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KADEC-KLEE AND RELATED PROPERTIES OF DIRECT SUMS OF BANACH SPACES

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Abstract. We characterize the Kadec-Klee property of various direct sums of Banach spaces, including ψ-direct sums. Corresponding characterizations of the uniform Kadec-Klee property, nearly uniform convexity and the drop property are also obtained.

1. Introduction

In two recent papers, Wiśnicki [9, 10] considered properties related to the weak fixed point property and their stability relative to certain types of direct sums. In [9], Wiśnicki proved that certain types of direct sums of uniformly noncreasy Banach spaces have the weak fixed point property, while in [10], it is shown that the certain direct sums of finite dimensional Banach spaces and Banach spaces with the super fixed point property again has the super fixed point property. The unifying theme in these papers is the type of direct sums.

The goal of this short note is to prove stability results for some properties that imply the weak fixed point property under direct sums of the type considered by Wiśnicki (see section 2 for specific details). As a consequence of our results, we will obtain stability results for the Kadec-Klee property, the uniform Kadec-Klee property, the near uniform convexity and the drop property in a variety of direct sums, including ψ-direct sums of Banach spaces.

2. Preliminaries and definitions

Throughout this note we will deal with the concept of absolute normalized norms. To begin, we need to recall some definitions.

A norm $\|\cdot\|$ on $\mathbb{C}^n$ is said to be *absolute* if

$$\|(z_1, z_2, \ldots, z_n)\| = \|(|z_1|, |z_2|, \ldots, |z_n|)\| \quad \text{for all } (z_1, z_2, \ldots, z_n) \in \mathbb{C}^n$$

and *normalized* if $\|(1, 0, \ldots, 0)\| = \|(0, 1, 0, \ldots, 0)\| = \cdots = \|(0, 0, \ldots, 0, 1)\| = 1$. The collection of all absolute normalized norms on $\mathbb{C}^n$ is denoted by AN_n.

According to Saito, Kato and Takahashi [8] we recall the following: We define the subset Δ_n of $\mathbb{R}^{n-1}$ by

$$\Delta_n = \{(s_1, s_2, \ldots, s_{n-1}) : s_1 + \cdots + s_{n-1} \leq 1, s_i \geq 0 \text{ for all } 1 \leq i \leq n-1\}.$$

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For each norm $\|\cdot\|$ in AN_n, we define a function ψ on Δ_n by

$$\psi(s_1, s_2, \dots, s_{n-1}) = \left\|(1 - \sum_{i=1}^{n-1} s_i, s_1, s_2, \dots, s_{n-1})\right\| \text{ for each } (s_1, s_2, \dots, s_{n-1}) \in \Delta_n.$$

It is easy to show that ψ is a continuous convex function on Δ_n that satisfies the following conditions;

(A_0)

$$\psi(0, 0, \dots, 0) = \psi(1, 0, 0, \dots, 0) = \dots = \psi(0, 0, \dots, 0, 1) = 1,$$

$$\begin{aligned} \psi(s_1, s_2, \dots, s_{n-1}) \geq & (s_1 + s_2 + \dots + s_{n-1}) \\ (A_1) \qquad & \times \psi\left(\frac{s_1}{s_1 + \dots + s_{n-1}}, \cdots, \frac{s_{n-1}}{s_1 + \dots + s_{n-1}}\right) \text{ if } s_1 + \dots + s_{n-1} \neq 0, \end{aligned}$$

(A_2)

$$\psi(s_1, s_2, \dots, s_{n-1}) \geq (1 - s_1)\psi\left(0, \frac{s_2}{1 - s_1}, \cdots, \frac{s_{n-1}}{1 - s_1}\right) \quad \text{if } s_1 \neq 1,$$

$$\vdots \qquad \qquad \vdots$$

(A_n)

$$\psi(s_1, s_2, \dots, s_{n-1}) \geq (1 - s_{n-1})\psi\left(\frac{s_1}{1 - s_{n-1}}, \cdots, \frac{s_{n-2}}{1 - s_{n-1}}, 0\right) \quad \text{if } s_{n-1} \neq 1.$$

On the other hand, if we denote by Ψ_n, the collection of all continuous convex functions on Δ_n that satisfies conditions $(A_0), (A_1), \dots, (A_n)$, then each element $\psi \in \Psi_n$ defines a norm $\|\cdot\|_\psi$ on $\mathbb{C}^n$ belonging to AN_n by

$$\begin{aligned} &\|(z_1, \dots, z_n)\|_\psi \\ &= \begin{cases} (|z_1| + \dots + |z_n|)\psi\left(\frac{|z_2|}{|z_1| + \dots + |z_n|}, \cdots, \frac{|z_n|}{|z_1| + \dots + |z_n|}\right) & \text{if } (z_1, \dots, z_n) \neq (0, \dots, 0) \\ 0 & \text{if } (z_1, \dots, z_n) = (0, \dots, 0). \end{cases} \end{aligned}$$

For Banach spaces $X_1, X_2, \dots, X_n$ and $\psi \in \Psi_n$, the ψ-direct sum, $(X_1 \oplus \dots \oplus X_n)_\psi$, is the direct sum of $X_1, X_2, \dots, X_n$ equipped with the norm

$$\|(x_1, x_2, \dots, x_n)\|_\psi = \|(\|x_1\|, \|x_2\|, \dots, \|x_n\|)\|_\psi \quad \text{with } x_i \in X_i \text{ for } 1 \leq i \leq n$$

(Kato, Saito and Tamura [4]).

A ψ-direct sum can also be viewed as part of a more general notion of a Z-direct sum. In this setting, we let Z be a finite dimensional normed space $(\mathbb{R}^n, \|\cdot\|_Z)$, which has a monotone norm; that is,

$$\|(a_1, \dots, a_n)\|_Z \leq \|(b_1, \dots, b_n)\|_Z$$

if $0 \leq a_i \leq b_i$ for each $i = 1, \dots, n$. We write $(X_1 \oplus \dots \oplus X_n)_Z$ for the Z-direct sum of the Banach spaces $X_1, \dots, X_n$ equipped with the norm

$$\|(x_1, \dots, x_n)\| = \|(\|x_1\|_{X_1}, \dots, \|x_n\|_{X_n})\|_Z\,,$$

where $x_i \in X_i$ for each $i = 1, \dots, n$.

Note that in defining $(X_1 \oplus \cdots \oplus X_n)_Z$, we only needed to know the behavior of the Z-norm on $\mathbb{R}^n_+$. As a result, we may assume that the Z-norm is absolute; that is,

$$\|(a_1, \ldots, a_n)\|_Z = \|(|a_1|, \ldots, |a_n|)\|_Z \quad \text{for all } (a_1, \ldots, a_n) \in \mathbb{R}^n.$$

Clearly, a ψ-direct sum is a Z-direct sum, where Z is $\mathbb{R}^n$ equipped with the norm $\|\cdot\|_\psi$.

We will say that Z is *strictly monotone in the j-th coordinate* if

$$\|(a_1, \ldots, a_n)\|_Z < \|(b_1, \ldots, b_n)\|_Z$$

whenever $0 \leq a_i \leq b_i$ for each $i = 1, \ldots, n$ and $0 \leq a_j < b_j$. Note that by the triangle inequality and the assumption that the Z-norm is absolute, Z is strictly monotone in the j-th coordinate if

$$\|(a_1, \ldots, a_n)\|_Z < \|(b_1, \ldots, b_n)\|_Z$$

whenever $0 \leq a_i = b_i$ for each $i \neq j$ and $0 = a_j < b_j$.

3. The results

Our first two results give characterizations of the Kadec-Klee property of Z-direct sums and ψ-direct sums. Recall that a Banach space X has the *Kadec-Klee property* if whenever $\{x_n\}$ is a sequence in X which converges weakly to an element $x \in X$ and $\|x_n\| \to \|x\|$ as $n \to \infty$, then $\{x_n\}$ converges to x in norm. A Banach space X has the *Schur property* if weak and norm sequential convergences coincide in X.

Theorem 3.1. *Let $X_1, \ldots, X_n$ be Banach spaces. Then $(X_1 \oplus \cdots \oplus X_n)_Z$ has the Kadec-Klee property if and only if*

(1) *X_i has the Kadec-Klee property for each $1 \leq i \leq n$, and*
(2) *$A \cup B = \{1, 2, \ldots, n\}$, where $A = \{i : X_i$ has the Schur property$\}$ and $B = \{i : Z$ is strictly monotone in the i-th coordinate$\}$.*

Proof. Suppose that conditions (1) and (2) hold. Let $\{(x_1^k, \ldots, x_n^k)\}_{k=1}^\infty$ be a norm 1 sequence in $(X_1 \oplus \cdots \oplus X_n)_Z$ converging weakly to a norm 1 element $(x_1, \ldots, x_n)$. Naturally, $\{x_j^k\}_{k=1}^\infty$ converges weakly to x_j for each $1 \leq j \leq n$. Consider a subsequence of $\{(x_1^k, \ldots, x_n^k)\}_{k=1}^\infty$, again labeled as $\{(x_1^k, \ldots, x_n^k)\}_{k=1}^\infty$ for simplicity.

Weak lower semi-continuity of norms yields $\liminf_{k\to\infty} \|x_j^k\| \geq \|x_j\|$ for each $1 \leq j \leq n$. If X_j has the Schur property, then $\liminf_{k\to\infty} \|x_j^k\| = \lim_{k\to\infty} \|x_j^k\| = \|x_j\|$. Suppose there exists $1 \leq j \leq n$ such that $\liminf_{k\to\infty} \|x_j^k\| > \|x_j\|$. Then X_j fails to have the Schur property and thus Z is strictly monotone in the j-th coordinate. Consequently,

$$\begin{aligned} 1 = \|(x_1, \ldots, x_n)\| &= \|(\|x_1\|, \ldots, \|x_n\|)\|_Z \\ &< \|(\liminf_k \|x_1^k\|, \ldots, \liminf_k \|x_n^k\|)\|_Z \\ &\leq \liminf_k \|(\|x_1^k\|, \ldots, \|x_n^k\|)\|_Z \\ &= \liminf_k \|(x_1^k, \ldots, x_n^k)\| = 1. \end{aligned}$$

This is a contradiction and so we must have $\liminf_{k\to\infty} \|x_j^k\| = \|x_j\|$ for each $1 \leq j \leq n$. Choose subsequences $\{x_j^{k_l}\}_{l=1}^\infty$ of $\{x_j^k\}_{k=1}^\infty$, for each $1 \leq j \leq n$, so that $\lim_{l\to\infty} \|x_j^{k_l}\| = \|x_j\|$ for each $1 \leq j \leq n$. Therefore, since each X_j has the Kadec-Klee property, $\{x_j^{k_l}\}_{l=1}^\infty$ converges to x_j in norm for each $1 \leq j \leq n$, and thus $\{(x_1^{k_l}, \dots, x_n^{k_l})\}_{l=1}^\infty$ converges in norm to $(x_1, \dots, x_n)$. Thus we have shown that each subsequence of $\{(x_1^k, \dots, x_n^k)\}_{k=1}^\infty$ has a further subsequence that converges to $(x_1, \dots, x_n)$ in norm, and so $\{(x_1^k, \dots, x_n^k)\}_{k=1}^\infty$ converges to $(x_1, \dots, x_n)$ in norm.

Conversely, if (1) does not hold, then it is easily seen that $(X_1 \oplus \cdots \oplus X_n)_Z$ fails to have the Kadec-Klee property. If (2) does not hold, then there exists $1 \leq j \leq n$ with $j \notin A \cup B$. For this j, X_j fails to have the Schur property and Z fails to be strictly monotone in the j-th coordinate. Choose a weakly null norm 1 sequence $\{x_j^k\}_{k=1}^\infty$ in X_j. For each $i \neq j$, choose a norm 1 element x_i in X_i. Since Z is not strictly monotone in the j-th coordinate, there exists $(a_1, \dots, a_n)$ in Z so that $a_i \geq 0$ for all $i \neq j$, $a_j > 0$ and $\|(a_1, \dots, a_n)\|_Z = \|(b_1, \dots, b_n)\|_Z$, where $b_i = a_i$ for all $i \neq j$ and $b_j = 0$. Note that the sequence $\{(a_1x_1^k, \dots, a_nx_n^k)\}_{k=1}^\infty$ converges weakly in $(X_1 \oplus \cdots \oplus X_n)_Z$ to $(b_1x_1, \dots, b_nx_n)$, where $x_i^k = x_i$ for all k and $i \neq j$. Also note that $\|(a_1x_1^k, \dots, a_nx_n^k)\| = \|(b_1x_1, \dots, b_nx_n)\|$ for all $k \in \mathbb{N}$. However, $\|(a_1x_1^k, \dots, a_nx_n^k) - (b_1x_1, \dots, b_nx_n)\| = a_j\|(0, \dots, 0, 1, 0, \dots, 0)\|_Z > 0$ for all k, and so $\{(a_1x_1^k, \dots, a_nx_n^k)\}_{k=1}^\infty$ does not converge in norm to $(b_1x_1, \dots, b_nx_n)$. Thus $(X_1 \oplus \cdots \oplus X_n)_Z$ fails to have the Kadec-Klee property. $\square$

Remark 3.2. It follows from the proof of Theorem 4 of [2] that $(\mathbb{R}^n, \|\cdot\|_\psi)$ is strictly monotone in the j-th coordinate if and only if ψ satisfies (sA_j), where (sA_1), (sA_2), $\dots$, (sA_n) are defined as follows;

$$\psi(s_1, s_2, \dots, s_{n-1}) > (s_1 + s_2 + \cdots + s_{n-1})$$

$$(sA_1) \qquad \times\psi\left(\frac{s_1}{s_1 + \cdots + s_{n-1}}, \cdots, \frac{s_{n-1}}{s_1 + \cdots + s_{n-1}}\right) \text{ if } 0 < s_1 + \cdots + s_{n-1} < 1,$$

(sA_2)

$$\psi(s_1, s_2, \dots, s_{n-1}) > (1 - s_1)\psi\left(0, \frac{s_2}{1 - s_1}, \cdots, \frac{s_{n-1}}{1 - s_1}\right) \qquad \text{if } 0 < s_1 < 1,$$

$$\vdots \qquad\qquad \vdots$$

(sA_n)

$$\psi(s_1, s_2, \dots, s_{n-1}) > (1 - s_{n-1})\psi\left(\frac{s_1}{1 - s_{n-1}}, \cdots, \frac{s_{n-2}}{1 - s_{n-1}}, 0\right) \qquad \text{if } 0 < s_{n-1} < 1.$$

Consequently, we have the following corollary

Corollary 3.3. *Let $X_1, \dots, X_n$ be Banach spaces and let $\psi \in \Psi_n$. Then $(X_1 \oplus \cdots \oplus X_n)_\psi$ has the Kadec-Klee property if and only if*

1. *X_i has the Kadec-Klee property for each $1 \leq i \leq n$, and*
2. *$A \cup B = \{1, 2, \dots, n\}$, where $A = \{i : X_i$ has the Schur property$\}$ and $B = \{i : \psi$ satisfies $(sA_i)\}$.*

A uniform version of Theorem 3.1 can be easily proved; for completeness, we state it and its attendant corollary. Recall that a Banach space X is said to have the *uniform Kadec-Klee property* if for each $\varepsilon > 0$, there is $\delta = \delta(\varepsilon) > 0$ such that for any weakly convergent sequence $\{x_n\}$ in the unit ball of X with $\|x_n - x_m\| \geq \varepsilon$ for all $n \neq m$, $\|x\| \leq 1 - \delta$, where x is the weak limit of $\{x_n\}$.

Theorem 3.4. *Let $X_1, \ldots, X_n$ be Banach spaces. Then $(X_1 \oplus \cdots \oplus X_n)_Z$ has the uniform Kadec-Klee property if and only if*

(1) *X_i has the uniform Kadec-Klee property for each $1 \leq i \leq n$, and*
(2) *$A \cup B = \{1, 2, \ldots, n\}$, where $A = \{i : X_i$ has the Schur property$\}$ and $B = \{i : Z$ is strictly monotone in the i-th coordinate$\}$.*

Corollary 3.5. *Let $X_1, \ldots, X_n$ be Banach spaces and let $\psi \in \Psi_n$. Then $(X_1 \oplus \cdots \oplus X_n)_\psi$ has the uniform Kadec-Klee property if and only if*

(1) *X_i has the uniform Kadec-Klee property for each $1 \leq i \leq n$, and*
(2) *$A \cup B = \{1, 2, \ldots, n\}$, where $A = \{i : X_i$ has the Schur property$\}$ and $B = \{i : \psi$ satisfies $(sA_i)\}$.*

Proof of Theorem 3.4. We will follow the argument in the proof of Theorem 1 of [5]. First, one should notice that strict monotonicity in the i-th coordinate is a uniform property; namely, for each $\varepsilon > 0$ there is $\eta > 0$ such that $\|(a_1, \ldots, a_{i-1}, a_i - b_i, a_{i+1}, \ldots, a_n)\|_Z \leq 1 - \eta$ whenever $\|(a_1, \ldots, a_n)\|_Z \leq 1$ where $0 \leq a_j$ for all $j \in \mathbb{N}$ and $\varepsilon \leq b_i \leq a_i$ (see [5, Lemma 2]).

Suppose that conditions (1) and (2) hold. Let $\varepsilon > 0$ be given. Let $\{(x_1^k, \ldots, x_n^k)\}_{k=1}^\infty$ be a sequence in the unit ball of $(X_1 \oplus \cdots \oplus X_n)_Z$ converging weakly to $(x_1, \ldots, x_n)$ and assume that $\|(x_1^k, \ldots, x_n^k) - (x_1^l, \ldots, x_n^l)\| \geq \varepsilon$ for all $k \neq l$. Just as in [5, Theorem 1], we may assume (by passing to a subsequence if necessary) that $\|x_j^k\| = a_j$ for all $1 \leq j \leq n$ and for all $k \in \mathbb{N}$, for some $(a_1, \ldots, a_n) \in \mathbb{R}^n$. Moreover, we can also assume that $x_j^k = x_j$ for all $k \in \mathbb{N}$ and for all $j \in A$. Note that there exists $j_0 \in B$ so that $\|x_{j_0}^k - x_{j_0}^l\| \geq \varepsilon / \|(1, \ldots, 1)\|$ for all $k \neq l$ and hence $a_{j_0} \geq \varepsilon / 2\|(1, \ldots, 1)\|$. Since X_{j_0} has the uniform Kadec-Klee property and $\{x_{j_0}^k\}_{k=1}^\infty$ converges weakly to x_{j_0}, we find that $\|x_{j_0}\| \leq (1 - \delta) a_{j_0}$ for some $\delta > 0$. Since norms are weak lower semi-continuous and $(a_1, \ldots, a_n)$ is in the unit ball of Z, we obtain

$$\begin{aligned} \|(x_1, \ldots, x_n)\| &= \|(\|x_1\|, \ldots, \|x_n\|)\|_Z \\ &\leq \|(a_1, \ldots, a_{j_0 - 1}, (1 - \delta) a_{j_0}, a_{j_0 + 1}, \ldots, a_n)\|_Z \\ &\leq 1 - \eta, \end{aligned}$$

where the last inequality follows from the uniform nature of strict monotonicity in the j_0-th coordinate.

For the converse, if $(X_1 \oplus \cdots \oplus X_n)_Z$ has the uniform Kadec-Klee property, then arguing just as in Theorem 3.1, we easily see that (1) and (2) hold. This completes the proof. $\square$

A Banach space X is *nearly uniformly convex* if for each $\varepsilon > 0$, there is $\delta = \delta(\varepsilon) > 0$ such that for any sequence $\{x_n\}$ in the unit ball of X with $\|x_n - x_m\| \geq \varepsilon$ for all $n \neq m$, $\operatorname{co}\{x_n : n \in \mathbb{N}\} \cap (1 - \delta) B_X \neq \emptyset$, where $\operatorname{co}\{x_n : n \in \mathbb{N}\}$ is the convex hull of the sequence $\{x_n\}$, and B_X is the closed unit ball of X. The concept of nearly uniform convexity was introduced by Huff in [3], where he also proves that a Banach space is nearly uniformly convex if and only if it is reflexive and has the uniform Kadec-Klee property. Combining Theorem 3.4 with Huff's result, and using the fact that reflexive Schur spaces are finite dimensional, we obtain a characterization of nearly uniform convexity in Z-direct sums. We note that this result has also been proved by Kutzarova and Landes [5].

Theorem 3.6. *Let $X_1, \dots, X_n$ be Banach spaces. Then $(X_1 \oplus \cdots \oplus X_n)_Z$ is nearly uniformly convex if and only if*

(1) *X_i has is nearly uniformly convex for each $1 \le i \le n$, and*
(2) *$A \cup B = \{1, 2, \dots, n\}$, where $A = \{i : X_i$ is finite dimensional$\}$ and $B = \{i : Z$ is strictly monotone in the i-th coordinate$\}$.*

Of course, using Corollary 3.5, we obtain a companion characterization of nearly uniform convexity in ψ-direct sums.

Corollary 3.7. *Let $X_1, \dots, X_n$ be Banach spaces and let $\psi \in \Psi_n$. Then $(X_1 \oplus \cdots \oplus X_n)_\psi$ is nearly uniformly convex if and only if*

(1) *X_i is nearly uniformly convex for each $1 \le i \le n$, and*
(2) *$A \cup B = \{1, 2, \dots, n\}$, where $A = \{i : X_i$ is finite dimensional$\}$ and $B = \{i : \psi$ satisfies $(sA_i)\}$.*

Another property related to the Kadec-Klee property is the *drop property* which was introduced by Daneš in [1] (see also [7]). Let X be a Banach space with closed unit ball B_X. If $\|x\| > 1$, then the drop $D(x, B_X)$ induced by x and B_X is the convex hull of x and B_X. X has the drop property if, for each closed set S disjoint from B_X, there exists a point x in S such that the drop $D(x, B_X)$ intersects S only at x. It was Montesinos in [6] who proved that a Banach space has the drop property if and only if it is reflexive and has the Kadec-Klee property. Theorem 3.1 yields the following two results.

Theorem 3.8. *Let $X_1, \dots, X_n$ be Banach spaces. Then $(X_1 \oplus \cdots \oplus X_n)_Z$ has the drop property if and only if*

(1) *X_i has the drop property for each $1 \le i \le n$, and*
(2) *$A \cup B = \{1, 2, \dots, n\}$, where $A = \{i : X_i$ is finite dimensional$\}$ and $B = \{i : Z$ is strictly monotone in the i-th coordinate$\}$.*

Corollary 3.9. *Let $X_1, \dots, X_n$ be Banach spaces and let $\psi \in \Psi_n$. Then $(X_1 \oplus \cdots \oplus X_n)_\psi$ has the drop property if and only if*

(1) *X_i has the drop property for each $1 \le i \le n$, and*
(2) *$A \cup B = \{1, 2, \dots, n\}$, where $A = \{i : X_i$ is finite dimensional$\}$ and $B = \{i : \psi$ satisfies $(sA_i)\}$.*

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