Yokohama Publishers
ISSN 1349-8169 ONLINE JOURNAL

Pacific Journal of Optimization
Vol. 3, No. 3, pp. 439–458, September 2007

OPERATION OF DISTRIBUTED GENERATION UNDER STOCHASTIC PRICES*

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Abstract: The deregulation of electricity industries provides incentives for microgrids, entities that use small-scale distributed generation (DG) to meet local energy loads, to evolve independently of the centralised grid. We examine the impact of start-up costs on the operating costs and policies of DG given stochastic electricity and fuel prices by formulating a stochastic dynamic programme for the microgrid, which minimises its expected discounted cost, and solving it using least-squares Monte Carlo simulation. The microgrid's expected cost saving from using gas-fired DG relative to meeting its electric load via off-site purchases is the implied DG option value. Numerical examples indicate that although start-up costs do not significantly increase operating costs, they have a profound impact on the optimal DG operating schedule as the microgrid must incorporate not only current, but also future, expected start-up costs into its decisions. Consequently, the microgrid hesitates to turn DG units on, preferring to wait until the electricity price exceeds the natural gas generating cost by a significant margin. We demonstrate that ignoring this tradeoff results in drastically higher expected costs and fewer self-generation opportunities. Hence, optimal control policies are crucial as the loss in value from operating DG sub-optimally may deter the formation of microgrids.

Key words: *distributed generation, stochastic dynamic programming, real options, simulation*

Mathematics Subject Classification: *68M99, 90C39, 91B28*

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1 Introduction

The first major reorganisation in over a century of the familiar power system may be beginning. In the emerging paradigm, a significant fraction of energy conversion from primary fuels to electricity takes place closer to loads, i.e., as distributed generation (DG). By comparison, the current power system is characterised by large-scale centralised power generation, transmission at high voltage over long distances, and final delivery to customer sites via the low-voltage distribution network (see [18]). Under the emerging paradigm, the traditional centralised grid (or *macrogrid*) still delivers large quantities of energy to end-users, but electricity supply is augmented by new local entities employing DG and enjoying some measure

*This work described in this paper was funded by the Assistant Secretary of Energy Efficiency and Renewable Energy, former Distributed Energy Program of the US Department of Energy under Contract DE-AC02-05CH11231. Prior contributions to microgrid research by many colleagues at Ernest Orlando Lawrence Berkeley National Laboratory (Berkeley Lab) and within the Consortium for Electric Reliability Technology Solutions (CERTS) have provided much of the inspiration for this paper. We are also grateful for the feedback from the attendees of the 2006 International Association for Energy Economics International Conference in Potsdam, Germany (7-10 June 2006). The feedback from two anonymous referees has greatly improved this paper. All remaining errors are the authors' own.

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of control independence from the traditional grid, i.e., *microgrids* (see [1], [2], [11], [15], and [16]).

Microgrids promise four major benefits internal to its participants, namely: 1. the possibility of self-generating electricity at a cost below the delivered macrogrid cost; 2. the application of combined heat and power (CHP) technology; 3. the opportunity to tailor the quality of power delivered to suit the requirements of end-uses, here called heterogeneous power quality and reliability (PQR) (see [13]); and 4. the more favourable environment potentially created for energy efficiency and small-scale renewable generation investments.

Many other potential societal benefits of DG have been suggested (more detail on these benefit streams can be found in [10] and [14]), although this paper addresses only the first benefit. It is instructive to expand on all the four outlined above, which should motivate the importance of the line of research pursued in this paper:

1. Self-generation must beat only the delivered retail electricity price, which is typically two to three times the average wholesale price, against which central-station generators compete; that is, DG can compete successfully against grid power even though diseconomies of small-scale are likely to be strong.

2. While the simple cycle efficiency of generation at modern central-station power plants will normally exceed any likely competing technology available in small scales, usefully employing the roughly two-thirds of primary energy lost in conversion and delivery using CHP can change the overall efficiency competition considerably. CHP can potentially hand microgrids a lower overall carbon footprint (even with similar fuel) as well as lower cost. Since transporting electricity is much more convenient than transporting heat, placing generation where economically attractive heat sinks exist may be a desirable generation configuration and one that suggests a high degree of generator dispersion. Also note that in warm climates, the most economic use of waste heat may well be building space conditioning; that is, heating to the extent necessary directly and using thermally activated cooling cycles. The latter application can be particularly attractive because it displaces expensive grid electricity purchases at times of high overall demand.

3. While technical analysis of electricity service PQR can be highly sophisticated, by contrast, analysis of the economics of the PQR of end-uses is at best rudimentary. Locally matching the PQR delivered to the requirements of end-uses can potentially meet our goals at a lower cost than high universal macrogrid PQR. In fact, controlling PQR locally to end-uses may permit loosening of grid standards, thereby lowering necessary operating standards and costs.

4. The decision-maker in a microgrid is offered a powerful opportunity to jump some of the hurdles we face in the macrogrid. As the purchaser of electricity and other fuel inputs, the adopter of generating technologies, and also as possibly the selector of technologies on the demand side, he or she holds a unique vantage point that seems absent in the macrogrid. The alternatives on both demand and supply sides have a chance at being even-handedly considered, and alternatives that have difficulty competing on the macrogrid, such as diffuse renewables, perhaps have a better chance of being chosen; in other words, some of the market failures of the macrogrid might be mitigated.

Although this paper deals with only a small part of the potential DG benefit stream, two challenges should also mentioned:

1. While DG currently contributes an infinitesimal share of urban pollution (NO_x, CO, and particulate pollution, noise, etc.) relative to mobile- and larger-point sources, a significant penetration of (especially thermal) DG could have a detrimental affect on the urban environment. Thermal generation of electricity far removed from population concentrations at scales that make emissions control manageable, and its delivery via high-voltage transmission, results in low overall human exposures that set a high standard for close-to-people DG to compete against. On the other hand, limits on favourable station sites, rights-of-way for transmission, adequate cooling water supply, etc., threaten to limit the ability of the macrogrid to meet the ever-growing demand for electricity.

2. While traditional CHP has been implemented in energy-intensive industrial, food processing, and very large commercial installations, e.g., college campuses, most electricity demand growth in developed economies occurs in the residential and small commercial sectors to which operation of DG may represent a significant logistical burden. Absent well-developed competitive markets for equipment, installation, control, and maintenance of DG systems, rapid deployment is unlikely. For most decision-makers in these sectors, energy costs represent a small share of budgets and are unlikely to motivate the significant investment and tolerance for the operational headaches that DG adoption could cause.

In this paper, we examine the operation of a hypothetical microgrid that has already installed DG to meet some of its load and purchases electricity from a wholesale spot market as needed.[†] We gain insight into the optimal DG operational policy by taking a real options approach (see [8] and [17]). Specifically, we recognise that owning a flexible DG unit entitles the microgrid to a strip of embedded options to vary its output according to the relative prices of wholesale electricity and natural gas, the fuel on which existing DG predominantly runs. In large developed economies, natural gas is a pure commodity that is competitively traded. Given the weather-sensitive nature of much of its demand and the risks of supply disruption and general shortages, volatile prices are highly likely. Electricity, on the other hand, is generated by multiple fuels often bought on long-term contracts, so prices tend to be more muted in the long term, but perhaps subject to more short-term volatility due to capacity constraints and its non-storability. In solving the microgrid's cost-minimisation problem over a fixed time horizon, we obtain not only an optimal operating policy for the DG unit, but also its implied option value, i.e., the maximum amount the microgrid would be willing to pay to rent the unit for the given time horizon. Since we abstract from transmission and distribution costs, this latter estimate is a lower bound for the DG unit's value. We also do not account for other DG benefits, such as CHP, or allow for optimisation over its investment decision (see [20], [21], and [22] for a more comprehensive analysis in a deterministic setting). Rather, our focus here is on measuring the impact of stochastic electricity and fuel prices as well as DG start-up costs on the value of the DG unit and its operational policy. We find that optimal control policies will be crucial for the viability of microgrids.

The structure of this paper is as follows:

[†]In reality, most microgrids would face stable and relatively high electricity tariff rates from incumbent utilities. For example, in the service territory of Pacific Gas and Electric (PG&E) in California, the energy charge in year 2004 was around \$100/MWh, along with a demand charge of \$14350/MW during peak hours and a monthly fixed fee of \$175 (see [22]). Consideration of these additional costs would probably make the DG units more attractive than they are in the present analysis.

- Section 2 formulates the microgrid's problem and outlines the simulation algorithm to solve it
- Section 3 explores how the costs and operating schedules of DG units are affected by start-up costs
- Section 4 summarises the findings of this paper and offers directions for future research

2 Problem Formulation

2.1 Microgrid's Decision-Making Problem

We consider the decision-making problem over a time horizon, T (in years), of a microgrid that has installed R on-site gas-fired DG units, where G_r is the capacity of unit r (in MW) to meet a load, D_t (in MW), during hour t, where $t \in \{1, 2, \ldots, 8760\}$, and also retains the option to purchase electricity from the spot market. Here, we assume that the load has both a flat base and a flat peak component, i.e., $D_t = D_B + D_P I_{\{t \in T_P\}}$, where D_B is the base load, D_P is the additional load during peak hours, and T_P is the set of all peak hours, e.g., $T_P = \{[8(1+3(j-1)), 8(1+3(j-1))+11], j = 1, 2, \ldots, 365\} = \{[8, 19], \ldots, [8744, 8755]\}$ represents the hours 0800 to 2000 during each day over one year.[‡] Such load profiles are quite common as primary business activities occur during the daytime. Due to such a load profile, it may be natural to consider two DG units. However, we proceed to formulate the microgrid's problem more generally for R units before setting $R = 2$ in the numerical example. The natural logarithm of the electricity spot price during hour t is X_t (in \$/MWh), while that of the natural gas price is Y_t (in \$/GJ). These two follow (positively correlated) stochastic processes as some central-grid electricity is generated using natural gas as an input fuel.[§] Furthermore, the heat rate of DG unit r is H_r (in GJ/MWh), where it is likely that $G_r > G_{r'} \Rightarrow H_r < H_{r'}$, i.e., larger DG units are more efficient than smaller ones. Finally, the start-up cost of DG unit r is U_r (in \$/h per start), which incorporates the variable operating and maintenance (O&M) costs of the DG units, thereby taking into account the additional wear associated with frequent changes in operating status.

Since both electricity and natural gas prices are stochastic, the microgrid's decision each period is to determine whether or not to use DG. Specifically, during each period t, the microgrid's state is described by the $R \times 1$ vector, $\underline{m}_t$, where the rth element of $\underline{m}_t$ is 1 if generator r is on at time t and 0 otherwise. As the problem will be linear in all decision variables and homogenous, the property of its solution is that each DG unit can run either at full capacity or not at all. This also reflects the reality that part-load operation of DG is inefficient. Correspondingly, the microgrid's operating decision at time t is $\underline{z}_t \in M$, where M is the set of all possible states and $|M| = 2^R$ with R DG units.

Based on this structure, the microgrid's minimum expected discounted cost to go at time t given state $\underline{m}_t$ and price vector (X_t, Y_t) is $V_t(\underline{m}_t; X_t, Y_t)$. If $C_t(\underline{z}_t, \underline{m}_t; X_t, Y_t)$ is the operating cost in period t associated with making decision $\underline{z}_t$ while in state $\underline{m}_t$ and facing price vector (X_t, Y_t), then for $t = 1, 2, \ldots, K-1$:

$$V_t(\underline{m}_t; X_t, Y_t) = \min_{\underline{z}_t \in M} \{C_t(\underline{z}_t, \underline{m}_t; X_t, Y_t) + \beta E_t[V_{t+1}(\underline{m}_{t+1} = \underline{z}_t; \tilde{X}_{t+1}, \tilde{Y}_{t+1})]\} \tag{2.1}$$

[‡] A stochastic component to the load could also be included in our analysis.

[§] We assume that microgrids are effectively price takers because their loads are small relative to the quantities traded in energy markets. In addition, as microgrids tend to be quite heterogeneous because of their varied sizes, main business activities, on-site requrements for heat and power, etc., collusion among them to influence market prices would be unlikely. Thus, electricity and natural gas prices will be modelled as correlated exogenous stochastic processes.

Here, $\beta = e^{-\delta\Delta t}$ is the one-period discount factor, where δ is the annual risk-adjusted interest rate and $\Delta t = T/K$ is the length of the time period in years (the total time horizon divided by the number of decision-making periods). Note that the state vector at time $t+1$ is simply the operating decision selected at time t. The cost function, $C_t(\underline{z}_t, \underline{m}_t; X_t, Y_t)$, is specified as follows for $t = 1, 2, \ldots, K$:

$$C_t(\underline{z}_t, \underline{m}_t; X_t, Y_t) = \Delta t\nu[e^{X_t}(D_t - \underline{G}'\underline{z}_t) + e^{Y_t}\underline{G}'\underline{z}_t + \underline{U}'(\underline{z}_t - \underline{m}_t)^+] \tag{2.2}$$

Equation 2.2 describes the cost function given the decision taken for a particular state, where ν is a factor that converts the length of the time period Δt into hours from years.¶ Note that since $\underline{G}'\underline{z}_t$ is the total DG capacity operating at time t, $D_t - \underline{G}'\underline{z}_t$ is the residual electricity demand, which is purchased from the electricity spot market. Furthermore, the rth element of the vector $(\underline{z}_t - \underline{m}_t)^+$ is equal to 1 if and only if DG unit r is turned on from an idle state at time t. Thus, $\underline{U}'(\underline{z}_t - \underline{m}_t)^+$ is the start-up cost incurred by taking decision $\underline{z}_t$ at time t. As there is no operational dependence among the R units, the cost function is linear and separable in the R units. Using Equation 2.2, the microgrid's stochastic dynamic programme (SDP) may be written and solved subject to the following terminal condition:

$$V_K(\underline{m}_K; X_K, Y_K) = \min_{\underline{z}_K \in M}\{C_K(\underline{z}_K, \underline{m}_K; X_K, Y_K)\} \tag{2.3}$$

Thus, the minimised expected discounted cost of meeting the microgrid's energy needs is $\beta V_1(\underline{m}_1; X_1, Y_1)$.

In the absence of start-up costs, the microgrid's optimal decision-making rule is a myopic one: simply use the cheaper source of energy without taking into account future states. In this case, owning DG is similar to holding a strip of cross-commodity European options‖ for each hour of the year. The option to use DG is then exercised as long as the per MWh cost of DG generation is less than the price of electricity in the spot market. Consequently, the implied option value of DG may be determined by summing (or by integrating, in a continuous-time case) the hourly option values over the entire year (see [7]). In case of path-dependent features, closed-form expressions for option values may not available, thereby necessitating the use of Monte Carlo simulation (see [3]).

While simulation is an efficient procedure for pricing those European options without closed-form solutions, its drawback is that since it is a forward-induction procedure, simulation is not always applicable to options for which optimal exercise policies are needed in advance, such as early-exercise or compound options.** Indeed, for many real options, which have such features, the current state of the system does not include information on the expectation of future events. For example, standard simulation would not be applicable to the microgrid's problem if its DG has start-up costs or operating constraints because then the current state of the DG units alters the payoff structure of any remaining options to generate. It is, therefore, necessary to estimate the impact of current decisions on future cash flows given the current state. Typically, a backward-induction procedure, based on a lattice that discretises the underlying securities' stochastic processes, is used to price such real options (see [4], [6], [8], and [19]). However, if either the number of underlying securities

¶Technically, since $\nu\Delta t = 1$, we do not need to include this term in Equation 2.2. However, we do so to indicate clearly that we have converted from years to hours correctly.

‖European options may be exercised only at the date of expiration. By contrast, American options may be exercised at any time up to and including the expiration date.

**Compound options entitle the holder to gain access to embedded flexibility within the project, e.g., if the microgrid installs a DG unit and then has the additional option to upgrade to incorporate CHP applications, then the initial DG investment is a compound real option. For this reason, compound options are often referred to as being "options on options."

is large or the stochastic processes are too complex to be discretised adequately, then such lattice-based methods are computationally intractable.

In order to resolve this dilemma, a recently developed procedure, known as least-squares Monte Carlo (LSMC) simulation, prices early-exercise options by first generating a large number of sample paths for the underlying securities' prices and then estimating a conditional expectation payoff function via least-squares cross-sectional regressions (see [12]). Specifically, at each time step, the cash flows from continuing to hold on to the option are regressed on a function of current security prices to yield estimated response parameters. These may then be used to estimate a payoff continuation function conditional on current security prices. For a large number of states or underlying stochastic prices, LSMC simulation may be generalised to yield, for example, an estimated conditional continuation function for each state by regressing the cash flows from continuation in each state on a function of all the prices (see [24] and [26] for applications of LSMC simulation to real options). We proceed in Section 2.2 to outline the LSMC simulation algorithm needed to solve the microgrid's SDP.

2.2 LSMC Simulation Algorithm

The first step in solving the microgrid's SDP (Equations 2.1 through 2.3) using LSMC simulation is to generate N sample paths for the electricity and natural gas prices, with $(s_t^{(n)}, w_t^{(n)})$ representing the period t prices for sample path n. In this context, the $R \times 1$ vector of decision variables during time t and sample path n is $\underline{z}_t^{(n)}$, where $\mathbf{z}$ is a $N \times K \times R$ tensor and $\mathbf{z}_t$ is a $N \times R$ matrix. The value and continuation functions are defined and discussed as follows:

- $V_t(\underline{m}_t; s_t^{(n)}, w_t^{(n)})$: minimum expected discounted cost to go in period t given state $\underline{m}_t$ along sample path n (also known as the *value function*)
- $\Phi_t(s_t^{(n)}, w_t^{(n)}; \underline{m}_t, \underline{z}_t^{(n)}) \equiv E_t[V_{t+1}(\underline{z}_t^{(n)}; s_{t+1}^{(n)}, w_{t+1}^{(n)})]$: expected continuation value in period t for sample path n given state $\underline{m}_t$ and operational decision $\underline{z}_t^{(n)}$
- $\hat{\Phi}_t(s_t^{(n)}, w_t^{(n)}; \underline{m}_t, \underline{z}_t^{(n)}) = \underline{f}(s_t^{(n)}, w_t^{(n)})\hat{\underline{b}}_t(\underline{m}_{t+1} = \underline{z}_t^{(n)})$: estimated continuation value in period t for sample path n given state $\underline{m}_t$ and the decision taken is $\underline{z}_t^{(n)}$, where the estimated response parameter vector at period t given state $\underline{m}_{t+1}$ in period $t+1$ from a cross-sectional, least-squares regression of period $t+1$ value functions in state $\underline{m}_t$ on a function of period t prices is $\hat{\underline{b}}_t(\underline{m}_{t+1}) = [(\mathbf{f}(\underline{s}_t, \underline{w}_t))^T(\mathbf{f}(\underline{s}_t, \underline{w}_t))]^{-1}(\mathbf{f}(\underline{s}_t, \underline{w}_t))^T \underline{V}_{t+1}(\underline{m}_{t+1}; \underline{s}_{t+1}, \underline{w}_{t+1})$

Here, since $\mathbf{V}$ is a $N \times K \times |M|$ tensor, $\underline{V}_t(\underline{m}_t; \underline{s}_t, \underline{w}_t)$ is a $N \times 1$ vector, corresponding to the value vector during period t in state $\underline{m}_t$ given the $N \times 2$ price vector $(\underline{s}_t, \underline{w}_t)$. Similarly, $\hat{\mathbf{\Phi}}$ is a $N \times K \times |M| \times |M|$ tensor, with the last two dimensions having $|M|$ possible values each since both the number of possible states and number of available decisions are equal to the cardinality of the state space. Therefore, the expected discounted cost to go of the microgrid during period t given state $\underline{m}_t$ is:

$$V_t(\underline{m}_t; s_t^{(n)}, w_t^{(n)}) = \min_{\underline{z}_t^{(n)} \in M} \{C_t(\underline{z}_t^{(n)}, \underline{m}_t; s_t^{(n)}, w_t^{(n)}) + \beta E_t[V_{t+1}(\underline{z}_t^{(n)}; s_{t+1}^{(n)}, w_{t+1}^{(n)})]\} \tag{2.4}$$

where $C_t(\underline{z}_t^{(n)}, \underline{m}_t; s_t^{(n)}, w_t^{(n)})$ is the cost associated with taking decision $\underline{z}_t^{(n)}$. The dimensions of $\underline{f}(s_t^{(n)}, w_t^{(n)})$ and $\hat{\underline{b}}_t(\underline{m}_t)$ depend on the number of elements in $\underline{f}$. Following [12], we let

$\underline{f}(s_t^{(n)}, w_t^{(n)})$ be a 1×6 vector setting $\underline{f}(s_t^{(n)}, w_t^{(n)}) = [\ 1 \ \ s_t^{(n)} \ \ w_t^{(n)} \ \ s_t^{(n)2} \ \ w_t^{(n)2} \ \ s_t^{(n)} w_t^{(n)}\ ]$, which implies that $\hat{\underline{b}}_t(\underline{m}_t)$ is a 6×1 vector.[††] Consequently, $\mathbf{f}(\underline{s}_t, \underline{w}_t)$ is a $N \times 6$ matrix.

As indicated in Figure 1, the LSMC simulation approach proceeds by first generating N sample paths for the stochastic electricity and natural gas prices. Next, in line 2, a function of the prices at each time period t and sample path n is constructed. In line 3, the terminal condition for the value function is set: the decision is simply to minimise the immediate cost. Then, after the estimated continuation function is initialised to zero in line 4, the main recursion begins proceeds backwards in time as follows:

- Line 7: the response parameter vector for period t and future state $\underline{m}_{t+1}$ is estimated by least-squares regression of period $t+1$ value functions in state $\underline{m}_{t+1}$ on the function of the period t prices
- Line 9: the estimated continuation function at period t for state $\underline{m}_t$ given the decision taken is $\underline{z}_t^{(n)}$ is obtained by multiplying the function of the period t prices by the response parameter vector for time period t and future state $\underline{m}_{t+1} = \underline{z}_t^{(n)}$
- Line 10: the optimal operational decision at time t is that which minimises the expected discounted cost to go using the *estimated* continuation function
- Line 11: the value function is updated by using the cost associated with optimal decision $\underline{z}_t^{(n)*}$ and the *actual* future value

  Line 15: the minimum expected cost is simply the discounted average value of the N costs at time 1 in initial state $\underline{m}_1$

In particular, line 10 uses estimated continuation functions to make DG operating decisions, where the immediate cash outflow from the decision is simply the immediate cost of meeting the load. Therefore, the optimal decision is to minimise the cost from making a decision plus the estimated continuation value of proceeding optimally thereafter from the future state in period $t+1$. Note that while estimated continuation functions are used to decide DG operation, the value functions are recursively updated by employing *actual* continuation functions. Hence, by working recursively backwards through all of the time periods, the average minimised value of the microgrid's operating cost may be determined as the average over all N sample paths of costs in period 1 when starting from a position in which all DG units are off, i.e., starting with the terminal condition of line 2 in Figure 1, the LSMC simulation procedure works backwards, updating $V_t(\underline{m}_t; s_t^{(n)}, w_t^{(n)})$ using the recursion in Equation 2.4 until the answer is obtained as in line 15 of Figure 1.

3 Numerical Example

In this section, we examine the properties of the microgrid's DG system via hourly analysis over one test year. We assume that $\delta = 0.045$ per annum, $T = 1$ year, $\Delta t\nu = 1$ hour, $D_B = 0.50$ MW, $D_P = 0.20$ MW, $T_P = \{[8(1+3(j-1)), 8(1+3(j-1))+11], j = 1, 2, \cdots, 365\}$, and $R = 2$ with $G_1 = 0.50$ MW, and $G_2 = 0.20$ MW (see Figure 2 for the daily load profile). This implies that $M = \{[\ 0 \ \ 0\ ]', [\ 0 \ \ 1\ ]', [\ 1 \ \ 0\ ]', [\ 1 \ \ 1\ ]'\}$. Using the data from [22] on 0.50 MW and 0.20 MW reciprocating engines, we use $H_1 =$

[††] We also tried a basis function with third-order powers, but the fit was not much better, e.g., the coefficient of determination improved to 0.45 on average from 0.43. See [23] for an analysis of basis functions used in the LSMC simulation approach.

`Generate` $\mathbf{s}, \mathbf{w}$

$\underline{f}(s_t^{(n)}, w_t^{(n)}) = [\ 1 \quad s_t^{(n)} \quad w_t^{(n)} \quad {s_t^{(n)}}^2 \quad {w_t^{(n)}}^2 \quad s_t^{(n)} w_t^{(n)}\ ]$, $n = 1, 2, \ldots, N$, $t = 1, 2, \ldots, K$

$V_K(\underline{m}_K; s_K^{(n)}, w_K^{(n)}) = \min_{\underline{z}_K^{(n)} \in M} \{C_K(z_K^{(n)}, \underline{m}_K; s_K^{(n)}, w_K^{(n)})\}$, $\underline{m}_K \in M$, $n = 1, 2, \ldots, N$

$\hat{\Phi}_t(s_t^{(n)}, w_t^{(n)}; \underline{m}_t, \underline{z}_t^{(n)}) = 0$, $t = 1, 2, \ldots, K-1$, $\underline{m}_t, \underline{z}_t^{(n)} \in M$, $n = 1, 2, \ldots, N$

`For` $t = K-1, \ldots, 1$

  `For` $\underline{m}_t \in M$

    $\underline{\hat{b}}_t(\underline{m}_{t+1}) = [(\mathbf{f}(\underline{s}_t, \underline{w}_t))^T (\mathbf{f}(\underline{s}_t, \underline{w}_t))]^{-1} (\mathbf{f}(\underline{s}_t, \underline{w}_t))^T \underline{V}_{t+1}(\underline{m}_{t+1}; \underline{s}_{t+1}, \underline{w}_{t+1})$

    `For` $n = 1, 2, \ldots, N$

      $\hat{\Phi}_t(s_t^{(n)}, w_t^{(n)}; \underline{m}_t, \underline{z}_t^{(n)}) = \underline{f}(s_t^{(n)}, w_t^{(n)}) \underline{\hat{b}}_t(\underline{m}_{t+1} = \underline{z}_t^{(n)})$;

      $\underline{z}_t^{(n)*} = \arg\min_{\underline{z}_t^{(n)} \in M} \{C_t(\underline{z}_t^{(n)}, \underline{m}_t; s_t^{(n)}, w_t^{(n)}) + \beta \hat{\Phi}_t(s_t^{(n)}, w_t^{(n)}; \underline{m}_t, \underline{z}_t^{(n)})\}$;

      $V_t(\underline{m}_t; s_t^{(n)}, w_t^{(n)}) = C_t(\underline{z}_t^{(n)*}, \underline{m}_t; s_t^{(n)}, w_t^{(n)}) + \beta V_{t+1}(\underline{z}_t^{(n)*}; s_{t+1}^{(n)}, w_{t+1}^{(n)})$;

    `End`

  `End`

`End`

`Min Cost` $= \beta \frac{\sum_{n=1}^{N} V_1(\underline{m}_1; s_1^{(n)}, w_1^{(n)})}{N}$;

Figure 1: Solution Procedure for LSMC Simulation

10.3 GJ/MWh and $H_2 = 11.1$ GJ/MWh representing approximately 35% and 32% higher heating value conversion efficiencies, respectively. These are attainable by engines in this size range burning a heavy fuel as indicated in [9]. There are other costs associated with DG, such as turnkey and variable O&M costs. We do not consider the former explicitly since the microgrid is assumed to have installed DG already, and we are addressing only the operational problem. Nevertheless, the implied option value of the DG units from our analysis could provide a lower bound on the annuity the microgrid should pay to install DG. Unlike larger steam-boiler generators, smaller engine-powered generators of the type considered here can be started quite quickly (in a matter of minutes) without significant overall loss of fuel; still, starts impose added wear and tear on all generators, smaller ones included. Consequently, a start-up cost has been estimated for these generators by allocating the variable O&M costs. From the data in [22], the variable O&M costs are \$12/MWh and \$15/MWh for the large and small DG units, respectively. Since it is optimal for each DG unit to be operated either at full capacity or not at all, the hourly variable O&M costs are obtained by multiplying the per MWh costs by the respective capacities. This yields $U_1 =$ \$6/h and $U_2 =$ \$3/h per start, which may be thought of as the *additional* O&M expense that must be borne due to starting up a DG unit.

Finally, we assume that the microgrid functions in an idealised deregulated market, where all electricity and natural gas must be purchased at spot prices. Following [8], we assume that short-term evolution of the natural logarithms of both electricity and natural gas prices can best be described by correlated mean-reverting Ornstein-Uhlenbeck (OU) processes. Specifically:

$$dX_t = \kappa_X(\theta_X - X_t)dt + \sigma_X dS_t \tag{3.1}$$

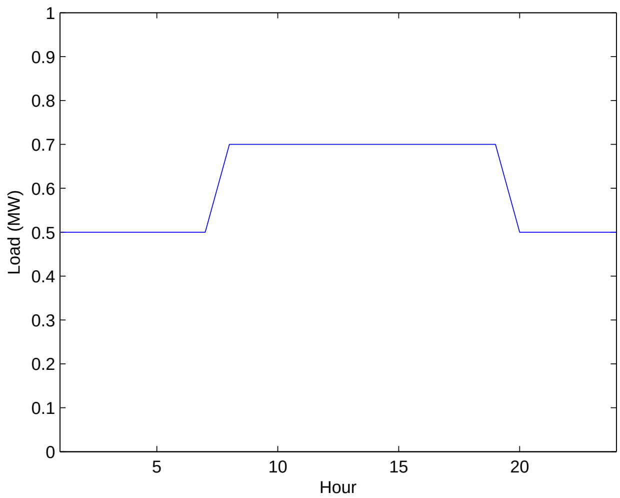

Figure 2: Daily Microgrid Load Profile

$$dY_t = \kappa_Y(\theta_Y - Y_t)dt + \rho\sigma_Y dS_t + \sqrt{1-\rho^2}\sigma_Y dW_t \tag{3.2}$$

Here, for process i, θ_i is the long-term mean, κ_i is the rate of mean reversion, σ_i is the annualised volatility for process i, and ρ is the instantaneous correlation coefficient between $\{X_t, t \geq 0\}$ and $\{Y_t, t \geq 0\}$. Furthermore, $\{S_t, t \geq 0\}$ and $\{W_t, t \geq 0\}$ are independent standard Brownian motion processes. The OU processes in Equations 3.1 and 3.2 may be simulated as follows using two independent standard normal random variables ϵ_X and ϵ_Y:

$$X_{t+\Delta t\nu} = X_t + \kappa_X(\theta_X - X_t)\Delta t + \sigma_X\epsilon_X\sqrt{\Delta t} \tag{3.3}$$

$$Y_{t+\Delta t\nu} = Y_t + \kappa_Y(\theta_Y - Y_t)\Delta t + \sigma_Y\rho\epsilon_X\sqrt{\Delta t} + \sqrt{1-\rho^2}\sigma_Y\epsilon_Y\sqrt{\Delta t} \tag{3.4}$$

Using the data from [8] (reproduced in Table 1) and initial prices of \$21.7/MWh and \$3.16/GJ for electricity and natural gas, respectively, we generate $N = 1000$ sample paths.[‡‡] The effective cost of generation from the DG units is obtained by multiplying the natural gas price by the appropriate heat rate. An example of the simulated paths is shown in Figure 3.

θ_X	θ_Y	σ_X	σ_Y	κ_X	κ_Y	ρ
3.2553	0.87	0.79	0.60	3	2.25	0.30

Table 1: Parameter Data for Correlated OU Price Processes

Without loss of generality, we assume that the microgrid that has installed only one DG unit.[*] A 95% confidence interval for the microgrid's operating cost may be constructed

[‡‡]While doubling the number of sample paths reduces the standard deviation of the estimators, we find that $N = 1000$ sample paths are sufficient for the conclusions of this paper.

[*]This reduces the set of allowable states to $M = \{0, 1\}$. The SDP in Equations 2.1 through 2.3 is modified accordingly. Examining a multiple-unit system would be possible, but would not provide any more intuition.

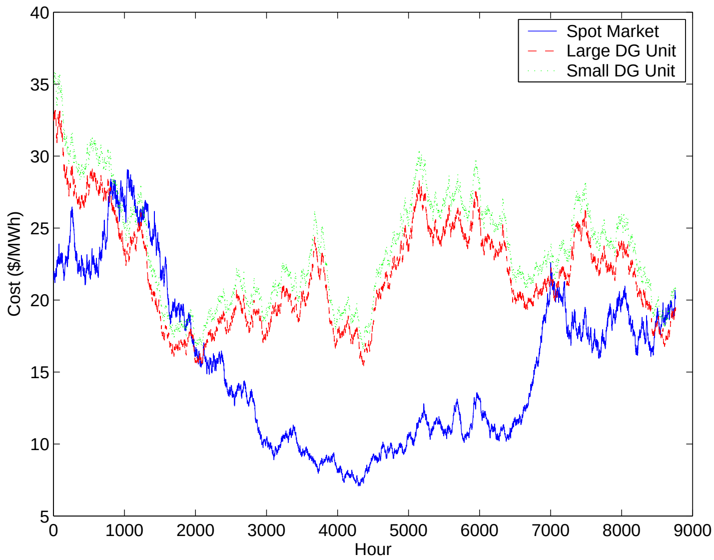

Figure 3: Simulated Electricity Price and On-Site Generation Costs

using the Central Limit Theorem as follows:

$$\overline{V} \pm z_{0.975}\sqrt{\frac{\mathcal{S}^2(V)}{N}} \tag{3.5}$$

In Equation 3.5, the sample mean, $\overline{V} = \beta\frac{\sum_{n=1}^{N} V_1(m_1;s_1^{(n)},w_1^{(n)})}{N}$, is the average of the minimised discounted costs over N sample paths, and $\frac{\mathcal{S}^2(V)}{N}$ is its sample variance. We assume that the initial operating state of the DG unit is "off," i.e., $m_1 = 0$. In order to find the option value, F, of this investment over the test year, we first solve the SDP described in Equations 2.1 through 2.3 and then subtract its minimised cost, V, from that of a microgrid that has no installed capacity, V^0, i.e., one that meets its entire load via spot purchases. The estimated option value is then the difference between the mean minimum costs without and with DG installed, i.e., $\overline{F} = \overline{V^0} - \overline{V}$.

Using this procedure and Equation 3.5, we minimise the operating costs of the single DG units under three cases:

- No start-up costs (NS): achieve cost minimisation by following a myopic policy of making the decision that minimises only immediate costs without considering future value functions, i.e., ignoring the second term in Equation 2.1 when selecting from among the alternative choices.

- Start-up costs (SU): include start-up costs and proceed optimally using the LSMC simulation algorithm as outlined in Equation 2.4 and Figure 1.

- Start-up costs with a myopic policy (SM): include start-up costs, but use the myopic policy of case NS to make decisions, i.e., the microgrid considers only current cash flows related to its operating and start-up costs in making decisions by ignoring the second term in Equation 2.1.

Via these three cases, we illustrate that although start-up costs do not significantly increase the operating costs of DG units, they must be taken into account by altering the operating policy to consider the tradeoff between current costs and future expected start-up costs. Indeed, we shall show that if a microgrid ignores this tradeoff and follows a myopic operating policy as in case NS by not taking into account future cash flows, then the operating cost is statistically significantly increased with less on-site generation of electricity.

From Table 2, it can be seen first of all that the inclusion of start-up costs increases the average operating cost of DG only slightly as long as the correct tradeoff is made between current cost minimisation and future start-up costs. In particular, there is a 0.09% increase in the operating cost of the 0.20 MW DG unit when start-up costs are imposed (see Table 3). For the 0.50 MW DG unit, the impact of start-up costs is higher at 0.22% due to the larger unit's greater on-site generation. This increase is not statistically significant at the 95% level as indicated by the overlapping confidence intervals for $\overline{V}$ in cases NS and SU in Table 4. On the other hand, if the microgrid does not adjust its operating policy to account for future start-up costs and continues to follow a myopic policy when start-up costs are imposed (as in case SM), then the average operating cost of DG increases by a much larger amount, i.e., 1.47% and 2.74% for the small and large DG units, respectively. As indicated in Table 4, this increase is statistically significant at the 95% level for the larger unit as the confidence intervals for $\overline{V}$ between cases SU and SM are disjoint.

Case	0.20 MW DG	0.50 MW DG
NS	$\overline{V} = 127.06$, $\mathcal{S}(V) = 0.72$	$\overline{V} = 120.75$, $\mathcal{S}(V) = 0.62$
SU	$\overline{V} = 127.18$, $\mathcal{S}(V) = 0.72$	$\overline{V} = 121.01$, $\mathcal{S}(V) = 0.62$
SM	$\overline{V} = 128.92$, $\mathcal{S}(V) = 0.73$	$\overline{V} = 124.07$, $\mathcal{S}(V) = 0.63$

Table 2: Sample Mean and Standard Deviation for Individual DG Operating Costs (in thousand $)

Case	0.20 MW DG	0.50 MW DG
SU	0.09	0.22
SM	1.47	2.74

Table 3: Average Percentage Cost Increase Relative to Case NS

Case	0.20 MW DG	0.50 MW DG
NS	[125.65, 128.47]	[119.54, 121.97]
SU	[125.77, 128.59]	[119.80, 122.23]
SM	[127.49, 130.36]	[122.82, 125.31]

Table 4: Confidence Intervals for Individual DG Operating Costs (in thousand $)

The implied DG option value is also more strongly affected by the start-up costs when the myopic policy is followed (see Table 5). For example, the decrease in implied option value is negligible for the SU case, but becomes quite significant (at 44% and 32% for the 0.20 and 0.50 MW units, respectively) for the SM case. More telling, however, is the average fraction of electricity generated by DG in each case (see Table 6). While there is a modest decrease in on-site generation in case SU as the microgrid becomes more hesitant to turn the unit on in the first place, there is a drastic reduction in DG usage with case SM since an active DG unit is turned off too easily based on its immediate (and not future) profitability, thereby making it more difficult to justify turning an idle unit on. We note, furthermore, that the smaller DG unit's operations are more affected by the start-up costs due to its relative inefficiency. This is the case even though the larger DG unit's operating cost increases by a greater percentage, which is a consequence of its being attractive enough to be used frequently by the microgrid. Indeed, since the microgrid's average operating cost is always lower when using the larger DG unit, the imposition of start-up costs results in a relatively greater impact on the bottom line. Conversely, since the smaller DG unit is used infrequently (e.g., it provides only 12% of the microgrid's electricity even when no start-up costs exist), the loss in its implied option value is relatively greater when start-up costs are imposed. Hence, the microgrid's average cost increases by a greater percentage when start-up costs are imposed when using the large DG unit, but its greater utilisation rate also means that its implied option value decreases by a smaller percentage than that of the small DG unit.

Case	0.20 MW DG	0.50 MW DG
NS	4.20	10.51
SU	4.09	10.25
SM	2.34	7.20

Table 5: Average Implied Option Values of DG (in thousand $)

Case	0.20 MW DG	0.50 MW DG
NS	11.88	29.70
SU	7.82	20.29
SM	1.12	4.90

Table 6: Average Percentage of Electricity Generated by DG

In order to determine the operational implications of start-up costs, we find start-up and shut-down price thresholds for the DG unit. We pick a representative hour during the test year to illustrate how the thresholds shift as a result of the start-up costs. Figure 4 is a scatterplot of the electricity prices and the DG generating costs at which the 0.20 MW DG unit is turned on (indicated by the blue circles) or turned off (indicated by the red crosses). Note that the region in which it is optimal to turn on the DG unit is disjoint from that in which it is optimal to turn off the DG unit. Indeed, the shared boundary between these two regions reflects the fact that a myopic policy is optimal, i.e., in the absence of start-up costs, it is optimal to start-up (shut-down) the DG unit immediately if the ratio of the DG generating cost to the electricity price falls below (exceeds) a critical value. For example, if the current DG generating cost is $20/MWh and the DG unit is off, then the microgrid should wait until the electricity price is just above $20/MWh before switching the unit on. Similarly, if the unit is on, then the microgrid should switch the unit off when the electricity price drops to \$20/MWh.

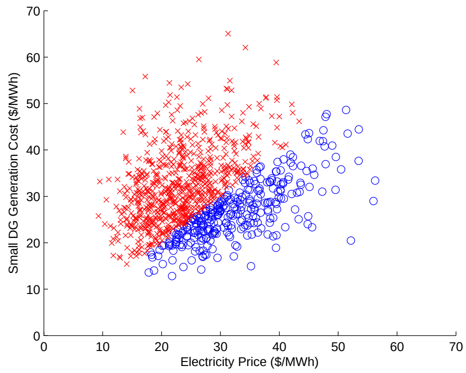

Figure 4: Operational Thresholds for 0.20 MW DG Unit (Case NS, Hour 4000)

As discussed previously, the presence of start-up costs implies that a myopic policy is no longer optimal since the microgrid needs to consider not only the current relative costs of meeting its load, but also the future costs, which may include start-up costs. Therefore, a similar scatterplot for case SU results in a zone of inaction between the two action regions (see Figure 5) in which it is optimal to wait and maintain the *status quo*, i.e., keep the DG unit on (off) if it is currently on (off). Effectively, for a given DG generating cost, the electricity price threshold at which to turn on (off) a DG unit that is currently off (on) is higher (lower) than in the NS case. For example, if the current DG generating cost is \$20/MWh and the DG unit is off (on), the microgrid waits until the electricity price is around \$23/MWh (\$17/MWh) before turning the unit on (off). The intuition for this hesitancy is that the microgrid wants to avoid a situation in which it turns on (off) a marginally cheaper (more expensive) DG unit only to have to turn it off (on) the following hour. In fact, both operational thresholds are affected because once a DG unit is on, the prospect of shutting it down only to have to turn it back on (and thus, pay the start-up cost) delays the shut-down decision. It is preferable to incur slightly higher energy costs in the current period by keeping a relatively expensive DG unit on than to pay start-up costs or face even higher costs in the future. Furthermore, the zone of inaction appears to widen as both the electricity price and DG generating cost increase, e.g., if the latter is \$40/MWh, then the electricity price needs to be nearly \$50/MWh (\$35/MWh) for an off (on) unit to be turned on (off).

We quantify the "on" and "off" thresholds by fitting lines to the frontiers of the two sets and find that the "on" threshold has the equation:

$$GENCOST = 0.91009 \times ELPRICE - 2.2782 \tag{3.6}$$

Here, $GENCOST$ and $ELPRICE$ refer to the natural gas generation cost and electricity price, respectively. The equation for the "on" threshold is constructed in two steps:

1. We fit a preliminary line using the two extreme points of the "on" cluster in Figure 5. Specifically, we take the northeastern-most point and the southwestern-most point in the cluster and draw a line between them.
2. Then, we discard all of the points in the cluster that lie below the preliminary fitted line and fit an OLS regression line to the remaining points, i.e., the ones that lie above the preliminary fitted line from step 1.

This is an *ad hoc* procedure and may have more severe drawbacks if used in analysing more sophisticated boundaries. For example, the selection of the extreme points in the first stage of this exercise could influence the ultimate fitted boundaries. However, we feel that it is sufficient to demonstrate the effect of the start-up costs on the threshold price and construct the "off" threshold similarly (except that we retain the points that lie below the preliminary fitted line):

$$GENCOST = 1.0434 \times ELPRICE + 2.8062 \tag{3.7}$$

Both regressions have coefficients of determination of over 0.98. We note from Equations 3.6 and 3.7 that the "on" threshold gets flatter as the electricity price increases, while the "off" threshold gets steeper. In Figure 5, the thresholds are sketched in as dotted lines. This increasing wedge between the two action zones could simply reflect a constant percentage by which the electricity price must exceed (fall below) the DG generating cost before action may be taken optimally. On the other hand, at higher prices, there may be more uncertainty about future prices, which then causes the microgrid to be more cautious in its decision making. In either case, although the operating cost of the DG unit is not significantly affected by start-up costs, their impact on the microgrid's operating policy is more profound.

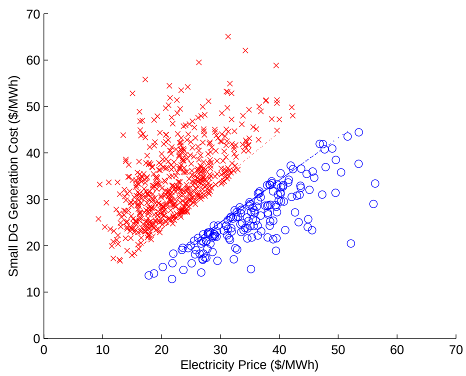

Figure 5: Operational Thresholds for 0.20 MW DG Unit (Case SU, Hour 4000)

We find that if start-up costs are present, but the microgrid continues to follow the myopic operating policy of case NS, then the operating cost of its DG unit is increased. From the scatterplot in Figure 6, it now becomes clear why this is the case: following a myopic policy does not alter the "off" threshold relative to case NS, but shifts the "on" threshold much further to the right. This is because by ignoring future start-up costs, the microgrid readily turns off an active DG unit according to the threshold given in Figure 4, which subsequently puts it in a situation where it is not cost effective to use DG in the future unless the electricity price increases drastically so that the immediate energy cost savings outweigh the start-up costs of DG. In contrast, if it takes future start-up costs into account as in case SU, then it turns an active DG unit off only if the immediate cost savings outweigh *future* expected start-up costs. Hence, a microgrid that follows an optimal operating policy shifts its "off" and "on" thresholds slightly relative to case NS, whereas one that follows a myopic policy ignores not only future expected start-up costs in turning off an active DG unit (thereby maintaining the "off" boundary from case NS), but also future expected cost savings from turning on an idle DG unit (thereby causing the "on" boundary to shift more to the right than in case SU).

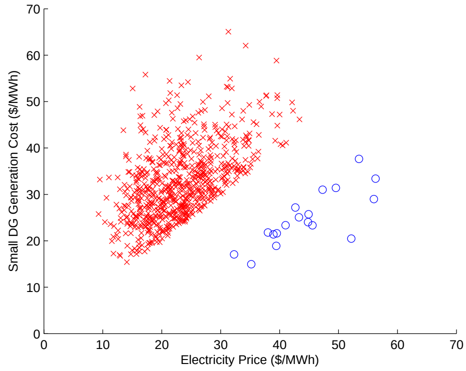

Figure 6: Operational Thresholds for 0.20 MW DG Unit (Case SM, Hour 4000)

The impact of start-up costs on the larger 0.50 MW DG unit is similar: a zone of inaction appears in case SU relative to case NS that results in a more hesitant microgrid (see Figures 7 and 8). We quantify the "on" and "off" thresholds via regression:

$$GENCOST = 0.8973 \times ELPRICE - 0.9038 \tag{3.8}$$

$$GENCOST = 1.0469 \times ELPRICE + 1.9158 \tag{3.9}$$

These thresholds are plotted as dotted lines in Figure 8. By subtracting Equation 3.6 from Equation 3.7 and Equation 3.8 from Equation 3.9, we can also quantify the width of the inaction zones as functions of the electricity price. As can be seen from Figure 9, the zone of inaction appears to be narrower with the 0.50 MW DG unit than with the 0.20 MW unit.† In effect, the greater efficiency and relatively lower start-up cost of the larger DG unit imply that it is more flexible than the smaller one. Nevertheless, as indicated in Figure 10, if the large DG unit were operated in a myopic manner, then it, too, would lose a significant fraction of its option value as it would not trade off future expected cash flows with current ones. Hence, substantial alteration in the DG unit's operating policy is required in the presence of start-up costs even if these constraints have little significant impact on the microgrid's costs. Finally, Figure 11 indicates that the option value of the DG unit is increasing in the electricity price volatility as higher electricity prices become more likely. Furthermore, the undervaluation of the DG unit in the myopic case becomes less of an issue when electricity price volatility is high because the greater occurrence of high electricity prices enables on-site generation to be dispatched more frequently even with incorrect accounting of future expected costs.

While the conclusions about the impact of start-up costs on operational policy are well known in the real options literature (see for example [5]), they are especially relevant for DG

†In order to test this hypothesis statistically, it should be possible to do a standard test for differences in means. However, as the slopes of the regression lines depend on *ad hoc* selection of extreme points, the conclusions from any hypothesis tests would be circumspect. A more promising, if speculative, alternative would be to use support vector machines (SVMs), which are based on statistical learning theory and are designed to separate two clusters of data points that represent different sub-populations (see [25]).

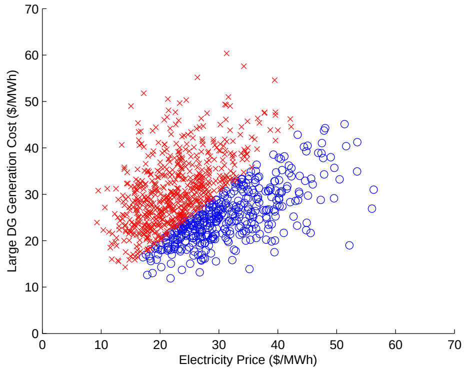

Figure 7: Operational Thresholds for 0.50 MW DG Unit (Case NS, Hour 4000)

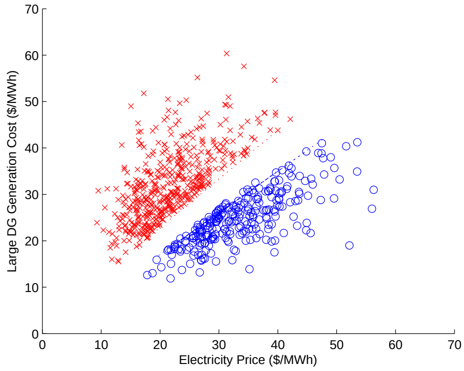

Figure 8: Operational Thresholds for 0.50 MW DG Unit (Case SU, Hour 4000)

projects. This is because the viability of DG is going to revolve around the effectiveness of their control algorithms. Indeed, given the non-trivial costs of automated control systems, examining the tradeoffs between DG costs and benefits is essential for microgrids to develop. Under stochastic prices, finding an optimal operating policy becomes even more critical for microgrids since without one, their advantage over central-station generation narrows dramatically. We have, thus, attempted to provide such a framework and illustrated via numerical examples some of the pitfalls that microgrid managers should avoid in practice.

4 Conclusion

The ongoing deregulation of electricity industries worldwide provides opportunities for microgrids to evolve according to the needs of end-use consumers by incorporating DG and CHP applications where beneficial. Limits on carbon emissions, such as by cap-and-trade allowance markets, would make microgrids more economically attractive because CHP permits higher overall fuel efficiency. Microgrids will, nonetheless, face complex control problems for many reasons, only one of which is addressed herein. Furthermore, the cost of control is non-trivial at small scales of operation and coördination with other building operations is

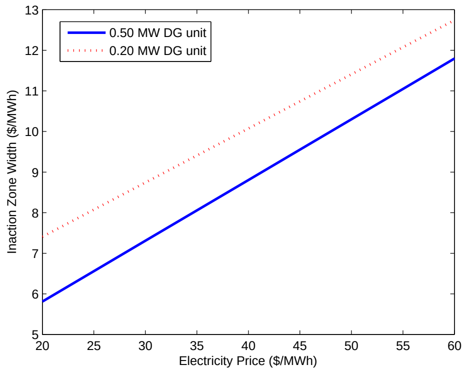

Figure 9: Width of Inaction Zones (Case SU, Hour 4000)

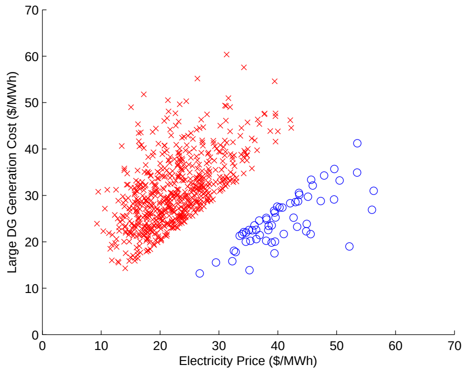

Figure 10: Operational Thresholds for 0.50 MW DG Unit (Case SM, Hour 4000)

also required, so finding efficient algorithms for microgrid operations could be a critical aspect of their overall viability. In this paper, we make a stylised attempt to examine the impact of modest start-up costs on DG value and operation within a stochastic setting.

By taking a real options approach, we find that although the impact of start-up costs on the expected operating cost of DG is minor, their operational implications are certainly more profound. In effect, the presence of start-up costs forces the microgrid to trade off current cash flows with estimates of future expected cash flows before making any operational decisions since future cash flows are affected by current actions and states. By contrast, without start-up costs, the microgrid may proceed to make decisions in a myopic manner, i.e., consider only current cash flows and states in making its decision because future cash flows are independent of current actions and states. Factoring this dependency into the microgrid's decisions causes there to be a zone of inaction between the "on" and "off" thresholds for DG as it becomes preferable to wait before the electricity price (DG generating cost) more than exceeds the DG generating cost (electricity price) before turning the DG unit on (off). Indeed, this hesitancy results from the fact that the microgrid must now include *future* expected start-up costs as implicit opportunity costs of turning on an idle DG unit. Since the additional cost makes switching to the "on" state less attractive, the microgrid maintains

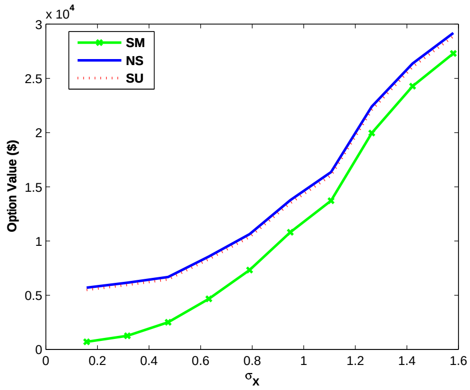

Figure 11: Option Value Sensitivity Analysis for 0.50 MW DG Unit

the *status quo* over an intermediate range of prices. Similarly, the presence of start-up costs causes the microgrid to postpone turning off an active DG unit because future expected start-up costs are subtracted from the current cost savings of using electricity purchases, thereby leading to inaction as long as the electricity price is not relatively low. If such a tradeoff is ignored, i.e., the microgrid proceeds myopically in the presence of start-up costs, then the zone of inaction widens, resulting in significantly higher costs. In particular, the "off" threshold is not affected since future expected start-up costs are ignored. However, the "on" threshold is shifted far to the right as the subtraction of current start-up costs from current cost savings of using DG without accounting for future expected cost savings from an "on" DG unit reduces the benefits of DG. Therefore, from our model, a microgrid manager can infer not only the option values of its DG, but also identify an optimal operating policy.

In order to focus on such operational implications, we have neglected many real-world features in our stylised model. For instance, we do not consider the transmission and distribution costs of electricity purchases, which would make DG units more valuable than is implied in this work. Our analysis could also benefit from more rigorous treatment of DG O&M costs as we allocate only incremental variable O&M costs as start-up costs. Omission of CHP applications also significantly understates the value of DG. This feature may also affect DG operating schedules depending on the degree to which electric and heat loads are coincident. We propose to incorporate CHP as an option to upgrade in future work. In this context, the investment decision should also be modelled, although it is more common for such decisions to be driven by long-term price factors rather than the short-term ones we address here (see [17]). Finally, from a modelling perspective, since we examine a relatively short time horizon, we should incorporate seasonality, daily peak prices, and spikes in both price processes.

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Manuscript received 15 August 2006
revised 24 April 2007
accepted for publication 6 Jun 2007

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