mean log Price of electricity: $m_t(t)$

t	1	2	3	4	5	6	7	8
$m_t(t)$	1.8874	2.6557	1.9348	2.3402	3.5027	3.8568	3.7583	4.6602
t	9	10	11	12	13	14	15	16
$m_t(t)$	4.8613	4.71	5.8114	4.7363	5.044	5.7383	5.9166	4.7126
t	17	18	19	20	21	22	23	24
$m_t(t)$	3.7233	1.4573	1.322	2.5106	3.6167	0.6446	1.6033	1.8328

electricity prices and gas prices are generated by two mean reverting processes following (3.4). The corresponding coefficients in Table 1 are realistic in that they are based on the authors' industrial experience within PJM and can also be found in reference [7]. The correlation coefficient between electricity and gas is 0.078744. And electricity hourly prices also follow a certain on-peak vs. off-peak pattern as Table 2.

To compare the algorithm performance of LSMC with the lattice method in [7], seven cases are tested corresponding to seven operating periods using the same gas-fueled generating unit, ranging from 1 day (24 hours) to 7 days (168 hours). The average CPU times are recorded in Table 3.

Fig. 2 shows that the LSMC method is much more efficient than the lattice method. However, the LSMC method obtains 1 to 1.9% lower value than that from the lattice method in Fig. 3, which can be further reduced by adding more subintervals among each hour with the increase of CPU time. This discrepancy increases as the time horizon increases. This is because the regression function is only an approximation, not an exact fit. So there are errors in assessing the asset values. In addition, the errors may add up during the forward and backward iterations. Nevertheless, this test serves as a calibration of the LSMC method for valuing a generation asset.

4.2 A Fuel-switching Unit

To value a fuel-switching unit, an additional (fuel) price uncertainty must be considered. As mentioned previously, the lattice approach is prohibitive because of huge memory space and the corresponding large CPU time required.

Table 3: Average CPU time (seconds)

T (hours)	24	48	72	96	120	144	168
Lattice	0.78	1.64	2.83	4.22	5.86	7.86	10.09
LSMC	0.23	0.47	0.69	0.94	1.16	1.38	1.61

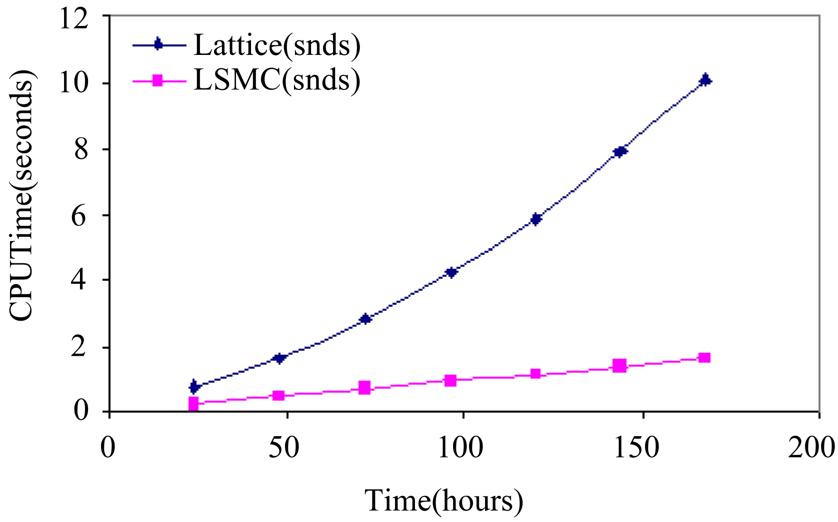

Figure 2: CPU time of the lattice model and the LSMC approach.

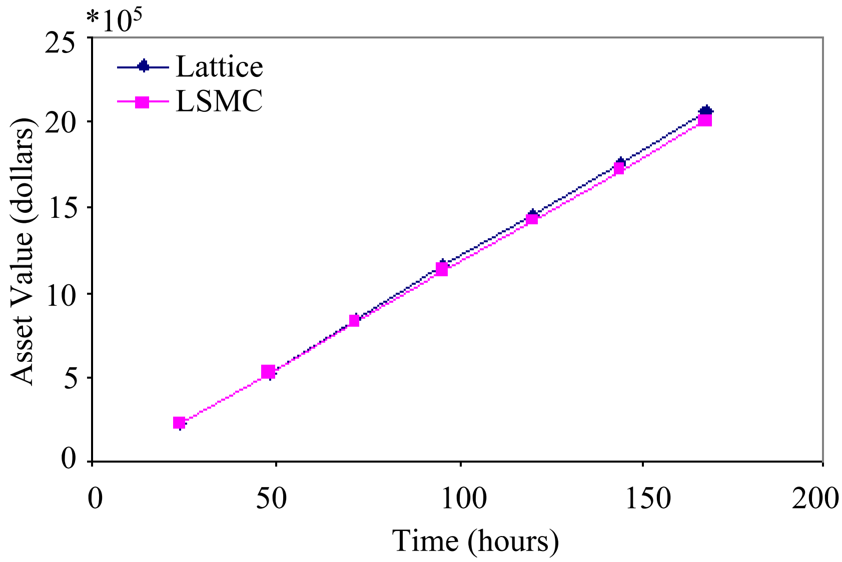

Figure 3: The asset value of the lattice model and the LSMC approach.

Table 4: Asset value (dollars)

T (hours)	24	48	72	96	120	144	168
Lattice (w/o FSO)	221343	525641	835970	1145564	1453553	1759987	2065077
LSMC (w/o FSO)	219318	520810	826594	1130352	1431635	1730252	2025840
LSMC (w FSO)	229492	547244	873881	1202886	1535541	1868028	2201012

Table 5: Asset Value (dollars)

ρ	0.0	0.1	0.2	0.3	0.4	0.5
Asset value	2208745	2202111	2195412	2188820	2182207	2175539

For simplicity, we assume that when the generating unit is fired by oil, the input-output characteristics remain the same as in (4.1a) and (4.1b), although in reality the coefficients of the quadratic function in (4.1a) should vary. That is,

$$H_1(q_t) = H_2(q_t) \tag{4.2}$$

and

$$C_2(q_t) = H_2(q_t) \times P_t^{F_2} \tag{4.3}$$

Hourly electricity prices, gas prices and oil prices are generated by three mean reverting process following (3.4). The correlation coefficient between electricity and oil is 0.033024 and the correlation between gas and oil is 0.19704.

In Fig. 4 and Table 4, it can be seen that the fuel-switching capability can increase the asset value. There exist a 7% (when T=168) additional value due to the fuel switching option, comparing with the baseline using the lattice method. Since the LSMC approach underestimates the asset value (2%) in the baseline case (compared with the lattice method), it may also underestimate the asset value with the fuel-switching options. If this argument is true, than the fuel switching option may increase the asset value as high as 10%. To verify this conjecture, we build a three-factor lattice (with a small T, $T = 12$). We compare the asset values using both the 3-factor lattice method and the LSMC method, the result of lattice method is 1% higher than that of the LSMC method. Since the error increases over T, it is fair to estimate that the error in the 1-week case ($T = 168$), the error (underestimate) is at least 2%.

4.3 Sensitivity Analysis on $\rho_{gas,oil}$

Intuitively the value of the fuel-switching option increases as the correlation between the fuel prices decreases. That means more chances for one fuel to be in-the-money and the other out-of-the-money.

To study the impacts of fuel price correlation on the option value, we take the same fuel-switching unit and repeatedly run the program with different ρ between gas and oil prices. The following results for one-week case (T=168) are obtained.

It can be seen that the asset value decreases linearly as the correlation coefficient $\rho_{gas,oil}$ increases. For an example, under an extreme case, the correlation between gas and oil price

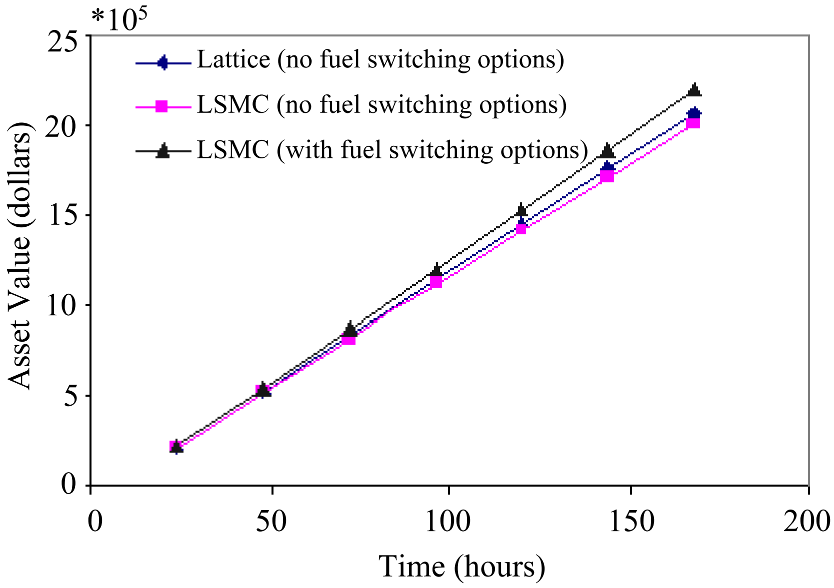

Figure 4: The asset value with and without fuel-switching options.

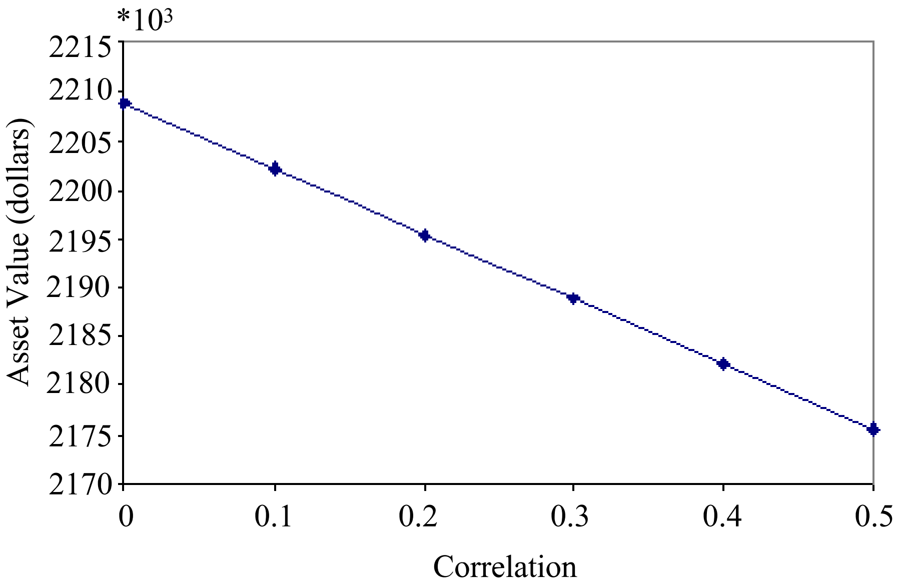

Figure 5: The asset value vs. $\rho_{gas,oil}$.

is -1. That is the increase of gas price will cause the decrease of oil price correspondingly. Under this condition, the power producer will make decisions in switching fuel from natural gas to fuel oil to cut down fuel cost, so as to increase the asset value of the fuel-switching unit. In the present energy market, changes in oil prices in this country are almost irrelevant to natural gas. Our recent analysis based on futures price data, indicates that the price correlation between the oil and gas is less than 0.1, which means the fuel-switching units are still in great need in the current power market, even before considering their value for emission abatement.

5 Conclusions

In this paper, we use the LSMC method to value the fuel-switching option of a generation asset considering the operational constraints. We estimate that the option value can be very significant (at about 10% over a one-week period in the example we considered). It provides another evidence that operational flexibility should not be overlooked when valuing an generation asset. In this sense, fuel-switching options is one of the most important factors in determining electricity generation asset value. The value of fuel-switching option is affected by the correlation coefficient between natural gas and fuel oil prices to some extent.

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