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COALITION FORMATION IN CONVEX TU-GAMES BASED ON POPULATION MONOTONICITY OF RANDOM ORDER VALUES

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Abstract. Two important problems in cooperative games are the coalition formation of players and the allocation schemes of profits among the players (so-called solutions of the games). The random order values, which include the Shapley value as a special case, are fundamental point-valued solutions in transferable utility games (TU-games). We prove the population monotonicity, i.e., the monotonicity of allocated individual values with respect to coalitions, of the random order values in convex games. We also discuss coalition formation in TU-games as hedonic games based on the random order values. We prove that the coalition structure obtained from the top coalition algorithm satisfies some stability properties in hedonic games.

1. Introduction

In the theory of cooperative games there are two important problems. One is the problem of finding reasonable solutions for games which are allocation schemes of profits among players. Roughly speaking there are two types of solutions: set-valued solutions and point-valued solutions. Typical examples of the former are the core and the stable set. Those of the latter are the Shapley value and some variations of it (see for example Mondere and Samet [5]). In this paper we focus on the random order values which are equivalent to the efficient probabilistic values (i.e. quasi values).

The other important problem in the theory of cooperative games is formation of coalitions, i.e., discussion on what is the coalition structure formed in the games. One approach to this problem is dealing with hedonic games which are coalition formation games under each player's individual preferences over coalitions including herself (Banerjee et al. [1], Bogomolnaia and Jackson [2], Sung and Dimitrov [7] and so on). Some conditions under which reasonable coalition structures can be obtained algorithmically. One of those conditions is the top coalition property introduced by Banerjee et al. [1].

In this paper we deal with convex transferable utility games as cooperative games, which have some nice properties. We generalize the population monotonicity property proved for the Shapley value by Sprumont [6] to the random order values. We assume that each individual preference is specified by the random order value and prove stability of the coalition structure obtained by the top coalition algorithm based on this fact.

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The paper is organized as in the following. In section 2 we discuss random order values for transferable utility games. The main result is the population monotonicity property of the random order values. Section 3 is devoted to a brief review of hedonic games with the top coalition property. Finally in section 4 we consider hedonic games with the individual preferences based on the random order values and show that the top coalition algorithm effectively produces the stable coalition structures for those games.

2. Cooperative games and random order values

In this paper we deal with cooperative games, particularly transferable utility games. A transferable utility game (TU-game) is a pair (N, v), where $N = \{1, 2, \ldots, n\}$ is a finite set of players and $v : 2^N \to \mathbf{R}$ is a function satisfying $v(\emptyset) = 0$. When $S \subseteq N$, we may consider the subgame (S, v) of (N, v) by restricting the domain of v from 2^N to 2^S.

In the following, we use abbreviated notations such as $v(\{i\}) = v(i)$, $S \cup \{i\} = S \cup i$, $S \setminus \{i\} = S \setminus i$ and so on. We use two kinds of symbols for set inclusion: $S \subseteq T$ means that S is a subset of T, while $S \subset T$ implies that S is a proper subset of T. We denote the number of elements in S by $|S|$.

In this paper we focus on convex TU-games which have some nice properties.

Definition 2.1. A TU-game (N, v) is said to be **convex** if

$$v(S) + v(T) \leq v(S \cup T) + v(S \cap T) \text{ for all } S, T \subseteq N.$$

Proposition 2.2. *A TU-game (N, v) is convex if and only if*

$$v(S \cup i) - v(S) \leq v(T \cup i) - v(T) \text{ and for all } S, T \subseteq N \text{ with } S \subseteq T \text{ and for all } i \notin T.$$

For $S \subseteq N$ let $\Pi(S)$ be the set of all orderings on S. Practically $\pi(i)$ means the order of player $i \in S$ in the ordering $\pi \in \Pi(S)$. Given an ordering $\pi \in \Pi(S)$ on $S \subseteq N$, let

$$P(\pi, i, S) = \{j \in S \mid \pi(j) < \pi(i)\}$$

and define the marginal contribution of player $i \in S$ in the ordering π for the game (S, v) by

$$m_i^\pi(v, S) = v(P(\pi, i, S) \cup i) - v(P(\pi, i, S)).$$

Now let $\Lambda(S)$ be the set of all nonnegative weight systems on $\Pi(S)$, i.e.,

$$\Lambda(S) = \{\lambda = (\lambda_\pi)_{\pi \in \Pi(S)} \in \mathbf{R}^{|\Pi(S)|} \mid \lambda_\pi \geq 0, \sum_{\pi \in \Pi(S)} \lambda_\pi = 1\}.$$

Definition 2.3. A value is a point-valued solution of TU-games, i.e., a function ξ which associates an $|S|$ dimensional real vector with each game (S, v). In particular, a value with a nonnegative weight system $\lambda \in \Lambda(S)$ defined by

$$\xi_i^\lambda(v, S) = \sum_{\pi \in \Pi(S)} \lambda_\pi m_i^\pi(v, S), \quad i \in S$$

is called a random order value. We may often call the vector $\xi(v, S)$ the value (of v for S) too.

Proposition 2.4 (Weber [8])**.** *A value satisfies four axioms, Linearity, Null player property, Efficiency, Monotonicity, if and only if it is a random order value.*

Most typical example of random order values is the Shapley value $\varphi(v, S)$, which is specified by the weight system $\lambda_\pi = \frac{1}{|S|!}$ for all $\pi \in \Pi(S)$, i.e.,

$$\varphi_i(v, S) = \frac{1}{|S|!} \sum_{\pi \in \Pi(S)} m_i^\pi(v, S).$$

Definition 2.5. Let $S' \subset S \subseteq N$ and $\pi \in \Pi(S)$. A restriction of π onto S' is the ordering $\pi' \in \Pi(S')$ uniquely determined by

$$\pi'(i) < \pi'(j) \iff \pi(i) < \pi(j), \text{ for all } i, j \in S'.$$

On the contrary, given an ordering $\pi' \in \Pi(S')$, an ordering $\pi \in \Pi(S)$ is said to be consistent with π' if π' is the restriction of π onto S'. The set of all orderings on S consistent with π' is denoted by $\Pi(S; \pi')$.

Remark 2.6. The family $\{\Pi(S; \pi') \mid \pi' \in \Pi(S')\}$ is a partition of $\Pi(S)$, where each $\Pi(S; \pi')$ consists of $\frac{|S|!}{|S'|!}$ elements.

Definition 2.7. Let $S' \subset S \subseteq N$ and $\lambda \in \Lambda(S)$. A weight system $\lambda' \in \Lambda(S')$ is said to be induced from λ if

$$\lambda'_{\pi'} = \sum_{\pi \in \Pi(S;\pi')} \lambda_\pi, \text{ for all } \pi' \in \Pi(S').$$

Remark 2.8. Let $S' \subset S \subseteq N$. Then the weight system $\frac{1}{|S'|!}\mathbf{e} = (\frac{1}{|S'|!}, \ldots, \frac{1}{|S'|!}) \in \Lambda(S')$ is induced from the weight system $\frac{1}{|S|!}\mathbf{e} = (\frac{1}{|S|!}, \ldots, \frac{1}{|S|!}) \in \Lambda(S)$.

Now we consider a game (N, v), then for each $S \subset N$ and $\lambda \in \Lambda(S)$ we can define the random order value $\xi^\lambda(v, S)$. We obtain the following theorem which is a generalization of the population monotonicity of the Shapley value proved by Sprumont [6] (see also Branzei et al. [3]).

Theorem 2.9. *Let (N, v) be a TU-game, $S' \subset S \subseteq N$ and $\lambda \in \Lambda(S)$. If v is a convex game and $\lambda' \in \Lambda(S')$ is induced from λ, then*

$$\xi_i^{\lambda'}(v, S') \leq \xi_i^\lambda(v, S) \text{ for all } i \in S'.$$

Proof. Let $\pi' \in \Pi(S')$ and $\pi \in \Pi(S; \pi')$. Then it is clear that $P(\pi', i, S') \subseteq P(\pi, i, S)$ and hence for any $i \in S'$

$$\begin{aligned} m_i^{\pi'}(v, S') &= v(P(\pi', i, S') \cup i) - v(P(\pi', i, S')) \\ &\leq v(P(\pi, i, S) \cup i) - v(P(\pi, i, S)) \\ &= m_i^\pi(v, S) \end{aligned}$$

since v is a convex game. Therefore, for any $i \in S'$

$$\begin{aligned}
\xi_i^{\lambda'}(v,S') &= \sum_{\pi' \in \Pi(S')} \lambda'_{\pi'} m_i^{\pi'}(v,S') \\
&= \sum_{\pi' \in \Pi(S')} \sum_{\pi \in \Pi(S;\pi')} \lambda_\pi m_i^{\pi'}(v,S') \\
&\leq \sum_{\pi' \in \Pi(S')} \sum_{\pi \in \Pi(S;\pi')} \lambda_\pi m_i^{\pi}(v,S) \\
&= \sum_{\pi \in \Pi(S)} \lambda_\pi m_i^{\pi}(v,S) \\
&= \xi_i^{\lambda}(v,S).
\end{aligned}$$

This completes the proof. □

The case of the Shapley value can be obtained as a corollary of the above theorem.

Corollary 2.10. *Let (N,v) be a convex TU-game and $S' \subset S \subseteq N$. Then*

$$\varphi_i(v,S') \leq \varphi_i(v,S) \ \text{ for all } i \in S'.$$

We investigate more results for the random order values. The following lemma shows a relationship between the marginal contributions and restricted additivity of a game.

Lemma 2.11. *Let (N,v) be a convex TU-game, $S' \subset S \subseteq N$ and $S'' = S \setminus S'$. If $m_i^{\pi'}(v,S') = m_i^{\pi}(v,S)$ for all $\pi' \in \Pi(S')$, $\pi \in \Pi(S;\pi')$ and $i \in S'$, then $v(T' \cup T'') = v(T') + v(T'')$ for $T' \subseteq S'$ and $T'' \subseteq S''$ and $m_j^{\pi''}(v,S'') = m_j^{\pi}(v,S)$ for all $\pi'' \in \Pi(S'')$, $\pi \in \Pi(S;\pi'')$ and $j \in S''$.*

Proof. Let $T' \subseteq S'$ and $T'' \subseteq S''$. If $T' = \emptyset$, the first result is obvious. Hence we assume that $T' = \{i_1,\ldots,i_l\}$ $(l \geq 1)$. Take orderings $\pi' \in \Pi(S')$ and $\pi \in \Pi(S;\pi')$ such that

$$\begin{array}{ll}
\pi'(j) < \pi'(k) & \forall j \in T',\ \forall k \in S' \setminus T', \\
\pi'(i_1) < \pi'(i_2) < \cdots < \pi'(i_l), & \\
\pi(j) < \pi(k) & \forall j \in T'',\ \forall k \in S', \\
\pi(j) < \pi(k) & \forall j \in S',\ \forall k \in S'' \setminus T''.
\end{array}$$

Then

$$\begin{aligned}
v(T' \cup T'') &= v(T' \cup T'') - v(T' \cup T'' \setminus i_l) + v(T' \cup T'' \setminus i_l) \\
&\quad - v(T' \cup T'' \setminus \{i_l, i_{l-1}\}) + \cdots + v(T'' \cup i_1) - v(T'') + v(T'') \\
&= m_{i_l}^{\pi}(v,S) + m_{i_{l-1}}^{\pi}(v,S) + \cdots + m_{i_1}^{\pi}(v,S) + v(T'') \\
&= m_{i_l}^{\pi'}(v,S') + m_{i_{l-1}}^{\pi'}(v,S') + \cdots + m_{i_1}^{\pi'}(v,S') + v(T'') \\
&= v(T') - v(T' \setminus i_l) + v(T' \setminus i_l) - v(T' \setminus \{i_l, i_{l-1}\}) + \cdots \\
&\quad + v(i_1) - v(\emptyset) + v(T'') \\
&= v(T') + v(T'').
\end{aligned}$$

Now let $\pi'' \in \Pi(S'')$, $\pi \in \Pi(S;\pi'')$ and $j \in S''$. Then

$$\begin{array}{rcl} m_j^{\pi}(v,S) & = & v(P(\pi,j,S)\cup j) - v(P(\pi,j,S)) \\ & = & v((P(\pi,j,S)\cap S'')\cup j) + v(P(\pi,j,S)\cap S') \\ & & -v(P(\pi,j,S)\cap S'') - v(P(\pi,j,S)\cap S') \\ & = & v(P(\pi'',j,S'')\cup j) - v(P(\pi'',j,S'')) \\ & = & m_j^{\pi''}(v,S''), \end{array}$$

as was to be proved. □

Theorem 2.12. *Let (N,v) be a TU-game, $S' \subset S \subseteq N$, $S'' = S \setminus S'$, and $\lambda \in \Lambda(S)$ with $\lambda > 0$. If (N,v) is a convex game, $\lambda' \in \Lambda(S')$ and $\lambda'' \in \Lambda(S'')$ are induced from λ, and*

$$\xi_i^{\lambda'}(v,S') = \xi_i^{\lambda}(v,S) \ \textit{for all}\ i \in S',$$

then

$$\xi_j^{\lambda''}(v,S'') = \xi_j^{\lambda}(v,S) \ \textit{for all}\ j \in S''.$$

Proof. Since $\lambda_\pi > 0$ for all $\pi \in \Pi(S)$, in view of the proof of Theorem 2.9, $m_i^{\pi'}(v,S') = m_i^{\pi}(v,S)$ for all $\pi' \in \Pi(S')$, $\pi \in \Pi(S;\pi')$ and $i \in S'$. Then, due to Lemma 2.11, $m_j^{\pi''}(v,S'') = m_j^{\pi}(v,S)$ for all $\pi'' \in \Pi(S'')$, $\pi \in \Pi(S;\pi'')$ and $j \in S''$. Therefore, as in the proof of Theorem 2.9, we can easily show that

$$\xi_j^{\lambda''}(v,S'') = \xi_j^{\lambda}(v,S) \ \text{for all}\ j \in S''.$$

This completes the proof. □

Thus if the random order values are all equal for some players, then they are also equal for the remaining players.

3. Hedonic games and top coalition property

In this section we deal with coalition formation among players in terms of hedonic games. Let $N = \{1,2,\ldots,n\}$ be a finite set of players as before. Each player i is endowed with a preference $\succeq_i$ over the set $\mathcal{A}^i = \{S \subseteq N \mid i \in S\}$ of all possible coalitions she may belong to. Each $\succeq_i$ is assumed to be a complete pre-ordering, i.e.,

$$\begin{array}{l} R \succeq_i S,\ S \succeq_i T \implies R \succeq_i T,\ \ \forall R,S,T \subseteq N, \\ S \succeq_i T \text{ or } T \succeq_i S,\ \ \forall S,T \subseteq N. \end{array}$$

A hedonic game is a pair $\langle N,\succeq\rangle$, where $\succeq$ is a profile of players' preferences, i.e., $\succeq = (\succeq_1,\succeq_2,\ldots,\succeq_n)$. An outcome Θ for $\langle N,\succeq\rangle$ is a partition of the player set N. For each partition Θ of N and for each player $i \in N$, we denote by $\Theta(i)$ the coalition in Θ containing i. The following are some notions of stability of a partition of the players (Dimitrov and Sun [4], Sung and Dimitrov [7]).

Definition 3.1. Given a hedonic game $\langle N,\succeq\rangle$ and a partition Θ of N, we say that

- Θ is core stable if, for each nonempty $S \subseteq N$,

$$\Theta(i) \succeq_i S \ \text{for some}\ i \in S;$$

- Θ is strictly core stable if, for each nonempty $S \subseteq N$,

$$\Theta(i) \succeq_i S \ \text{for each}\ i \in S \text{ if } S \succeq_i \Theta(i) \text{ for each } i \in S;$$

- Θ is Nash stable if, for each $S \in \Theta \cup \{\emptyset\}$ and for each $i \in N$,

$$\Theta(i) \succeq_i S \cup i;$$

- Θ is individually stable if, for each $S \in \Theta \cup \{\emptyset\}$ and for each $i \in N$,

$$S \succ_j S \cup i \text{ for some } j \in S \text{ if } S \cup i \succ_i \Theta(i).$$

Observe that strict core stability implies core stability and individual stability, and that Nash stability implies individual stability.

Let $i \in N$ and $S \in \mathcal{A}^i$. We denote by $Ch(i,S) \subseteq 2^S \cap \mathcal{A}^i$ the set of all maximal elements under $\succeq_i$, i.e.,

$$Ch(i,S) = \{T \in 2^S \cap \mathcal{A}^i \mid T \succeq_i R \text{ for any } R \in 2^S \cap \mathcal{A}^i\}.$$

Observe that each $T \in Ch(i,S)$ satisfies $i \in T \subseteq S$. Moreover, for each $T, R \subseteq 2^S \cap \mathcal{A}^i$, we have $T \succ_i R$ if $T \in Ch(i,S)$ and $R \notin Ch(i,S)$.

The following property was introduced by Banerjee et al. [1]

Definition 3.2. A preference profile $\succeq$ of a hedonic game $\langle N, \succeq \rangle$ is said to satisfy the top coalition property if the following holds: For each $S \subseteq N$, there exists $T \subseteq S$ such that

$$T \in Ch(i,S) \text{ for any } i \in T.$$

A hedonic game $\langle N, \succeq \rangle$ in which $\succeq$ satisfies the top coalition property is called a TC hedonic game.

If the hedonic game $\langle N, \succeq \rangle$ is a TC hedonic game, we can obtain a partition of N by using the following top coalition algorithm TCA.

Algorithm TCA

Step 1. Let $R^1 := N$, $\Theta = \emptyset$ and $k := 1$.
Step 2. Choose S^k such that $S^k \in Ch(i, R^k)$ for all $i \in S^k$.
Step 3. Let $R^{k+1} := R^k \setminus S^k$ and $\Theta = \Theta \cup \{S^k\}$.
Step 4. If $R^{k+1} \neq \emptyset$, then let $k := k+1$ and return to Step 2. If $R^{k+1} = \emptyset$, then go to Step 5.
Step 5. Stop with the outcome Θ.

The partition obtained by applying the algorithm TCA to the hedonic game $\langle N, \succeq \rangle$ is denoted by $\Theta^{TC}_{\langle N, \succeq \rangle}$.

Theorem 3.3 ([1]). *Let $\langle N, \succeq \rangle$ be a TC hedonic game. Then the partition $\Theta^{TC}_{\langle N, \succeq \rangle}$ is core stable.*

We may consider a modified version of the algorithm TCA by choosing S^k with the largest size in Step 2. The obtained algorithm is called the maximal top coalition algorithm and is denoted by MTCA. The obtained partition is denoted by $\Theta^{MTC}_{\langle N, \succeq \rangle}$.

Theorem 3.4. *Let $\langle N, \succeq \rangle$ be a TC hedonic game. Then the partition $\Theta^{MTC}_{\langle N, \succeq \rangle}$ is individually stable.*

Proof. Let $S \in \Theta^{MTC}_{\langle N,\succeq\rangle} \cup \{\emptyset\}$ and $i \in N$. Suppose that $\Theta(i) = S^k$ in Algorithm MTCA. First we consider the case $S = \emptyset$. In this case it does not occur that $S \cup \{i\} = \{i\} \succ_i \Theta(i)$. Now let $S = S^m \neq \emptyset$. If $k = m$, $S \cup i = S^m \cup i = S^k \cup i = S^k \not\succ_i S^k$. Next suppose that $k < m$. Since $S^k \in Ch(i, R^k)$ and $S^m \cup i \subseteq R^k$, $S^m \cup i \not\succ_i S^k$. Finally suppose that $k > m$ and $S^m \cup i \succ_i S^k$. Since $i \in S^k$ and $S^k \cap S^m = \emptyset$, $i \notin S^m$. Since $S^m \in Ch(j, R^m)$ for all $j \in S^m$, $S^m \succeq_j S^m \cup i$ for all $j \in S^m$. If $S^m \sim_j S^m \cup i$ for all $j \in S^m$, then $S^m \cup i \in Ch(i, R^m)$, which contradicts that S^m is of the largest size. Therefore there exists some $j \in S^m$ such that $S^m \succ_j S^m \cup i$. This completes the proof. □

4. Hedonic games with the preferences based on the random order values

Now we consider a TU-game (N, v) and suppose that the total profit $v(S)$ is allocated according to the random order value $\xi^\lambda(v, S)$ if a coalition $S \subseteq N$ is formed. Hereafter, given a weighting system $\lambda \in \Lambda(N)$, we consider the random order value $\xi^{\lambda'}(v, S)$ with the weighting system $\lambda' \in \Lambda(S)$ induced from λ. Therefore we denote it by $\xi^\lambda(v, S)$. Then we can consider the hedonic game based on the random order value as follows.

Definition 4.1. Given a game (N, v) and a weighting system $\lambda \in \Lambda(N)$, the preference ordering $\succeq_i^{(v,\xi^\lambda)}$ of player i on $\mathcal{A}^i$ is defined by

$$S \succeq_i^{(v,\xi^\lambda)} T \iff \xi_i^\lambda(v, S) \geq \xi_i^\lambda(v, T), \quad \forall S, T \in \mathcal{A}^i.$$

Then we can obtain the following result directly from Theorem 2.9 and it implies that the hedonic game $\langle N, \succeq^{(v,\xi^\lambda)}\rangle$ satisfies the top coalition property if (N, v) is a convex game.

Theorem 4.2. *If a TU-game (N, v) is convex, then for any weighting system $\lambda \in \Lambda(N)$,*

$$S \succeq_i^{(v,\xi^\lambda)} T \text{ for any } i \in T, \text{ with } T \subseteq S \subseteq N.$$

Lemma 4.3. *Let $\langle N, v\rangle$ be a hedonic game and Θ be a partition of N. If $i \in \Theta(i) \in Ch(i, N)$ for each $i \in N$, then Θ is strictly core stable and Nash stable.*

Proof. Since $\Theta(i) \in Ch(i, N)$, from the definition of $Ch(i, N)$, we have $\Theta(i) \succeq_i T$ for any $T \in \mathcal{A}^i$. Therefore it is obvious that Θ is core stable, strictly core stable, Nash stable and individually stable. □

Theorem 4.4. *If a TU-game (N, v) is convex and a weighting system $\lambda \in \Lambda(N)$ is positive, then the partition $\Theta^{TC}_{\langle N, \succeq^{(v,\xi^\lambda)}\rangle}$ is strictly core stable and Nash stable (and therefore also core stable and individually stable).*

Proof. Let $\Theta^{TC}_{\langle N, \succeq^{(v,\xi^\lambda)}\rangle} = \{S^1, \ldots, S^l\}$. Since $S^1 \in Ch(i, N)$ for all $i \in S^1$, $\xi_i^\lambda(v, S^1) \geq \xi_i^\lambda(v, N)$ for all $i \in S^1$. Then, due to Theorem 2.9, $\xi_i^\lambda(v, S^1) = \xi_i^\lambda(v, N)$ for all $i \in S^1$. Because of Theorem 2.12, $\xi_j^\lambda(v, R^2) = \xi_j^\lambda(v, N)$ for all $j \in R^2 = N \setminus S^1$. Analogously $S^2 \in Ch(i, R^2)$ implies that $\xi_i^\lambda(v, S^2) = \xi_i^\lambda(v, R^2) = \xi_i^\lambda(v, N)$ for all

$i \in S^2$. Then, because of Theorem 2.12, $\xi_j^\lambda(v, R^3) = \xi_j^\lambda(v, R^2) = \xi_j^\lambda(v, N)$ for all $j \in R^3$. Thus, by continuing this process, we can prove generally that, for each $S^k (k = 1, \dots, l)$ and $i \in S^k$, $\xi_i^\lambda(v, S^k) = \xi_i^\lambda(v, N) \geq \xi_i^\lambda(v, T)$ for all $T \in \mathcal{A}^i$, i.e., $S^k \in Ch(i, N)$. Therefore, the theorem follows immediately due to Lemma 4.3. □

Finally we should note that the maximal top coalition algorithm always provides the grand coalition as its outcome for any convex game.

Remark 4.5. If a TU-game (N, v) is convex, then the partition $\Theta^{MTC}_{\langle N, \succeq^{(v,\xi^\lambda)} \rangle} = \{N\}$.

5. Conclusion

Any random order value for a convex TU-game satisfies the population monotonicity. A hedonic game (coalition formation game) based on a random order value for a convex TU-game satisfies the top coalition property. The top coalition algorithm provides a stable coalition structure for a hedonic game based on a random order value for a convex TU-game.

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