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EXISTENCE AND MULTIPLICITY OF SOLUTIONS FOR NONLINEAR BOUNDARY VALUE PROBLEMS

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Abstract. We present some results on the existence and multiplicity of solutions for boundary value problems involving equations of the type $-\Delta_p u = f(x,u) + \lambda g(x,u)$, where Δ_p is the p-Laplacian operator ($p>1$), λ is a real parameter and $f, g : \Omega \times \mathbb{R} \to \mathbb{R}$, $\Omega \subset \mathbb{R}^N$, are two Carathéodory functions. The approach is variational and mainly based on a critical point theorem by B. Ricceri.

1. Introduction

Let $\Omega \subset \mathbb{R}^N$ be a non empty open bounded set with boundary $\partial\Omega$ of class C^1. Let $f, g : \Omega \times \mathbb{R} \to \mathbb{R}$ be two Carathéodory functions and let $\lambda \in \mathbb{R}$. In this paper we are concerned with the existence and multiplicity of solutions for the problem

$$\begin{cases} -\Delta_p u = g(x,u) + \lambda f(x,u) & \text{in } \Omega \\ B(u) = 0 & \text{on } \partial\Omega \end{cases}$$

where the boundary operator is of the type $B(u) = \delta u + (1-\delta)\frac{\partial u}{\partial \nu}$ with $\delta \in \{0,1\}$ and ν being the outer unit normal to $\partial\Omega$. For $\delta = 1$, $B(u) = 0$ is the Dirichlet boundary condition $u_{|\partial\Omega} = 0$ while if $\delta = 1$ we have the Neumann boundary condition $\frac{\partial u}{\partial \nu}_{|\partial\Omega} = 0$. Here $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian operator, where $p > 1$.

Variational methods have been extensively used to study the above boundary value problem. In recent years, an incentive in using this methods was given by some general variational results obtained by B. Ricceri in [19]. Applications of these results to differential and integral equations have been made by several authors. We cite, for instance, the papers [7, 8, 9, 10, 11, 12, 14, 15, 17, 20, 21] (see also reference therein).

The existence and multiplicity theorems we present here are deduced in part using the results in [19] jointly with regularity arguments and in part using some developments and ideas which have originated from [19].

Throughout this paper, if $p \in [1, +\infty[$ we put

$$\|u\|_p = \left(\int_\Omega |u|^p dx\right)^{\frac{1}{p}}$$

for all $u \in L^p(\Omega)$ and

$$\|u\|_\infty = \operatorname*{ess\,sup}_\Omega |u|$$

for all $u \in L^\infty(\Omega)$.

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Moreover, we recall that:

- a *weak solution* for the problem

$$(P_\lambda) \qquad \begin{cases} -\Delta_p u = g(x,u) + \lambda f(x,u) & \text{in } \Omega \\ u = 0 \ \ (\text{resp.} \frac{\partial u}{\partial \nu} = 0) & \text{on } \partial\Omega \end{cases}$$

  is any $u \in W_0^{1,p}(\Omega)$ (resp. $u \in W^{1,p}(\Omega)$) satisfying the equation

$$\int_\Omega |\nabla u(x)|^{p-2} \nabla u(x) \nabla v(x) dx = \int_\Omega (g(x,u(x)) + \lambda f(x,u(x))) v(x) dx$$

  for all $v \in W_0^{1,p}(\Omega)$ (resp. $v \in W^{1,p}(\Omega)$. Thus, the weak solutions are exactly the critical points of the energy functional

$$(1) \quad u \in W_0^{1,p}(\Omega) \text{ (resp. } u \in W^{1,p}(\Omega)) \longrightarrow \int_\Omega \left(\frac{1}{p} (|\nabla u|^p - \int_0^{u(x)} (g(x,t) + \lambda f(x,t)) dt \right) dx;$$

- a *strong solution* for the same problem is any $u \in W_0^{1,p}(\Omega) \cap W^{2,p}(\Omega) \cap C^0(\overline{\Omega})$ (resp. $u \in W^{2,p}(\Omega) \cap C^0(\overline{\Omega})$) satisfying equation $-\Delta_p u = g(x,u) + \lambda f(x,u)$ almost everywhere in Ω and the boundary condition pointwise.

2. The results

2.1. **Dirichlet boundary condition.** The results we present here are related to problem (P_λ) with the Dirichlet boundary condition $u_{|\partial\Omega} = 0$.

Our first theorem concerns the multiplicity of nonzero solutions for problem (P_λ) with g and $f(\cdot, 0)$ identically 0. Using an idea developed in [22] (see also [5, 6]), we can find conditions on f in order that the energy functional (1) have at least two local minima. This fact, jointly to a mountain pass argument, allow us to obtain at least two non-zero nonnegative solutions for problem (P_λ) for all $\lambda \geq \lambda^*$ where $\lambda^* \geq 0$ will turn out explicitly determined. The statement of the result is as follows

Theorem 1 (Theorem 3.4 of [2]). *Let $g \equiv 0$ and assume the following growth condition*

i) $p \leq N$ and $\sup_{t \in \mathbb{R}} \dfrac{|f(\cdot,t)|}{1+|t|^q} \in L^\infty(\Omega)$ for some $q > 0$ with $q < \frac{N(p-1)+p}{N-p}$ if $p < N$;

ii) $p > N$ and $\sup_{|t| \leq r} |f(\cdot,t)| \in L^1(\Omega)$ for all $r > 0$.

Moreover, suppose that:

iii) $f(x,0) = 0$ for almost all $x \in \Omega$;

iv) *there exist* $\xi_0, \xi_1 \in [0, +\infty[$ *with* $\xi_0 < \xi_1$ *and* $u_0 \in W_0^{1,p}(\Omega)$ *with* $u_0(x) \geq 0$ *for a.a.* $x \in \Omega$ *such that*

$$\int_0^{\xi_0} f(x,t)dt = \sup_{\xi \in [\xi_0, \xi_1]} \int_0^{\xi} f(x,t)dt \text{ for a.a. } x \in \Omega;$$

$$\overline{\eta} \stackrel{def}{=} \int_\Omega \left(\int_0^{u_0(x)} f(x,t)dt - \sup_{0 \leq \xi \leq \xi_0} \int_0^{\xi} f(x,t)dt \right) dx > 0.$$

v) *there exist* $C > 0$ *and* $s \in]0, p[$ *such that* $\sup_{\xi \geq 0} \frac{\int_0^\xi f(x,t)dt}{1 + |\xi|^s} \leq C$ *for a.a.* $x \in \Omega$.

Then for each $\lambda > \frac{\|\nabla u_0\|_p^p}{p\overline{\eta}}$*, there exist two nonzero nonnegative weak solutions* $u_\lambda, v_\lambda \in W_0^{1,p}(\Omega) \cap C^{1+\gamma}(\Omega)$ *(with* $\gamma \in]0,1[$*) of problem* (P_λ).

Moreover, one has $\sup_{\lambda \in K} \max\{\|\nabla u_\lambda\|_p, \|\nabla v_\lambda\|_p\} < +\infty$ *for every bounded set* $K \subset \left] \frac{\|\nabla u_0\|_p^p}{p\overline{\eta}}, +\infty \right[$.

Sketch of Proof. We limit ourselves to sketch the proof of the existence of solutions. Let λ as in the hypotheses. We put $f(x,t) = 0$ for all $t \leq 0$, $x \in \Omega$. So, the nonnegative solutions of problem (P_λ) with f so modified are nonnegative solutions of the original problem. By conditions $i) - ii)$ the energy functional Ψ_λ defined by (1) is sequentially weakly lower semicontinuous and continuously Gâteaux differentiable. By condition $v)$ we also infer that the same is coercive. Thus, if we consider the weakly closed set

$$E = \{u \in W_0^{1,p}(\Omega) : 0 \leq u(x) \leq \xi_1\},$$

where ξ_1 is as in the hypotheses, then $\inf_E \Psi$ is attained in a point u_1. Condition $iv)$ implies that u_1 is, in point of fact, a local minimum for Ψ_λ. Moreover by $\lambda > \frac{\|\nabla u_0\|_p^p}{p\overline{\eta}}$ we also infer that u_1 is not a global minimum. Therefore, Ψ_λ has at least two distinct critical points. Moreover, one can show that Ψ_λ satisfies the Palais-Smale condition. So, using a mountain pass theorem (see [18]), we also find a third critical point. A standard argument shows that these three critical points are nonnegative and, clearly, two of which must be nonzero. □

Among the existing results which are comparable with Theorem 1, we point out Theorem 1.2 of [16] where the assumptions are close to ours. However, these results are mutually independent. For example, Theorem 1 allows us to consider nonlinearity f such that $\sup_{\xi \in [0,\delta]} \int_0^\xi f(x,t)dt > 0$ for all $\delta > 0$, contrarily to Theorem 1.2 of [16].

Now we present a second multiplicity result for problem (P_λ) assuming $f(\cdot, 0)$, $g(\cdot, 0)$ identically 0 and g sublinear with respect to the second variable. In this case, it is possible to obtain the existence of at least two non-zero solutions for λ small enough assuming only a mild growth condition on f (condition $i)$ below) that is fulfilled, for instance, when $f \in C^0(\overline{\Omega} \times \mathbb{R})$.

Theorem 2 (Theorem 2.3 of [1]). *Assume the following conditions*

i) *there exists* $q > \frac{N}{2}$ *such that* $\sup_{|\xi| \leq r} |f(\cdot, \xi)| \in L^q(\Omega)$ *for all* $r > 0$ *;*

ii) $f(x,0)=0$ *for a.e.* $x\in\Omega$*;*

iii) *there exist* $a>0$ *and* $s\in]1,2[$ *such that* $|g(x,t)|\le a|t|^{s-1}$ *for all* $t\in\mathbb{R}$ *and a.e* $x\in\Omega$.

iv) *there exists a non empty open set* $D\subseteq\Omega$ *such that*

$$\liminf_{\xi\to 0}\frac{\inf_{x\in D}\int_0^\xi g(x,t)dt}{\xi^2}=+\infty.$$

Then, there exist $\sigma,\overline{\lambda}>0$ *such that, for every* $\lambda\in[-\overline{\lambda},\overline{\lambda}]$*, there exist a strong nonzero nonnegative solution* $u_\lambda\in W_0^{1,2}(\Omega)\cap W^{2,q}(\Omega)$ *and a strong nonzero nonpositive solution* $v_\lambda\in W_0^{1,2}(\Omega)\cap W^{2,q}(\Omega)$ *of problem* (P_λ) *with* $\max\{\|u_\lambda\|_{W^{2,q}(\Omega)},\|v_\lambda\|_{W^{2,q}(\Omega)}\}\le\sigma$.

Sketch of Proof. In order to find the non-zero nonnegative solution we consider the function g_0 defined by

$$g_0(x,\xi)=\begin{cases}g(x,\xi) & \text{if } (x,\xi)\in\Omega\times[0,+\infty[\\ 0 & \text{if } (x,\xi)\in\Omega\times[0,+\infty[\end{cases}$$

as well as the function f_0 defined by

$$f_0(x,\xi)=\begin{cases}f(x,\xi) & \text{if } (x,\xi)\in\Omega\times[0,C]\\ f(x,C) & \text{if } (x,\xi)\in\Omega\times[C,+\infty)\\ 0 & \text{if } (x,\xi)\in\Omega\times]-\infty,0[\end{cases}$$

where C is a fixed number greater than $(aC_0)^{\frac{1}{2-s}}m(\Omega)^{\frac{1}{q(2-s)}}$, being $m(\Omega)$ the Lebesgue-measure of Ω, $C_0=C_0(N,q,\Omega)$ a positive constant such that, for each $h\in L^q(\Omega)$ and for each weak solution $u\in W_0^{1,2}(\Omega)$ of the equation $-\Delta u=h$ on Ω, one has $\|u\|_\infty\le C_0\|h\|_q$ (see [13]). Now, consider the energy functional Ψ_λ defined by (1) (with $p=2$) where f,g are replaced by f_0,g_0 respectively. Applying Theorem 2.1 of [19] we find $\lambda_0>0$ such that, for all $\lambda\in[-\lambda_0,\lambda_0]$, Ψ_λ has a local minimum u_λ which is nonzero thanks to condition *iv)*. Standard regularity arguments show that $u_\lambda\in C^0(\overline{\Omega})$ and one can check that u_λ must be nonnegative as well. Moreover, taking λ_0 smaller if necessary, by Schauder estimates and the choice of C we have $\|u\|_\infty\le C$. So u_λ is a nonnegative solution of (P_λ). In order to obtain the nonpositive solution it is suffice to repeat the previous proof replacing the functions f,g with $\tilde{f}(x,t)=-f(x,-t)$ and $\tilde{g}(x,t)=-g(x,-t)$ respectively. Finally we can easily find a uniformly (with respect to λ) upper bound for the $W^{2,q}(\Omega)$-norm of the solutions again using Schauder estimates. □

2.2. **Neumann boundary condition.** The results we present here are related to problem (P_λ) with the Neumann boundary condition $\frac{\partial u}{\partial\nu}_{|\partial\Omega}=0$. The first one deals with the ordinary case $N=1$, with $\Omega=]0,1[$. In this case, the boundary condition becomes the two point condition $u'(0)=u'(1)=0$. We consider the nonlinearity f of the type $\alpha(x)h(t)$ and take $g(x,t)=-t$. Then, using a variational result stated in [4], we are able to find $\lambda^*>0$ such that for all $\lambda\ge\lambda^*$ problem (P_λ) admits at least three weak solutions. Moreover, the number λ^* will turn out explicitly determined.

We note that the result below is, in some aspects, comparable with Theorem 1 and, likely, it can be deduced by using arguments similar to ones used in the proof of Theorem 1. However, applying the result of [4], the proof becomes very easy.

Theorem 3 (Theorem 4 of [4]). *Let $h : \mathbb{R} \to \mathbb{R}$ be a continuous function. Let $\alpha \in L^1([0,1])$ with $\alpha \neq 0$ and $\alpha(x) \geq 0$ for a.a. $x \in [0,1]$. Put $H(t) = \int_0^t h(\xi)d\xi$ for every $t \in \mathbb{R}$, and assume that there exist $t_0, t_1 \in \mathbb{R}$ and $r, b, s > 0$, with $|t_0| < r < |t_1|$, $s < 2$ such that*

(i) $H(t_0) = \sup_{|t|\leq cr} H(t)$;

(ii) $H(t_1) > H(t_0)$

(iii) $\sup_{t\in\mathbb{R}} \frac{H(t)}{1+|t|^s} \leq b$

where c is the embedding constant of $W^{1,2}(]0,1[)$ in $C^0([0,1])$. Then, for every $\lambda > \frac{t_1^2-t_0^2}{2(H(t_1)-H(t_0))\int_0^1 \alpha(x)dx}$, the problem

$$\begin{cases} -u'' + u = \lambda\alpha(x)h(u) & \text{in }]0,1[\\ u'(0) = u'(1) = 0 \end{cases}$$

admits at least three weak solutions $u_\lambda, v_\lambda, w_\lambda \in W^{1,2}(]0,1[)$. Moreover, for every bounded set

$$K \subseteq \left] \frac{t_1^2 - t_0^2}{2(F(t_1) - F(t_0))\int_0^1 \alpha(x)dx}, +\infty \right[,$$

we have

$$\sup_{\lambda\in K} \max\{\|u_\lambda\|_{W^{1,2}(]0,1[)}, \|v_\lambda\|_{W^{1,2}(]0,1[)}, \|w_\lambda\|_{W^{1,2}(]0,1[)}\} < +\infty.$$

Sketch of Proof. The solutions of the above problem are the critical points of the energy functional Ψ_λ defined in (1) where $g(x,t) = -t$, $f(x,t) = \alpha(x)h(t)$ and $\Omega =]0,1[$. Condition (iii) assures that Ψ_λ is coercive for all $\lambda \geq 0$ and satisfies the Palais-Smale condition (see Example 38.25 of [23]). If we put $\Phi(u) = \int_0^1 \alpha(x)H(u(x))dx$ and consider the functions $u_0, u_1 \in W^{1,2}(]0,1[)$ identically equal to t_0 and t_1 respectively, one has $\Phi(u_0) < \Phi(u_1)$. Moreover, if $u \in W^{1,2}(]0,1[)$ is such that $\|u'\|_2^2 + \|u\|_2^2 < r^2$ we easily get

$$\Phi(u_0) \geq \sup_{\|u'\|_2^2+\|u\|_2^2<r^2} \Phi(u).$$

Now, the conclusion follows by applying Theorem 2 and Remark 1 of [4] to the functionals Φ and $u \in W^{1,2}(]0,1[) \to \frac{1}{2}\|u'\|_2^2 + \frac{1}{2}\|u\|_2^2$. □

The next result gives the existence of at least one solution under very general conditions on the nonlinearity g. Here, as in Theorem 3, we suppose f of the type $\alpha(x)h(t)$ and assume that the primitive H of h does not attain its maximum on a compact interval $[a,b]$ at the extreme points a, b. We assume $p = 2$.

Theorem 4 (Theorem 1 of [3]). *Let $[a,b] \subset \mathbb{R}$ be a compact interval, and $h : [a,b] \to \mathbb{R}$ a continuous function. Let H be a primitive of h and suppose that*

$$\max\{H(a), H(b)\} < \max_{[a,b]} H. \tag{2}$$

Moreover let $\alpha \in L^\infty(\Omega)$ *with* $\operatorname{ess\,inf}_\Omega \alpha > 0$. *Then, for every Carathéodory function* $g : \Omega \times [a,b] \to \mathbb{R}$ *satisfying*

$$\sup_{t\in[a,b]} |g(\cdot,t)| \in L^q(\Omega)$$

for some $q > N$, *there exist* $\overline{\lambda}, \sigma > 0$ *such that, for every* $\lambda \in [-\overline{\lambda}, \overline{\lambda}]$, *there exists a strong solution* $u_\lambda \in W^{2,q}(\Omega)$ *of problem* (P_λ), *where* $f(x,t) = \alpha(x)h(t)$, *fulfilling* $u_\lambda(x) \in]a,b[$ *for all* $x \in \overline{\Omega}$ *and* $\|\nabla u\|_2^2 + \|u\|_2^2 \le \sigma$.

Sketch of Proof. By condition (2) we infer that there exists a subinterval $[c,d] \subset [a,b]$ such that $\max\{H(c), H(d)\} < \max_{[c,d]} H$ and $h(c) > 0, h(d) < 0$. Now, extend $\alpha(\cdot)h(\cdot)$ and $g(\cdot,\cdot)$ to $\Omega \times \mathbb{R}$ putting $h(t) = h(c)$, $g(x,t) = g(x,c)$ for $t \le c$ and $h(t) = h(d)$, $g(x,t) = g(x,d)$ for $t \ge d$. Let $K \subset [c,d]$ the set of the global maxima of H. Then, if for $\xi \in K$ we denote by u_ξ the constant function identically equal to ξ, we see that each point of the set $\tilde{K} = \{u_\xi, \xi \in K\}$ is a global minimum for the functional

$$J(u) = \frac{1}{2}\|\nabla u\|^2 - \int_\Omega H(u(x))dx$$

in $W^{1,2}(\Omega)$. Moreover, if for $r > 0$ we put $\tilde{K}_r = \{u \in W^{1,2}(\Omega) : \inf_{\xi\in K}(\|\nabla u - \nabla u_\xi\|^2 + \|u - u_\xi\|^2) \le r^2\}$, then $\tilde{K}_r$ is weakly compact in $W^{1,2}(\Omega)$ and one has $\inf_{\partial\tilde{K}_r} J > \inf_{W^{1,2}(\Omega)} J$. This allows us to apply Theorem 2.1 of [19] to find, for all $r > 0$, a positive number λ_r such that for all $\lambda \in [-\lambda_r, \lambda_r]$ the functional $J(u) - \lambda \int_\Omega \left(\int_0^{u(x)} g(x,t)dt\right) dx$ has a local minimum which belongs to $\tilde{K}_r$. From this, using a regularity result jointly to a boot-strap argument, we find $\overline{\lambda} > 0$ such that, for all $\lambda \in [-\overline{\lambda}, \overline{\lambda}]$, there exists a local minimum u_λ of the previous functional satisfying $u_\lambda(x) \in]c,d[$ for all $x \in \Omega$. Consequently, u_λ turns out a weak solution of problem (P_λ). Finally, the fact that u_λ is a strong solution follows from the regularity result quoted above. □

Remark. Note that from Theorem 4 we can obtain in a straightforward way the existence of at least n strong solutions result assuming that condition (2) holds for n disjoint compact intervals.

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