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EXPONENTIAL H_∞ STABILIZING CONTROL OF A CLASS OF UNCERTAIN IMPULSIVE SWITCHED SYSTEMS

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Dedicated to Professor Shu-Cherng Fang for his 60th birthday.

Abstract: This paper is concerned with the robust H_∞ optimal stabilization problem for a class of impulsive switched linear systems with norm-bounded time-varying uncertainty. Based on a Riccati inequality approach, sufficient conditions of robust H_∞ exponential stability of the impulsive closed-loop systems are derived and those results are hence applied to design effective linear state feedback stabilizing controllers. A numerical example is presented to illustrate the obtained results and the control synthesis procedure.

Key words: *robust controller, H_∞, impulsive switched systems*

Mathematics Subject Classification: *93B52, 93B36*

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1 Introduction

There exist many dynamical processes with impulsive effects in physics, chemical engineering, biology and information science. For example, the population of a kind of insects can be controlled by leaving its natural enemies at some proper time instants and the operation process of chemical reactors can be regulated by impulsively adding chemicals to instantaneously change the solution's concentration. These dynamical behaviors with abrupt state change characteristics can be modeled by impulsive systems [1, 4, 10]. Besides the above impulsive models, dynamics of the total stock value of a particular investor can be modelled by an impulsive systems as well; see, for example, [5] and the references therein. Stability problems of these impulsive systems have been investigated in [1, 2] and various stability criteria have been provided. Based on these results, sufficient conditions of exponential stability of impulsive systems with time delays have been studied and presented in [6] by using the method of Lyapunov functionals.

In recent decades, an impulsive switched system, naturally describing dynamical processes with the interaction of impulses and switchings, has attracted more and more attention. Stability and control synthesis problems of impulsive switched systems have been studied in [7–9, 11]. In particular, a variety of linear quadratic controllers are devised to achieve guaranteed cost control performance in [7, 9] and the corresponding guaranteed cost will be obtained as well. In this paper, we consider the robust H_∞ optimal stabilizing control problem of impulsive switched linear systems with time-varying uncertainty. We hence design an appropriate feedback controller such that the controlled impulsive switched closed-loop system is exponentially stable and the robust H_∞ optimal performance will be satisfied at the same time. A Riccati inequality approach will be applied to construct these linear feedback controllers.

The rest of the paper is organized as follows. In Section 2, we present the robust H_∞ optimal stabilizing control problem of an impulsive switched linear system with time-varying uncertainty. In Section 3, sufficient conditions for the existence of exponential H_∞ optimal stabilizing controllers are derived by using Lyapunov stability theory and then the associate feedback controllers are obtained by solving algebraic Riccati inequalities. A numerical example is then presented to illustrate the feedback controller's design procedure in Section 4. Finally, this paper is concluded in Section 5.

2 System Description and Problem Statement

A linear impulsive switched system in which impulses occur at fixed time instants can be described by

$$\begin{cases} \dot{x}(t) = A_{i_k}(t)x(t) + C_{i_k}(t)u(t) & t \neq t_k \\ \Delta x(t) = I_k(t,x) & t = t_k,\ k = 1,2,\ldots,\ i_k \in \{1,2,\ldots,m\} \end{cases} \tag{2.1}$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control input, $A_{i_k}(t)$ and $C_{i_k}(t)$ are state matrices and control matrices of system (2.1) with appropriate dimensions, $I_k : R^n \to R^n$ are continuous functions, and t_k $(k = 1, 2, \cdots) \to \infty$ as $k \to \infty$. $\Delta x(t) = x(t^+) - x(t^-)$, $x(t^-) = \lim\limits_{h\to 0+} x(t-h)$, and $x(t^+) = \lim\limits_{h\to 0+} x(t+h)$. It means that the solution of impulsive switched system (2.1) is left continuous. When the above system experiences external disturbances, the uncertain impulsive switched system can be expressed in form of

$$\begin{cases} \dot{x}(t) = A_{i_k}x(t) + B_{i_k}w(t) + C_{i_k}u(t) & t \neq t_k \\ \Delta x(t) = I_k(t,x) = D_k x(t) & t = t_k \\ z(t) = E_{i_k}x(t) & \\ x(t) = 0, & t = t_0 = 0 \end{cases} \tag{2.2}$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control input, $w(t) \in R^p$ is the disturbance input, and $z(t) \in R^q$ is controlled output. $A_{i_k} \in R^{n\times n}$, $B_{i_k} \in R^{n\times p}$, $C_{i_k} \in R^{n\times m}$, $E_{i_k} \in R^{q\times n}$ are constant real matrices that describe known nominal system. D_k are a series of matrices with appropriate dimensions.

Definition 2.1. For a given $\gamma > 0$, the impulsive closed-loop system (2.2) is said to be robustly H_∞ exponentially stable if for any admissible uncertainty, the following conditions are satisfied under any switching law :

(i) Exponential stability: the resulting impulsive closed-loop system (2.2) is exponentially stable when $w(t) = 0$.

(ii) Robust H_∞ performance: when a positive constant γ is given as the objective performance, $\|z(t)\| \leq \gamma \|w(t)\|$ will be satisfied.

Lemma 2.2 ([3]). *Given a matrix $G \in R^{p\times q}$ such that $G^T G \leq I$ then*

$$2x^T G y \leq x^T x + y^T y \tag{2.3}$$

for all $x \in R^p$ and $y \in R^q$. In the case that G is an identity matrix, (2.3) reduces to $2x^T y \leq x^T x + y^T y$. Here, for two symmetric matrices A and B, $A \leq B$ ($A \geq B$) means the eigenvalues of $A - B$ are non-positive (non-negative, respectively).

Lemma 2.3 ([8]). *Given a positive definite matrix $P \in R^{n\times n}$ and a symmetric matrix $Q \in R^{n\times n}$ then*

$$\lambda_{\min}(P^{-1}Q)x(t)^T Px(t) \le x(t)^T Qx(t) \tag{2.4}$$

for all $x(t) \in R^n$.

The objective of this paper is to design a linear state feedback controller

$$u(t) = F_{i_k}x(t) \tag{2.5}$$

where $F_{i_k} \in R^{m\times n}$ are constant matrices such that the resulting impulsive closed-loop system is robustly H_∞ exponentially stable, i.e., the following closed-loop system

$$\begin{cases} \dot{x}(t) = (A_{i_k} + C_{i_k}F_{i_k})x(t) + B_{i_k}w(t) & t \ne t_k \\ \Delta x(t) = I_k(t,x) = D_k x(t) & t = t_k \\ z(t) = E_{i_k}x(t) & \\ x(t) = 0, & t = t_0 = 0 \end{cases} \tag{2.6}$$

will be exponentially stable satisfying a given H_∞ optimal performance constraint for all admissible uncertainties.

3 Exponential H_∞ Control

Theorem 3.1. *Let β_k be the largest eigenvalue of $P_{i_k}^{-1}(I+D_k)^T P_{i_k}(I+D_k)$ and $0 \le \beta_k \le 1$, $k \in N$, hold. For a given $\gamma > 0$, the impulsive closed-loop system (2.6) is robustly H_∞ exponentially stable if there exist positive definite symmetric matrices $P_{i_k} \in R^{n\times n}$ such that*

$$(A_{i_k} + C_{i_k}F_{i_k})^T P_{i_k} + P_{i_k}(A_{i_k} + C_{i_k}F_{i_k}) + \gamma^{-2}P_{i_k}B_{i_k}B_{i_k}^T P_{i_k} + E_{i_k}^T E_{i_k} < 0. \tag{3.1}$$

Proof. For the case of $t \in (t_k, t_{k+1}]$, let $u(t) = F_{i_k}x(t)$ and the resulting impulsive closed-loop system can be given by (2.6). First, without loss of generality, consider the following quadratic Lyapunov function candidate

$$V(t) = x(t)^T P_{i_k}x(t). \tag{3.2}$$

The derivative of the Lyapunov function (3.2) along the closed-loop system (2.6) is

$$\begin{aligned} \dot{V}(x) &= \dot{x}(t)^T P_{i_k}x(t) + x(t)^T P_{i_k}\dot{x}(t) \\ &= x(t)^T A_{i_k}^T P_{i_k}x(t) + x(t)^T A_{i_k}P_{i_k}x(t) + 2u(t)^T C_{i_k}^T P_{i_k}x(t) + 2x(t)^T P_{i_k}B_{i_k}w(t). \end{aligned}$$

Then, applying the linear feedback controller (2.5) and using the following inequality

$$2x(t)^T P_{i_k}B_{i_k}w(t) \le \gamma^{-2}x(t)^T P_{i_k}B_{i_k}B_{i_k}^T P_{i_k}x(t) + \gamma^2 w(t)^T w(t)$$

derived from Lemma 2.2, we have

$$\dot{V}(t) \le x(t)^T((A_{i_k}+C_{i_k}F_{i_k})^T P_{i_k}+P_{i_k}(A_{i_k}+C_{i_k}F_{i_k})+\gamma^{-2}P_{i_k}B_{i_k}B_{i_k}^T P_{i_k})x(t)+\gamma^2 w(t)^T w(t) \tag{3.3}$$

$$\begin{aligned} \dot{V}(t) &< -x(t)^T E_{i_k}^T E_{i_k}x(t) + \gamma^2 w(t)^T w(t) \\ &= -\|z(t)\| + \gamma^2\|w(t)\| \end{aligned}$$

which leads to

$$\|z(t)\| < -\dot{V}(t) + \gamma^2\|w(t)\|$$

and

$$\int_0^\tau \|z(t)\|^2 dt < -\int_0^\tau \dot{V}(t)dt + \gamma^2 \int_0^\tau \|w(t)\|^2 dt \qquad \tau \in (t_k, t_{k+1}]. \tag{3.4}$$

According to the zero initial condition and (3.2), we can see $V(0) = 0$, $V(t_k) > 0$. When $0 \le \beta_k \le 1$, we have

$$\begin{aligned}\int_0^\tau \dot{V}(t)dt &= \int_0^{t_1} \dot{V}(t)dt + \int_{t_1}^{t_2} \dot{V}(t)dt + \ldots + \int_{t_{k-1}}^{t_k} \dot{V}(t)dt + \int_{t_k}^{\tau} \dot{V}(t)dt \\ &= V(t_1) - V(0) + V(t_2) - V(t_1^+) + \ldots + V(t_k) - V(t_{k-1}^+) + V(\tau) - V(t_k^+) \\ &\ge \sum_{i=1}^{k} [1-\beta_k]V(t_k) + V(\tau) \ge 0.\end{aligned}$$

Then, from the above inequality and (3.4), it follows that

$$\|z(t)\|_\tau^2 = \frac{1}{\tau}\int_0^\tau \|z(t)\|dt < \frac{\gamma^2}{\tau}\int_0^\tau \|w(t)\|dt = \gamma^2\|w(t)\|^2$$

which shows that the H_∞ optimal performance will be satisfied for a given $\gamma > 0$. Next, for the case of $w(t) = 0$, then

$$\dot{V}(t) < -\|z(t)\| + \gamma^2 \|w(t)\| = -x(t)^T Q_{i_k} x(t) \tag{3.5}$$

where $Q_{i_k} = E_{i_k}^T E_{i_k}$. Then, by using

$$x(t)^T Q_{i_k} x(t) \ge \lambda_{\min}(P_{i_k}^{-1} Q_{i_k}) x(t)^T P_{i_k} x(t)$$

derived by Lemma 2.3, and (3.5), we have

$$\dot{V}(t) + \eta_k V(t) < 0 \qquad t \in (t_k, t_{k+1}]$$

where $\eta_k = \lambda_{\min}(P_{i_k}^{-1} Q_{i_k}) x(t)^T P_{i_k} x(t) > 0$. At the impulsive switching time instants, it follows from (2.6) and (3.2) that

$$\begin{aligned}V(t_k^+) &= x(t_k^+)^T P_{i_k} x(t_k^+) \\ &\le \beta_k x(t_k)^T P_{i_k} x(t_k) \\ &= \beta_k V(t_k)\end{aligned}$$

Let $\eta = \min(\eta_k)$, then

$$\begin{aligned}V(t) &< V(t_k^+)\exp(-\eta(t-t_k)) \qquad t \in (t_k, t_{k+1}] \\ &\le \beta_k V(t_k)\exp(-\eta(t-t_k)).\end{aligned}$$

When $t \in (t_0, t_1]$,

$$V(t) < V(t_0)\exp(-\eta(t-t_0))$$

then,

$$V(t_1) < V(t_0)\exp(-\eta(t_1-t_0)).$$

When $t \in (t_1, t_2]$,

$$\begin{aligned}V(t) &< V(t_1^+)\exp(-\eta(t-t_1)) \\ &< \beta_1 V(t_1)\exp(-\eta(t-t_1)) \\ &< \beta_1 V(t_0)\exp(-\eta(t-t_0))\end{aligned}$$

Thus, when $t \in (t_k, t_{k+1}]$,

$$\begin{aligned} V(t) &< V(t_k^+)\exp(-\eta(t-t_k)) \\ &< \prod_{i=1}^{k} \beta_i V(t_0)\exp(-\eta(t-t_0)) \\ &< V(t_0)\exp(-\eta(t-t_0)). \end{aligned}$$

Therefore, the impulsive closed-loop system (2.6) is exponentially stable for the case that $w(t) = 0$. This completes the proof. □

Theorem 3.2. *Let β_k be the largest eigenvalue of $P_{i_k}^{-1}(I+D_k)^T P_{i_k}(I+D_k)$ and $0 \le \beta_k \le 1$, $k \in N$, hold. For a given $\gamma > 0$, the impulsive closed-loop system (2.6) is robustly H_∞ exponentially stable if there exist positive definite symmetric matrices $P_{i_k} \in R^{n\times n}$ such that*

$$A_{i_k}^T P_{i_k} + P_{i_k}A_{i_k} + P_{i_k}(\gamma^{-2}B_{i_k}B_{i_k}^T - \varepsilon C_{i_k}C_{i_k}^T)P_{i_k} + E_{i_k}^T E_{i_k} < 0. \tag{3.6}$$

Moreover, a suitable feedback controller can be expressed in the form of

$$u(t) = F_{i_k}x(t), F_{i_k} = -\frac{1}{2\varepsilon}C_{i_k}^T P_{i_k}.$$

Proof. A linear feedback controller is constructed in the form of $u(t) = F_{i_k}x(t)$, where $F_{i_k} = -\frac{1}{2\varepsilon}C_{i_k}^T P_{i_k}$, $\varepsilon > 0$. By applying the designed controller and using (3.1), we can get

$$A_{i_k}^T P_{i_k} + P_{i_k}A_{i_k} + P_{i_k}(\gamma^{-2}B_{i_k}B_{i_k}^T - \varepsilon C_{i_k}C_{i_k}^T)P_{i_k} + E_{i_k}^T E_{i_k} < 0$$

which shows that if condition (3.6) is satisfied, the impulsive closed-loop system (2.6) will be robustly H_∞ exponentially stable. This completes the proof. □

Without loss of generality, we normally choose the free parameter $\gamma = \varepsilon = 1$, then we can obtain the following corollary straightforwardly.

Corollary 3.3. *Let β_k be the largest eigenvalue of $P_{i_k}^{-1}(I+D_k)^T P_{i_k}(I+D_k)$ and $0 \le \beta_k \le 1$, $k \in N$, hold. The impulsive closed loop system (2.6) is robustly H_∞ exponentially stable if there exist positive definite symmetric matrices $P \in R_{i_k}^{n\times n}$ such that*

$$A_{i_k}^T P_{i_k} + P_{i_k}A_{i_k} + P_{i_k}(B_{i_k}B_{i_k}^T - C_{i_k}C_{i_k}^T)P_{i_k} + E_{i_k}^T E_{i_k} < 0. \tag{3.7}$$

Moreover, a suitable feedback controller can be presented by

$$u(t) = F_{i_k}x(t), F = -\frac{1}{2}C_{i_k}^T P_{i_k}.$$

When the impulses and switchings do not occur during the evolution process, the uncertain linear system will be

$$\begin{cases} \dot{x}(t) = Ax(t) + Bw(t) + Cu(t) \\ z(t) = Ex(t) \end{cases}. \tag{3.8}$$

For this case, we can have the following corollary.

Corollary 3.4. *For a given $\gamma > 0$, the linear system (3.8) is robustly H_∞ exponentially stable if there exist a constant $\varepsilon > 0$ and a positive definite symmetric matrix $P \in R^{n\times n}$ such that*

$$A^T P + PA + P(\gamma^{-2}BB^T - \varepsilon^{-2}CC^T)P + E^T E < 0.$$

Moreover, a suitable feedback control law is given by

$$u(t) = Fx(t), F = -\frac{1}{2\varepsilon^2}C^T P$$

4 A numerical Example

Consider the uncertain impulsive switched systems (2.2) with the following specifications.

$$A_1 = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}, B_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0.5 \end{bmatrix}, C_1 = \begin{bmatrix} 1.2 & 0.3 \\ 0.5 & 1.2 \end{bmatrix}, D_k = -0.5, E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and

$$A_2 = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}, B_2 = \begin{bmatrix} 0.2 & 1 \\ 1.2 & 0.8 \end{bmatrix}, C_2 = \begin{bmatrix} 1.3 & 0.3 \\ 0.6 & 1.5 \end{bmatrix}, D_k = -0.5, E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Then, the two subsystems will be

$$\begin{cases} \dot{x}(t) & = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1.2 & 0.3 \\ 0.5 & 1.2 \end{bmatrix} u(t) + \begin{bmatrix} 0 & 1 \\ 1 & 0.5 \end{bmatrix} w(t) \qquad t \neq t_k \\ \Delta x(t) & = -0.5x \qquad t = t_k \\ z(t) & = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(t) \end{cases}$$

and

$$\begin{cases} \dot{x}(t) & = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 1.3 & 0.3 \\ 0.6 & 1.5 \end{bmatrix} u(t) + \begin{bmatrix} 0.2 & 1 \\ 1.2 & 0.8 \end{bmatrix} w(t) \qquad t \neq t_k \\ \Delta x(t) & = -0.5x \qquad t = t_k \\ z(t) & = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(t) \end{cases}$$

respectively. Let $\gamma = \varepsilon = 1$, by using (3.7), we can obtain the following algebraic Riccati inequalities

$$P_1 \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}^T P_1 + P_1 \left(\begin{bmatrix} 0 & 1 \\ 1 & 0.5 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0.5 \end{bmatrix}^T - \begin{bmatrix} 1.2 & 0.3 \\ 0.5 & 1.2 \end{bmatrix} \begin{bmatrix} 1.2 & 0.3 \\ 0.5 & 1.2 \end{bmatrix}^T \right) P_1 + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} < 0 \quad (4.1)$$

and

$$P_2 \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}^T P_2 + P_2 \left(\begin{bmatrix} 0.2 & 1 \\ 1.2 & 0.8 \end{bmatrix} \begin{bmatrix} 0.2 & 1 \\ 1.2 & 0.8 \end{bmatrix}^T - \begin{bmatrix} 1.3 & 0.3 \\ 0.6 & 1.5 \end{bmatrix} \begin{bmatrix} 1.3 & 0.3 \\ 0.6 & 1.5 \end{bmatrix}^T \right) P_2 + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} < 0. \quad (4.2)$$

Then, we can find the following feasible solutions of (4.1) and (4.2),

$$P_1 = \begin{bmatrix} 1.7759 & 0.8800 \\ 1.6703 & 0.8437 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 4.0119 & -0.9226 \\ -0.4580 & 3.3750 \end{bmatrix}.$$

Hence, the required state feedback controllers can be devised as

$$u(t) = F_{i_k} x(t), \quad i_k = 1, \text{ or } 2 \tag{4.3}$$

$$F_1 = \begin{bmatrix} -1.4831 & -0.7389 \\ -1.2686 & -0.6382 \end{bmatrix}, \; F_2 = \begin{bmatrix} -2.4703 & -0.4128 \\ -0.2583 & -2.3929 \end{bmatrix}.$$

Therefore, the feedback controller (4.3) will exponentially stabilize the impulsive switched system (2.2) and meanwhile guarantees

$$\|z\|_\infty \leq \gamma \|x\|_\infty$$

5 Conclusion

We have developed a state feedback robust H_∞ optimal control technique for a class of impulsive switched systems with time varying uncertainty. Based on a positive definite solution of a modified algebraic Riccati inequality, the proposed robust H_∞ static state feedback controllers guarantee both exponential stability and robust H_∞ performance for a class of impulsive switched systems with norm-bounded time-varying uncertainty. An illustrative example have been given to demonstrate the applicability of the proposed approach.

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