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BUDGET-OF-UNCERTAINTY ROBUST APPROACH TO INTEGRATED FACILITY LOCATION AND PRODUCTION PLANNING PROBLEM UNDER DEMAND UNCERTAINTY*

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Abstract: In this research, the budget-of-uncertainty robust approach is applied to an integrated facility location and production planning problem with demand uncertainty over multiple periods. A firm establishes production facilities with different capacities and allocates the customers to these facilities in the initial period. Production and inventory plan for each opened facilities to meet the uncertain demand is determined in the following periods. Supposing uncertain demand in each period is unknown and within a bounded and symmetric multi-dimensional box, we propose the robust counterparts of the deterministic model considering the different degree of conservatism of the robust solution. It is shown that the different degree of conservatism lead to very different solution network topologies. The numerical results show that it may sacrifice not too much of optimality for the deterministic problem to ensure robustness of the solution. Simulation results imply that the difference between the theoretical bound of constraints violation and the empirically observed values is very small.

Key words: *facility location and production planning, budget-of-uncertainty, protection level, service level*

Mathematics Subject Classification: *90B15, 90C15, 90C90*

1 Introduction

Facility location models focus on determining the optimal facility location strategy and customers assignment to minimize the total cost including the facility opening cost and the shipping cost between the customers and the opened facilities. In fact, subsequently, the firm may decide production and inventory plan to satisfy the customers' demand according to the facility location and customer assignment strategy. However, without considering the production and inventory plan in the traditional facility location models, their optimal solutions may lead to very high (and unnecessarily) production and inventory costs. Romeijn et al. (2010)[21] combine an uncapacitated facility location problem with production and inventory decisions with deterministic demand in the multi-periods.

For the integrated facility location and production planning problem (IFLPP), various uncertainty issues should be taken into account. In practice, facility location project, which

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is an expensive work, is difficult to change or improve the location decision. Moreover, facility location strategy has a long term impact on production and inventory decision in the future. During the operational lifetime of a facility, some parameters of the problem (costs, demands and distances) may deviate from their estimated values. The solutions derived from the deterministic model may not be appropriate to the future situation. Hence, the uncertain factors in the multi-period should be taken into account in the facility location model to avoid the operational inefficiency in the future.

Optimization under uncertainty typically uses one of the two approaches: stochastic optimization or robust optimization. In stochastic optimization, the probability distributions of the random parameters are known, and the commonly adopted objective is to find a solution that minimizes the expected cost or maximizes the expected profit of a system. In robust optimization (RO), probabilities are unknown, and uncertain parameters are specified by continuous ranges or discrete scenarios; the objective function often attempts to optimize the worst-case performance of the system or the regret value. In this study, we take the latter approach, i.e. the exact probability distributions of the uncertain parameters are unknown and specified by symmetry intervals. In recent years, RO has emerged as a preeminent methodology to deal with the problems under uncertainty. The first step in this direction is taken by Soyster (1973)[25], referred to as a complete protection approach. He proposes a linear optimization model to construct a solution that is feasible for all data that belong in a convex set. But, the solutions obtained by his model are too conservative, because it may sacrifice too much of optimality to ensure robustness of the solution compared with the deterministic problem. A major step forward for developing a theory for robust optimization are taken independently by Ben-Tal and Nemirovski (1998, 1999, 2000)[4, 5, 6], El-Ghaoui and Lebret (1997)[13], and El-Ghaoui et al.(1998)[14]. To figure out the issue of over conservatism, these articles present less conservative models by considering uncertain linear problems with ellipsoidal uncertainties, which result in solving the robust counterparts of the deterministic problem in the form of conic quadratic problems. Baron et al. (2011)[3] apply the RO approach mentioned above to a multi-period fixed-charge network location problem. They consider two models of demand uncertainty: demand within a bounded and symmetric multi-dimensional box, and demand within a multi-dimensional ellipsoid. The two RO approaches mentioned above have some drawbacks: although Soyster's methodology admits the highest protection, it is too conservative; while the roust counterpart of Ben-Tal and Nemirovski (2000)[6] is a nonlinear model, which is not attractive for solving robust discrete optimization models. To deal with the above two drawbacks, Bertsimas and Sim (2004)[7] propose a robust approach that can adjust the degree of conservatism of the robust solutions flexibly by solving a linear program, which is called the budget-of-uncertainty robust approach.

In this research, we consider an integrated facility location and production planning problem that generalizes the traditional facility location models by taking into consideration the production and inventory cost in future production periods. This problem helps a firm establish a set of facilities with different production capacity and assign the customers to the opened facilities in the initial period. With the designed facility network, the production plan and inventory strategy for each facility is determined to meet the uncertain demand. The total demand served by a facility must be no more than its maximum capacity established at the start of the horizon. The goal of our problem is to minimize the total cost including facility opening cost, capacity establishing cost, delivery charge, production setup cost, production cost and inventory cost. Assuming that the demands are within a bounded and symmetric multi-dimensional box, to tackle this problem, we apply the concept of the budget of uncertainty. Comparing with the previous research on the robust facility location problem, our robust model retains the advantages of the linear framework of the deterministic problem, which is computationally tractable. The robust model offers full control on the degree of conservatism of the robust solution. Obviously, comparing with the deterministic problem, the robustness of the solution may increase the total cost of the objective function with uncertain demand. Hence, with the definition of measures of robustness, the trade-off between the cost and the robustness of the solution in our problem is deeply analyzed in the following sections.

The main contributions of this paper are summarized as follows: First, we combine the facility location problem with production and inventory decisions to avoid high (and unnecessarily) production and inventory costs; second, the uncertain demands are assumed to be within a bounded and symmetric multi-dimensional box, without an explicit probabilistic description of the uncertain parameters. This linear robust model can avoid to cope with a large number of scenarios; third, we provide insights into the network topology of the solutions. The trade-off between the degree of the conservatism of the robust solution and the total cost is investigated.

The remainder of this paper is structured as follows. Section 2 reviews the literature related to our problem. In section 3, we formulate the deterministic IFLPP problem. Section 4 presents the robust counterpart of the deterministic problem with uncertain demand in the multi-periods by using complete protection approach and the budget-of-uncertainty robust approach, respectively. In section 5, we compare the performance of RO approach under different degree of conservatism. Measures of the price of robustness are defined to investigate the trade-offs between the cost and the protection level. Final considerations are depicted in Section 6.

2 Literature Review

In this section, we briefly review some of the relevant studies that are related to our problem. The facility location problem is an important strategic decision and many models have been proposed. For a detailed introduction to this topic, we refer the reader to Daskin (2011)[11], Snyder and Daskin (2005)[23], and Jiang and Yuan (2012)[16]. In this section, we discuss facility location models under uncertain parameters.

In stochastic facility location problems, the most commonly used objective to deal with the uncertainty is optimizing the mean outcome of the system; e.g., minimizing the expected cost or maximizing the expected profit. Mirchandani et al.(1985)[20] discuss the 2-median problem on a tree with stochastic edge lengths described by discrete scenarios. The objective is to minimize the expected demand-weighted distance. Weaver and Church (1983)[26] present a Lagrangian relaxation algorithm for the stochastic P-Median Problem (PMP) on a general network. The set of scenarios and the probability of occurrence of each scenario are given. Each of the scenarios specifies a realization of the demands and travel costs. The objective is to minimize the expected travel cost, subject to the standard PMP constraints. Louveaux (1986)[19] studies how the two classical facility location models, the Simple Plant Location Problem (SPLP) and the PMP, are transformed in a two-stage stochastic program with recourse when demands, variable production and transportation costs are uncertainty. The relationship between the stochastic version of the SPLP and the stochastic version of the PMP is also discussed. Listes and Dekker (2005)[18] present a stochastic location model for production recovery network design problem by explicitly accounting for the uncertainties. They apply it to a case study involving the collection, recycling, and reuse of sand from demolition sites in the Netherlands. For more facility location models under uncertainty with stochastic optimization approach, the reader is referred to the review article by Snyder

(2007)[22].

In contrast, no probability information is known about the uncertain parameters in RO. The two most common objectives in RO are minimax cost and minimax regret, which are closely related to one another. Chen and Lin (1998)[8] consider the minmax regret 1-median problem on a tree network where edge lengths and node weights are uncertain, and the uncertainty is characterized by given intervals. They present an $O(n^3)$ algorithm to solve the model. Averbakh and Berman (2000)[2] consider the weighted 1-center problem on a network with uncertainty in node weights and edge lengths, and present an $O(n^6)$ algorithm for the problem on a tree. Averbakh (2003)[1] presents a general approach for finding minmax regret solutions for a class of combinatorial optimization problems with an objective function of minimax type and uncertain objective function coefficients. The approach they used is based on reducing an uncertain problem to a number of deterministic problems. The method is illustrated on minimax multi-facility location problems and maximum weighted tardiness scheduling problems. Current et al.(1997)[10] propose two approaches to analyze the dynamic location problems, focussing on situations where the total number of facilities to be located is uncertain (NOFUN, Number Of Facilities Uncertain). They analyze the NOFUN problem using two decision criteria: the minimization of expected opportunity loss, and the minimization of maximum regret.

In addition, several other robustness measures have also been applied to facility location problems. Kouvelis et al.(1992)[17] propose a method to find solutions where the relative regret under any scenario is limited to be less than some percentage. This measure is also applied in Gutierrez and Kouvelis (1995)[15] to the problem of selecting outsourcing suppliers with uncertain exchange rate. Snyder and Daskin (2006)[24] apply this measure (they refer to it as p-robustness) to the PMP and the UFLP. They combine the two objectives by minimizing the expected cost while bounding the relative regret in each scenario. In particular, the models seek the minimum-expected-cost solution that is p-robust. Both problems are solved by using variable splitting, with the Lagrangian sub-problem reducing to the multiple-choice knapsack problem. Daskin et al.(1997)[12] present the α-reliable minimax model that optimizes the worst-case performance over an endogenously generated set of scenarios. The collective probability of occurrence of the endogenously selected set is at least some user-specified value α, which we call the reliability level. In this way, the planner can be $l00\alpha\%$ sure that the regret will be no more than that found by the model. Chen et al.(2006)[9] present a model called the α-reliable mean-excess regret model. In contrast to the α-reliable minimax model where the regret that defines the α-quantile of all regrets is minimized, in the α-reliable mean-excess regret model, the objective is to minimize the expectation of the regrets associated with the scenarios in the tail, which has a collective probability of $1 - \alpha$.

The closest work to ours in applying robustness concepts to facility location problem is Baron et al.(2011)[3]. They apply robust optimization to the problem of locating facilities in a network under uncertain demand over multiple periods. Two models of demand uncertainty are considered: demand within a bounded and symmetric multi-dimensional box, and demand within a multi-dimensional ellipsoid. They show that both the box and ellipsoidal uncertainty cases can provide significant improvement over the solution to the problem compared to the deterministic model where demand is set at the mean value. However, the box demand uncertainty model results in the over-conservatism solution and the ellipsoid demand uncertainty model leads to conic quadratic program, which is not particularly attractive for solving the model. In this study, we apply the budget-of-uncertainty robust approach to deal with demand uncertainty in the IFLPP problem, which can adjust the degree of conservatism of the robust solutions flexibly by solving a linear problem.

3 Integrated Facility Location and Production Planning Model (IFLPP)

In this section, we consider an integrated facility location and production planning problem with deterministic demand. A firm seeks to locate some facilities at a set of candidate nodes and establish the maximum production capacity of these opened facilities. It also determines to allocate the demand to the opened facilities. These decisions are determined once at the start of a time horizon. Then, after establishing the facility network, the firm will make production and inventory decisions to fulfill the demand at each node in each period. Only the opened facilities can provide the production with their maximum capacity. We assume that inventory can be carried over from one period to the next.

Suppose that the capacity is infinitely divisible and the amount of production can be adjusted in each period without any cost. An opened facility can serve one or more demand nodes while each demand node is assigned to exactly one facility.

We formulate the deterministic problem by assuming that all parameters in the model are static and exactly known, including the future demands. Let $G(N, A)$ be a connected graph with node set N and arc set A. N represents the nodes set including the potential demand nodes and the candidate facilities. Let T be the length of the horizon, as well as the set of the periods. For the convenience of exposition, we define the following notations:

Data

f_i: The fixed cost of opening a facility at node i, $i{\in}N$

p_{i0}: The cost of establishing per unit capacity at node i at the beginning of the horizon, $i{\in}N$

p_{it}: The per unit production cost at node i in period t, $i{\in}N$,$t{\in}T$,

q_{it}: The production setup cost at facility i in period t, $i{\in}N$,$t{\in}T$,

h_{it}: The per unit holding cost at facility i in period t, $i{\in}N$,$t{\in}T$,

c_{ij}: The delivery cost from facility i to demand j, $i{\in}N$,$j{\in}N$,

d_{jt}: The demand at node j in period t, $j{\in}N$,$t{\in}T$,

M: Sufficient large number

Decision variables

y_i: 1 if a facility is located at candidate node i; otherwise 0, $i{\in}N$,

Q_{i0}: the maximum capacity of an open facility at node i, $i{\in}N$,

x_{ij}: 1, if demand j is assigned to a facility located at i; otherwise 0, $i{\in}N$, $j{\in}N$,

z_{it}: 1, if facility i is prepared to produce in period t; otherwise 0, $i{\in}N$, $t{\in}T$,

Q_{it}: The production at node i in period t, $i{\in}N$, $t{\in}T$,

I_{it}: The inventory at facility i in period t, $i{\in}N$, $t{\in}T$.

The deterministic integrated facility location and production planning problem we considered is formulated as:

$$(P0) \qquad \min \sum_{i\in N}(f_i y_i + p_{i0}Q_{i0}) + \sum_{i\in N}\sum_{j\in N} c_{ij}x_{ij} + \sum_{i\in N}\sum_{t\in T}(q_{it}z_{it} + p_{it}Q_{it} + h_{it}I_{it}) \tag{3.1}$$

$$\textit{Subject to} \quad Q_{i0} \le My_i \quad \forall i \in N \tag{3.2}$$

$$x_{ij} \le y_i \quad \forall i \in N, \forall j \in N \tag{3.3}$$

$$\sum_{i\in N} x_{ij} = 1 \quad \forall j \in N \tag{3.4}$$

$$z_{it} \le y_i \quad \forall i \in N, \forall t \in T \tag{3.5}$$

$$Q_{it} \le Mz_{it} \quad \forall i \in N, \forall t \in T \tag{3.6}$$

$$Q_{it} \leq Q_{i0} \quad \forall i \in N, \forall t \in T \tag{3.7}$$

$$\sum_{j \in N} d_{jt} x_{ij} \leq I_{i,t-1} + Q_{it} - I_{it} \quad \forall i \in N, \forall t \in T \tag{3.8}$$

$$I_{i0} = 0 \quad \forall i \in N \tag{3.9}$$

$$y_i \in \{0,1\} \quad \forall i \in N \tag{3.10}$$

$$x_{ij} \in \{0,1\} \quad \forall i \in N, \forall j \in N \tag{3.11}$$

$$z_{it} \in \{0,1\} \quad \forall i \in N, \forall t \in T \tag{3.12}$$

$$Q_{it} \geq 0 \quad \forall i \in N, \forall t \in T \tag{3.13}$$

$$I_{it} \geq 0 \quad \forall i \in N, \forall t \in T \tag{3.14}$$

The aim of the objective function (3.1) is to minimize the total cost of opening facility, establishing capacity, assignment, production setup, production and inventory. The first term is the cost incurred in opening facility and establishing capacity. The second term represents the cost of delivering production from facilities to demand nodes. The last term stands for the cost of production setup, production and inventory. Without loss of generality, we assume that the inventory level at the beginning of the planning horizon is null,i.e. $I_{i0} = 0$, $\forall i \in N$. Constraints (3.2) ensure that production capacity is only available at the opened facilities. Constraints (3.3) describe a facility must be set up if the demand node is assigned to it. Constraints (3.4) guarantee that each demand node must be assigned to a single facility. Constraints (3.5) ensure that only opened facility can be prepared to produce. Constraints (3.6) formulate that there is a non-null production level only if the production setup variable is 1 for each opened facility. Constraints (3.7) imply the production at a facility in each period is less than the maximum capacity originally established. Constraints (3.8) represent the material balance equation between production, inventory and demand. The set of constraints (3.10), (3.11), (3.12), (3.13) and (3.14) refer to the domain of the decision variables.

Once the facility is established, it will perform for a long time to fulfill the demand of the customers in future periods. The incurred cost can be explicitly represented by taking production planning decisions into consideration. If the values of all the parameters are precisely known, the problem (P0) could be solved as a mixed integer linear program (MILP) for the optimal decisions. However, the firm is inevitably exposed to many uncertain factors during the operational lifetime of a facility. That solution derived by the deterministic model would not necessarily be optimal and even unfeasible. Hence, it is important to consider uncertainty in this IFLPP modeling.

4 Robust Formulation of the IFLPP Model

For simplicity, we consider only demand uncertainty in this study. Other uncertainties such as production cost or inventory holding cost can readily be included by the same approach. Let $\tilde{d}_{jt}$ be the uncertain demand at node j in period t. Assume that demand $\tilde{d}_{jt}$ in each period is unknown and bounded on a symmetric interval around a known nominal value. Suppose $\tilde{d}_{jt}$ take values in the interval $[\bar{d}_{jt} - \hat{d}_{jt}, \bar{d}_{jt} + \hat{d}_{jt}]$, where $\bar{d}_{jt}$ is the mean or nominal value of $\tilde{d}_{jt}$ and $\hat{d}_{jt}$ is the maximal deviation from the nominal value. Define the box uncertainty set $U_{jt} = [\bar{d}_{jt} - \hat{d}_{jt}, \bar{d}_{jt} + \hat{d}_{jt}]$, then $\tilde{d}_{jt} \in U_{jt}$.

4.1 The Complete Protection Robust Formulation

In this subsection, the Soyster's approach is employed to formulate the robust counterpart of the problem (P0), which is called complete protection robust approach. Soyster (1973) proposes a linear optimization model to construct a solution that is feasible for all data realization that belongs in a convex set. We transform the problem (P0) into a new problem expressing uncertainty by substituting $\tilde{d}_{jt}$ for d_{jt} in constraints (3.8), then augmenting it with the constraints $\tilde{d}_{jt} \in U_{jt}$ for all $i \in N$ and $t \in T$.

The augmented constraints for (3.8) are:

$$\max_{\tilde{d}_{jt} \in [\bar{d}_{jt} - \hat{d}_{jt}, \bar{d}_{jt} + \hat{d}_{jt}]} \sum_{j \in N} \tilde{d}_{jt} x_{ij} \leq I_{i,t-1} + Q_{it} - I_{it} \quad \forall i \in N, \forall t \in T$$

Noting, $x_{ij} \geq 0 (\forall i \in N, \forall j \in N)$, the constraints imply

$$\sum_{j \in N} (\bar{d}_{jt} + \hat{d}_{jt}) x_{ij} \leq I_{i,t-1} + Q_{it} - I_{it} \quad \forall i \in N, \forall t \in T \tag{4.1}$$

The complete protection robust counterpart of the problem (P0) is:

$$\begin{aligned} (P1) \qquad & min \sum_{i \in N} (f_i y_i + p_{i0} Q_{i0}) + \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij} + \sum_{i \in N} \sum_{t \in T} (q_{it} z_{it} + p_{it} Q_{it} + h_{it} I_{it}) \\ Subject \ \ to \ & \sum_{j \in N} (\bar{d}_{jt} + \hat{d}_{jt}) x_{ij} \leq I_{i,t-1} + Q_{it} - I_{it}, \forall i \in N \quad \forall t \in T \\ & (3.2) - (3.7), (3.9) - (3.14) \end{aligned}$$

As the Soyster's methodology is equivalent to a linear optimization problem in which all uncertain parameters have been valued at their worst-case from the uncertainty set, the robust optimal objective is much worse than the optimal value of the deterministic problem. It may lead to more facility opened or capacity redundancy. That is, the complete protection robust approach places too much emphasis on good application of the solution and the tractability of the model at the expense of a severe deterioration in the objective function. It is considered over-conservative.

This extreme approach has been widely applied in some engineering applications of robustness, such as robust control theory, since doomed satellite launch or a destroyed unmanned robot has the high-profile repercussions. But it is less advisable in operations research and management science, because adverse events such as loss of customers do not result in the serious consequences that engineering failures may lead to. In the next subsection, we employ a robust method that is more appealing to the business practitioners that offers full control on the degree of conservatism of the robust solution.

4.2 The Budget-of-Uncertainty Robust Formulation

Although the Soyster(1973) robust approach admits the highest protection from parameter uncertainty under the model of data uncertainty U, i.e. complete protection, it is also the most conservative in practice in the sense that the robust solution has an objective function value much worse than the optimal value of the deterministic model. That is, the robust solution of Soyster's method affects the objective function excessively in order to guarantee the robustness of the solution. In this subsection, we present a novel robust formulation of the IFLPP problem, which retains the advantages of the linear framework of Soyster's method. More importantly, our model offers full control on the degree of conservatism of the robust solution. This control is achieved by imposing a so-called budget of uncertainty, denoted by Γ_{it}. We protect against the violation of constraints (3.8) deterministically, when only a prespecified number Γ_{it} of the uncertain coefficients changes; that is, we guarantee that the solution is feasible if less than Γ_{it} uncertain coefficients deviate from their nominal value. Moreover, we provide a probabilistic guarantee that even if more than Γ_{it} uncertain coefficients change, then the robust solution will be feasible with high probability. This robust methodology is referred to as the budget-of-uncertainty robust approach.

4.2.1 The Robust Modeling

We now quantify the concept mentioned above in mathematical terms. For each constraint in (3.8), we introduce a parameter Γ_{it}, not necessarily an integer, which takes values in the interval $[0, n]$,where n is the number of nodes in the network G. As would become clear below, the role of the parameter Γ_{it} is to adjust the degree of conservatism of the solution and is called the protection level. Speaking intuitively, it is unlikely that all of d_{jt} may deviate from their nominal value. Our system can be protected against all cases that up to $\lfloor \Gamma_{it} \rfloor$ of these uncertain coefficients are allowed to change, and one uncertain coefficient d_{jt} changes by $(\Gamma_{it} - \lfloor \Gamma_{it} \rfloor)\hat{d}_{jt}$. We consider the following formulation:

$$(P2) \qquad min \sum_{i\in N}(f_i y_i + p_{i0}Q_{i0}) + \sum_{i\in N}\sum_{j\in N} c_{ij}x_{ij} + \sum_{i\in N}\sum_{t\in T}(q_{it}z_{it} + p_{it}Q_{it} + h_{it}I_{it})$$

$$Subject\ \ to \quad \sum_{j\in N}(\bar{d}_{jt} + g_{it}(x, \Gamma_{it}) \le I_{i,t-1} + Q_{it} - I_{it} \quad \forall i \in N, \forall t \in T \tag{4.2}$$

$$\text{(3.2)-(3.7),(3.9)-(3.14)}$$

Where

$$g_{it}(x, \Gamma_{it}) = \max_{\{S\bigcup\{k\}|S\subseteq N,|S|=\lfloor \Gamma_{it}\rfloor,k\in N\setminus S\}} \sum_{j\in S}\left\{\hat{d}_{jt}x_{ij} + (\Gamma_{it} - \lfloor \Gamma_{it}\rfloor)\hat{d}_{kt}x_{ik}\right\} \tag{4.3}$$

$g_{it}(x, \Gamma_{it})$ is called the protection function. Obviously, $g_{it}(x, \Gamma_{it}) \ge 0$. If Γ_{it} is an integer, this protection function can be simplified as:

$$g_{it}(x, \Gamma_{it}) = \max_{\{S|S\subseteq N,|S|=\lfloor \Gamma_{it}\rfloor\}} \sum_{j\in S} \hat{d}_{jt}x_{ij}$$

In order to reformulate the model (P2) as a linear optimization model, some transformations should be done with the constraints (4.2). For each constraint in (4.2), we introduce auxiliary variables $D_{ijt}(\forall j \in N)$, and then the protection function $g_{it}(x, \Gamma_{it})$ can be written as the following linear optimization problem (G_{it}):

$$(G_{it}) \qquad g_{it}(x, \Gamma_{it}) = max\left\{\sum_{j\in N}\hat{d}_{jt}x_{ij}D_{ijt}\right\}$$

$$Subject\ \ to \ \sum_{j\in N} D_{ijt} = \Gamma_{it}$$

$$0 \le D_{ijt} \le 1 \quad \forall j \in N$$

If $\Gamma_{it} = 0$, then $D_{ijt} = 0 (\forall i \in N, \forall j \in N, \forall t \in T)$, $g_{it}(x, \Gamma_{it}) = 0$, the constraints (4.2) are equivalent to the constraints (3.8) in the deterministic problem, then the model (P2) is transformed to the model (P0). If $\Gamma_{it} = n$, then $D_{ijt} = 1 (\forall i \in N, \forall j \in N, \forall t \in T)$, $g_{it}(x, \Gamma_{it}) = \sum_{j \in N} \hat{d}_{jt} x_{ij}$, the constraints (4.2) are equivalent to the constraints (4.1), that is we have the Soyster's method, then the model (P2) is transformed to the model (P1). In other words, the problem (P2) naturally generalizes the problems (P0) and (P1).

Consider the dual of the problem (G_{it}) as follows:

$$
\begin{aligned}
(DG_{it}) \qquad & min \;\; \chi_{it}\Gamma_{it} + \sum_{j \in N} \beta_{ijt} \\
Subject\ to \;\; & \chi_{it} + \beta_{ijt} \geq \hat{d}_{jt} x_{ij} \quad \forall j \in N \\
& \chi_{it} \geq 0 \\
& \beta_{ijt} \geq 0 \quad \forall j \in N
\end{aligned}
$$

Where χ_{it} and β_{ijt} are the dual variables associated to the constraints in the problem (G_{it}). By strong duality, since the problem (G_{it}) is feasible and bounded for all $\Gamma_{it} \in [0, n]$, then the dual problem (DG_{it}) is also feasible and bounded and their objective values coincide.

By incorporating the problem (DG_{it}) into the constraints (4.2), we obtain the budget-of-uncertainty robust counterpart of the model (P0) with demand uncertainty is:

$$
(P2) \qquad min \sum_{i \in N} (f_i y_i + p_{i0} Q_{i0}) + \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij} + \sum_{i \in N} \sum_{t \in T} (q_{it} z_{it} + p_{it} Q_{it} + h_{it} I_{it})
$$

$$
\begin{aligned}
Subject\ to \quad & \sum_{j \in N} \bar{d}_{jt} x_{ij} + \chi_{it}\Gamma_{it} + \sum_{j \in N} \beta_{ijt} \leq I_{i,t-1} + Q_{it} - I_{it} \quad \forall i \in N, \forall t \in T && (4.4) \\
& \chi_{it} + \beta_{ijt} \geq \hat{d}_{jt} x_{ij} \quad \forall i \in N, \forall j \in N, \forall t \in T && (4.5) \\
& \chi_{it} \geq 0 \quad \forall i \in N, \forall t \in T && (4.6) \\
& \beta_{ijt} \geq 0 \quad \forall i \in N, \forall j \in N, \forall t \in T && (4.7)
\end{aligned}
$$

(3.2)-(3.7),(3.9)-(3.14)

It is obvious that the robust counterpart is a linear integer programming as the deterministic problem. The robust model (P2) can be readily solved through standard optimization tools, which is of course very appealing.

Let P_0^* and P_2^* denote the optimal cost resulting from (P0) and (P2), respectively. Simply note that

$$
\chi_{it}\Gamma_{it} + \sum_{j \in N} \beta_{ijt} \geq 0 \quad \forall i \in N, \forall t \in T
$$

So every solution that satisfies the constraints (4.2) also satisfies the constraints (3.8). Then it is easy to see that $P_0^* \leq P_2^*$. Hence, we call the difference $P_2^* - P_0^*$ the price of robustness.

4.2.2 The Service Level of the Facility

Let X^* be the optimal solution of the problem (P2). The budget-of-uncertainty robust approach can only ensure that X^* is deterministically feasible if at most Γ_{it} coefficients d_{jt} change in each constraint in (3.8). But, what happens when more than Γ_{it} coefficients d_{jt}

change? It is proved that even if more than Γ_{it} coefficients d_{jt} change, X^* is feasible for the constraints (3.8) with a high probability depending on the chosen Γ_{it}. According to the budget-of-uncertainty approach of Bertsimas and Sim (2003, 2004), the probability of X^* being infeasible for the constraints (3.8) can be calculated as follows

$$\begin{aligned} &Pr\left\{\textstyle\sum_{j\in N}\tilde{d}_{jt}x_{ij} \geq I_{i,t-1}+Q_{it}-I_{it}\right\} \leq B(n,\Gamma_{it}) \\ &= \tfrac{1}{2^n}\left\{(1-\mu)\textstyle\sum_{l=\lfloor\nu\rfloor}^{n}\binom{n}{l}+\mu\sum_{l=\lfloor\nu\rfloor+1}^{n}\binom{n}{l}\right\} \quad \forall i\in N, \forall t\in T \end{aligned} \tag{4.8}$$

where $n = |N|$, $\nu = \frac{\Gamma_{it}+n}{2}$, $\mu = \nu - \lfloor\nu\rfloor$. The bound in (4.8) may be difficult to compute due to the combinations, but the following expression yields an easy-to-compute bound and a very good approximation of (4.8):

$$Pr\left\{\sum_{j\in N}\tilde{d}_{jt}x_{ij} \geq I_{i,t-1}+Q_{it}-I_{it}\right\} \leq (1-\mu)C(n,\lfloor\nu\rfloor)+\sum_{l=\lfloor\nu\rfloor+1}^{n}C(n,l) \quad \forall i\in N, \forall t\in T \tag{4.9}$$

where

$$C(n,l) = \begin{cases} \frac{1}{2^n} & \text{if } l=0 \text{ or } l=n \\ \frac{1}{\sqrt{2\pi}}\sqrt{\frac{n}{(n-l)l}}e^{\ln\frac{n}{2(n-l)}+l\ln\frac{n-l}{l}} & \text{otherwise} \end{cases}$$

A good feature of the above probability bound (4.9) is that they are independent of the optimal solution X^* and a relatively small Γ_{it}, compared to n, gives a high probability for the feasibility of the robust solution, which also will be seen in the numerical example in the next section. The parameter Γ_{it} is known as the protection level in the sense that it offers full control on the probability of constraints violation and the conservatism of the robust solution.

By using the budget-of-uncertainty robust approach, the decision maker can make a full control on the degree of conservatism of the robust solution by choosing Γ_{it} appropriately. In addition, it provides a probabilistic guarantee that even if more than Γ_{it} coefficients change, the robust solution will be feasible with a high probability bound. More importantly, the robust counterpart of the original problem is a linear optimization problem, which is computationally tractable.

The service capability or reliability level, of facility $i(i = 1, 2, \ldots, n)$, is represented by the service level requirement. It can be described as the probability that the facility can fulfill the stochastic demand that assigned to it (Service Level), i.e.

$$ServiceLevel = Pr\left\{\sum_{j\in N}\tilde{d}_{jt}x_{ij} \leq I_{i,t-1}+Q_{it}-I_{it}\right\} \tag{4.10}$$

Then the service level of the facility in our problem is

$$ServiceLevel \geq 1-\left\{(1-\mu)C(n,\lfloor\nu\rfloor)+\sum_{l=\lfloor\nu\rfloor+1}^{n}C(n,l)\right\} \tag{4.11}$$

Obviously, given the protection level Γ_{it}, we can compute the service level of the facility according to the formula (4.11). The reverse problem is, given the service level required, how to make the facility location and production planning decisions under uncertain demand. Obviously, we can also use the formula (4.11) to compute the value of the protection level Γ_{it} and incorporate the value to the problem (P2). The optimal decisions made by solving the problem (P2) can reach the service level required.

Actually, we can use bisection method to compute the value of the protection level Γ_{it}. Let SL^* denotes the service level required, n_{max} denotes the maximum number of iterations and θ denotes the optimality tolerance specified by the decision maker. The detailed steps of using bisection method to compute Γ_{it} are described as follows:

Step 1: Let $l_0 = 0$, $h_0 = n$, $k = 0$, $\Gamma_{it} \leftarrow \frac{l+h}{2}$, $SL_k = 1 - \left\{(1-\mu)C(n, \lfloor\nu\rfloor) + \sum_{l=\lfloor\nu\rfloor+1}^{n} C(n,l)\right\}$, $k \leftarrow k+1$, go to step 3;

Step 2: $Mid = \frac{l_k+h_k}{2}$, $\Gamma_{it} \leftarrow Mid$, $SL_k = 1 - \left\{(1-\mu)C(n, \lfloor\nu\rfloor) + \sum_{l=\lfloor\nu\rfloor+1}^{n} C(n,l)\right\}$, $k \leftarrow k+1$;

Step 3: if $SL_k \le SL^*$, $l_k \leftarrow Mid$, $h_k \leftarrow h_{k-1}$; if $SL_k \ge SL^*$, $l_k \leftarrow l_{k-1}$,$h_k \leftarrow Mid$; else $\Gamma^*_{it} \leftarrow \Gamma_{it}$, stop;

Step 4: if $\left|\frac{SL_k - SL^*}{SL^*}\right| \le \theta$ or $k \ge n_{max}$, $\Gamma^*_{it} \leftarrow \Gamma_{it}$ stop; else, go to step 2.

5 Numerical Case Study

In this section, we conduct some numerical experiments to illustrate the differences among the optimal solutions provided by the deterministic model (P0), the complete protection robust model (P1) and the budget-of-uncertainty robust model (P2). We firstly compare the different solution topology under different protection level, by considering the number of facilities opened, their capacities, and the network topology. Furthermore, the trade-offs between the total cost and the protection level is deeply analyzed and the price of robustness in our model is investigated based on the service level. The models are coded in the General Algebraic Modeling System (GAMS) and solved using the optimization system ILOG-Cplex 12.5.

5.1 Test Environment

We randomly generate n nodes in the square [0,100]$\times$[0,100], representing both the candidate facility locations and the potential demand nodes. The nominal demand at each demand node in each period, d_{jt}, is independent of all others and is assumed constant over the T periods. Let $T = 10$. The nominal demand at each node in each period is drawn from the uniform distribution $Uniform[1000, 2000]$. Other input parameters are generated in Table 1. The delivery cost $c_{ij} = h \times e_{ij}, e_{ij}$ is the Euclidean distance between nodes i and j, h is the delivery cost of unit distance.

Assume $\hat{d}_{jt} = \bar{d}_{jt}\varepsilon_{jt}$, where $0 \le \varepsilon_{jt} \le 1$ measures the uncertainty size at node j in period t. The uncertainty sets are generated as follows. In the base case, we let r be the initial (first period) uncertainty of the demand, and let $\varepsilon_t = r + (1-r)\varepsilon_{t-1}$ with $\varepsilon_0 = 0$. The box uncertainty sets $U_{jt} = \bar{d}_{jt}[1 \pm \varepsilon_t]$. Further computation obtains that $\varepsilon_t = 1-(1-r)^t$. Since ε_t is concave, increasing function in t, U_{jt} is increasing in t. In Figure 1, the three lines depict the boundaries of the uncertain level when the initial uncertain level equal to 0.04, 0.05 and 0.06, respectively.

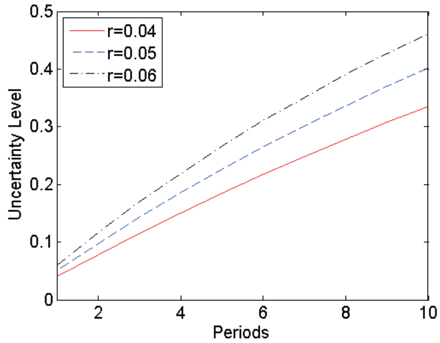


Figure 1: Uncertain Level Bound Given Different Initial Uncertain Levels

5.2 Comparing the Network Topology of the Solutions

We compare the number of facilities opened, the capacity established at each opened facility, and the assignment of the demand nodes to the opened facilities under different degree of conservatism in our robust model. To do so, we generate 20 nodes uniformly over the square of [0,100]×[0,100] and the demands are drawn from the uniform distribution $Uniform[1000, 2000]$. For the random generate example, we solve the model (P2) by setting $\Gamma_{it} = 0$, $\Gamma_{it} = 1$, $\Gamma_{it} = 2$ and $\Gamma_{it} = 20$, respectively. As the above analysis, the model (P2) equals to the non-protection model (P0) (i.e. deterministic model) when $\Gamma_{it} = 0$ and equals to the complete protection model (P1) when $\Gamma_{it} = 20$. In this case, the model (P1) reduces to a worst-case deterministic model. To avoid confusion, the model (P0) can be referred to as the mean deterministic model and the model (P1) can be referred to as the worst-case deterministic model.

In Table 2, we present the solutions of the model (P2) under four different values of Γ_{it}. The second line from the bottom is the mean number of demand nodes served by each opened facility, which is defined as

$$\frac{\sum_i \sum_j H\{x_{ij}\}}{\sum_i H\{y_i\}}$$

where $H\{\ \}$ is the indicator function. The last line is the facility location cost including the initial cost of opening the facilities and establishing their capacities. The topology of the network with different Γ_{it} is shown in the subfigures of Figure 2. In the figures, a red star represents a demand node (co-located with a candidate facility), a red circle denotes an opened facility, and a black line indicates that the facility is used to serve the demand at the node. The size of the blue circle is proportional to the capacity of the facility (the scale is 8:10000).

Table 1: Parameter Values

Parameter	Values
Fixed facility location cost f_i	2000
Production setup cost p_{i0}	5
Variable cost of establishing per unit capacity p_{it}	50
Per unit production cost q_{it}	1
Per unit inventory holding cost h_{it}	0.5
Per unit inventory holding cost d_{jt}	$uniform[1000, 2000]$

We make several observations. First, the robust models with different Γ_{it}-values ($0 < \Gamma_{it} < 20$) open less facilities compared with the two deterministic models (the models (P0) and (P1)). Moreover, with the increasing of $\Gamma_{it}(0 < \Gamma_{it} < 20)$, opening more facilities or establishing larger capacity are two strategies to deal with greater uncertainty. Second, for two extreme models (P0) when $\Gamma_{it} = 0$ and (P1) when $\Gamma_{it} = 20$, the mean number of demand nodes served by each facility is 4 and 3.33, respectively. However, when $0 < \Gamma_{it} < 20$ in the model (P2), the number is 6.67 and 10 when Γ_{it} is equal to 1 and 2, respectively. That is, when $0 < \Gamma_{it} < 20$, the average demand nodes served by each facility is less than that of the extreme models (P0) when $\Gamma_{it} = 0$ and (P1) when $\Gamma_{it} = 20$.

Table 2: Comparison Results of the Solutions Topology for Different Γ_{it}-Values

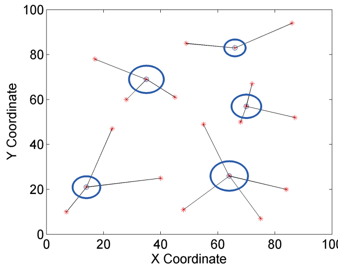
			The protection level			
			0	1	2	20
Number	X Label	Y Label	Capacity	Capacity	Capacity	Capacity
1	28	60	0	13276	0	9861
2	45	61	0	0	0	0
3	66	83	4670	0	0	5818
4	17	78	0	0	0	0
5	70	57	6498	0	19605	0
6	14	21	6065	0	0	0
7	64	26	8188	10165	0	0
8	49	85	0	0	0	0
9	75	7	0	0	0	4000
10	55	49	0	0	0	0
11	84	20	0	0	0	0
12	86	94	0	0	0	0
13	35	69	7591	0	0	0
14	68	50	0	0	0	11583
15	23	47	0	0	13627	0
16	7	10	0	0	0	1752
17	72	67	0	9695	0	0
18	40	25	0	0	0	6104
19	48	11	0	0	0	0
20	87	52	0	0	0	0
The total capacity established			33012	33136	33232	39117
The mean number of demand nodes each facility served			4	6.67	10	3.33
Facility location cost			175060	171680	170160	207580

5.3 The Price of Robustness

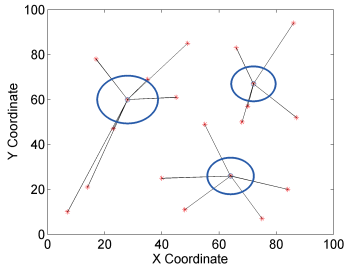
Robustness, viewed as the guarantee of a performance, comes at a cost. In the case of the integrated facility location and production planning problem, it is the probability that the demand can be fulfilled by the located facilities. In order to achieve robustness of the solution, a cost sacrifice will occur; that is, in order to withstand parameter uncertainty under the uncertain set U, the optimal total cost may be increased. But how much does the cost increase? And is it worth it? In order to investigate this trade-offs, we define measures of robustness and make deep analyses about them.

5.3.1 Measures of Cost

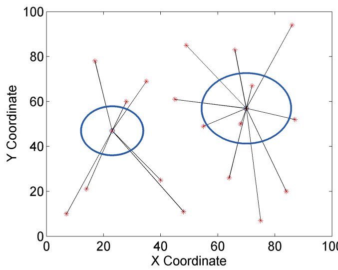
Let the initial uncertainty level r=0.04, 0.05 and 0.06. We solve the robust model (P2) for different values of Γ_{it} when the number of nodes n=20, 30 and 50. Figure 3 illustrates the effect of the protection level on the objective function value. It is obvious that the total cost increases when the value of Γ_{it} increases.



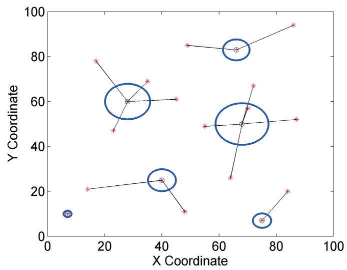
(a) Γ_{it}=0



(b) Γ_{it}=1



(c) Γ_{it}=2



(d) Γ_{it}=20

Figure 2: Network Representations of the Solution to Instance of (a) Γ_{it}=0 (The Mean Deterministic Model), (b) Γ_{it}=1,(c) Γ_{it}=2, (d) Γ_{it}=20 (The Worst-Case Deterministic Model)

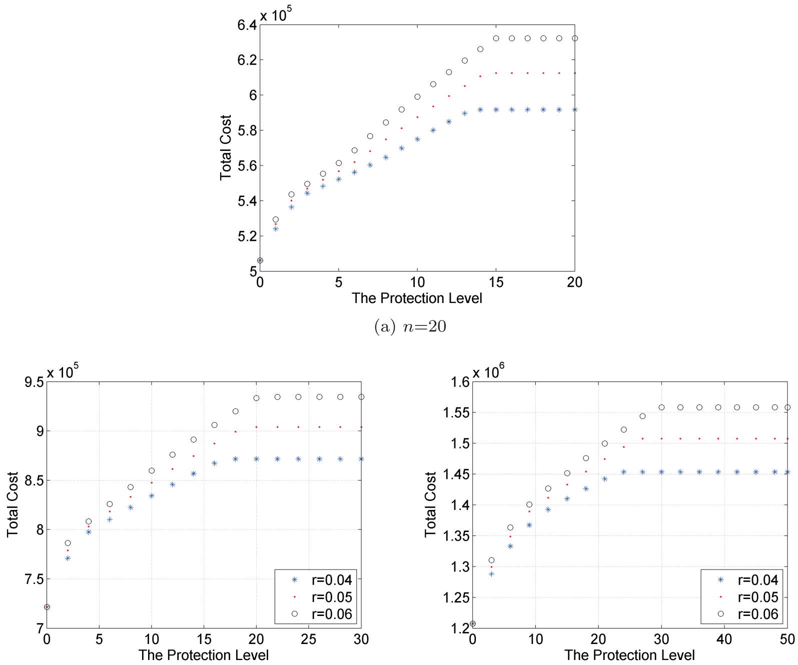
Further investigation reveals that the difference between the three curves get larger as the protection level increases. This is evident from the definition of Γ_{it}. For smaller Γ_{it}-values, the total amount of demands that may deviate from the nominal value we take into considered is small (when Γ_{it} is integer, there is at most Γ_{it} demands deviate from the nominal value simultaneously.). The uncertain level has relatively smaller effect on the total cost. But when Γ_{it}-values increases, the amount of demands that may deviate from the nominal value is larger. The uncertain level has relatively larger effect on the total cost. The results show that with the increase of the protection level, the system total cost get more and more sensitive to the uncertain level.

It can be seen from Figure 3 that the three curves are parallel to the horizontal ordinate when Γ_{it} is more than a certain value. This phenomenon can be explained by the constraints (3.8):

$$\sum_{j \in N} d_{jt} x_{ij} \leq I_{i,t-1} + Q_{it} - I_{it} \quad \forall i \in N, \forall t \in T$$

When we use the budget-of-uncertainty robust approach, we impose a Γ_{it}-value to each constraint in (3.8), $\Gamma_{it} \in [0, n]$, i.e. we consider Γ_{it} demands change simultaneously. The total number of nonzero terms on the left hand side of the constraints (3.8) may be less than n, as is the case in most instances, because for some j, $x_{ij} = 0$. Suppose the maximum number of nonzero terms on the left hand side of constraints (8) is k. When $\Gamma_{it} > k$, the objective value retains the same with the objective value when $\Gamma_{it} = k$.

The effects of the protection level on the facility location cost, the operational cost and the total cost are depicted in Figure 4. The facility location cost (the initial cost of opening the facilities and establishing their capacities) decrease firstly and then increase as the protection level increases. The operational cost (the delivery, production setup, production and inventory cost) increase firstly and then decrease when the protection level



(a) n=20

(b) n=30

(c) n=50

Figure 3: The Protection Level vs the Total Cost.

increases. The total cost (the facility location cost and the operational cost) strictly increase in the protection level.

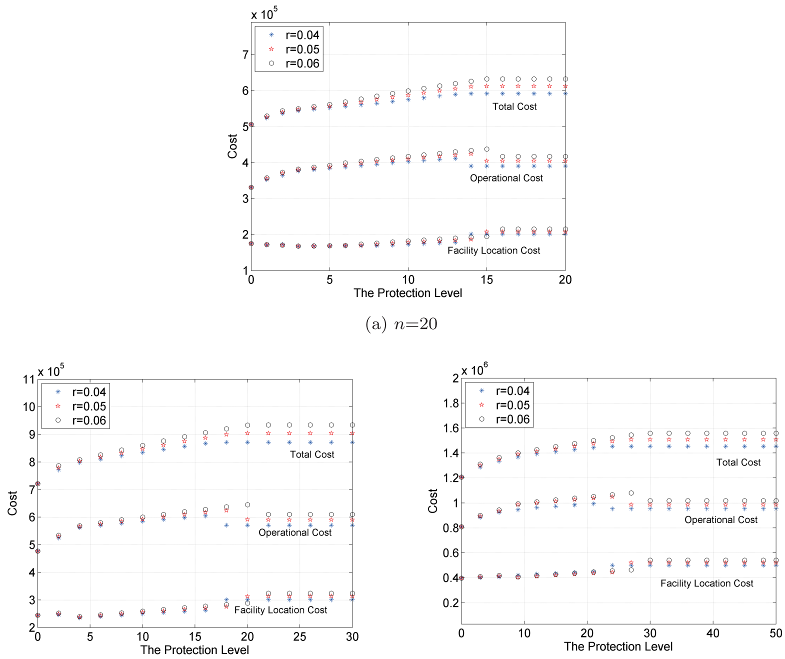
The above qualitative analyses show the trend of the curves and give us an intuitionistic picture of how the protection level effect on the cost. Next, we analyze the trade-offs between the service level and the cost quantitatively. Let P_0^*, P_1^* and P_2^* denote the optimal total cost resulting from the model (P0), (P1) and (P2), respectively. We introduce two measures for the price of robustness.

$$Price1 = \frac{P_2^* - P_0^*}{P_0^*} \tag{5.1}$$

$$Price2 = \frac{P_1^* - P_2^*}{P_1^*} \tag{5.2}$$

The $Price1$ given by the equation (5.1) measures the relative difference between the optimal objective of the mean deterministic problem and the objective function value evaluated at the robust optimal solutions. While the $Price2$ given by the equation (5.2) measures the relative difference between the optimal value of worst-case deterministic model and the objective function value evaluated at the robust optimal solutions. Obviously, $Price1 \geq 0$ and $Price2 \geq 0$.

The trade-off curves between $Price1$ and the facility service level are depicted in Figure 5. $Price1$ increases in the service level. As is shown in Figure 5, when the service level is too low or too high, the curves increase slowly. Specifically, when the service level approaches 1, the curves are almost parallel to the horizontal ordinate. But when the service level is between 0.6 and 0.9, the curves increase sharply. It is shown that large increases in service



(a) n=20

(b) n=30

(c) n=50

Figure 4: Facility Location Cost, Operational Cost and Total Cost under Different Protection Level.

level are not necessary with large increases in $Price1$. This result reveals that higher service level can be attained without large increases in $Price1$, that is, higher service level not always necessarily results in much sacrifice in the total cost. This is evident from Figure 5 since the tradeoff curves are steep when the service level is between 0.6 and 0.9. According to the trade-offs between $Price1$ and the service level, the decision maker can compute the $Price1$ based on the service level or obtain the service level depending on $Price1$.

To withstand the demand fluctuation, one extreme way is to design the network according to the maximal demand, i.e. the model (P1). The optimal solution of the model (P1) is feasible for any realization of the uncertain parameters, i.e. the service level of the facility is 100%. However, it may be costly. Obviously, the objective function of (P2) is smaller than that of (P1), but the service level resulting from (P2) is less than 100%. To measure the total cost decrease, we define $Price2$. The trade-off curves between $Price2$ and the service level are depicted in Figure 6. It can be seen that the $Price2$ increase when the service level decreases. When the service level is higher, especially the service level approaches 1, the curves are smooth, and it implies that large reduction in the total cost, compared to that of the worst-case problem, will lead to small decrease in the service level.

5.3.2 Measures of robustness

The budget-of-uncertainty robust approach ensures that the optimal solution, X^*, is deterministically feasible if at most Γ_{it} coefficients d_{jt} deviate from their nominal value in each constraint in (3.8); otherwise, X^* is feasible with a high probability depending on the chosen Γ_{it}. To examine the quality of the robust solution, we run 1000 simulations of random generated demand and compare robust solutions generated by varying the Γ_{it}-values. We define the two following measures of service level:

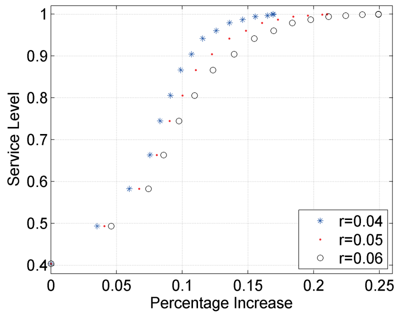


Figure 5: Service Level vs. Percentage Increase in Total Cost When n=20

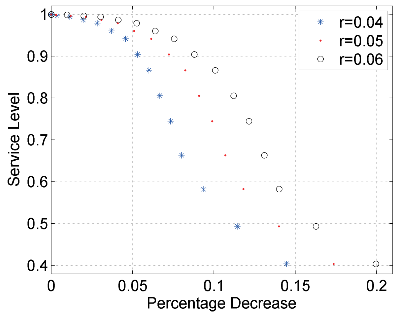


Figure 6: Service Level vs. Percentage Decrease in Total Cost When n=20

$$\begin{aligned} SL1 &= Pr\left\{\sum_{j\in N}\tilde{d}_{jt}x_{ij} \geq I_{i,t-1} + Q_{it} - I_{it}\right\} \\ &= (1-\mu)C(n,\lfloor\nu\rfloor) + \sum_{l=\lfloor\nu\rfloor+1}^{n} C(n,l), \forall i\in N, \forall t\in T \end{aligned} \tag{5.3}$$

$$SL2 = \frac{\sum_{j\in N}\sum_{t\in T} H\left\{d_{jt}x_{ij} - I_{i,t-1} - Q_{it} + I_{it}\right\}}{i\times t} \tag{5.4}$$

where $H\{\ \}$ is the indicator function. $H\{\ \}$ is 1 if $\sum_{j\in N} d_{jt}x_{ij} \geq I_{i,t-1} + Q_{it} - I_{it}$ and 0 otherwise. $SL1$ is the theoretical bound of constraints violation and $SL2$ is the actual probability of constraints violation. In Figure 7, we compare the theoretical bound in Equation (5.3) with the fraction of the simulated constraints violation given by Equation (5.4). The empirical results show that the theoretical bound is close to the empirically observed values. To measure the difference between the theoretical bound and the empirically observed values, we define the following measure:

$$Cost = \frac{abs(SL1 - SL2)}{n} \tag{5.5}$$

Where function $abs(\ )$ denotes the absolute value of $(SL1-SL2)$. From the above random generate simulations, we can compute $Cost = 0.1194\%$, which implies that the difference between the theoretical bound and the empirically observed values is very small.

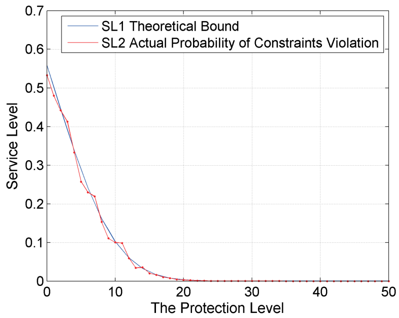


Figure 7: Simulation Study of the Probability of Constraints Violation as a Function of Γ_{it} When n=50

6 Conclusions

The objective of this study is to investigate potential methods by which robust optimization techniques could be used to solve the facility location and production planning problem in the multi-periods. Previous work on facility location problem under demand uncertainty hasnt taken the degree of conservatism of the solution into account. The budget-of-uncertainty robust approach we employed offers full control on the degree of conservatism of the robust solution by solving a linear program. The violation of constraints is protected against deterministically, when only a prespecified number, Γ, of the uncertain coefficients change. We compare the different solution topology under different protection level. The numerical example shows that with the increase of Γ_{it}-values($0 < \Gamma_{it} < n$), it opens more facilities or establishes more capacity. In order to investigate the trade-offs between the robustness and the cost, we define measures of robustness and make analyses about them. The numerical results show that large increases in service level are possible with small increases in total cost compared with that of the deterministic problem. In order to ensure robustness of the solution, it may sacrifice not too much of the total cost, that is to say, it may cost a little to buy robustness.

In this paper, we suppose that the nominal demand and its deviation in each period are known at the beginning of the planning horizon. Future work may consider the nominal demand and its deviation are evaluated at the beginning of each period based on the information of the last period. The robust counterpart of the deterministic problem is a mixed integer linear program (MILP) and we use the optimization system ILOG-Cplex 12.5 to solve it. In the numerical example, we set the maximal number of node is 50 and the planning period is 10. The optimization system is very effective when the test network is middle-sized and the time period is not long. But the processing time increases considerably when the network is large-sized and the time period is long. An effective heuristic algorithm should be explored for solving large scale robust model.

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