A NONSYMMETRIC LINEAR COMPLEMENTARITY PROBLEM TO SOLVE A QUASISTATIC ROLLING FRICTIONAL CONTACT PROBLEM

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Dedicated to the memory of Filippo Chiarenza.

ABSTRACT. A simple approach and an algorithm are proposed to solve the quasistatic rolling frictional contact problem between an elastic cylinder and a flat rigid body. The discretization is based on the boundary element method. The unilateral frictional contact problem (nonsmooth but monotone) is formulated in a compact form as a nonsymmetric linear complementarity problem which is solved using Lemke's algorithm.

1. INTRODUCTION

In the quasistatic case of a rolling elastic cylinder in frictional contact with a flat rigid body, the relative slip velocity is a very important parameter of the problem. To compute it, a direct method is presented in Abascal et al. [6, 7, 8], where two approaches are used considering the velocity as unknown variable in the LCP solving the frictional contact.

Here, we propose another approach based in solving the static frictional contact problem and, afterwards, computing the velocity using an explicit appropriate relation. The boundary element method (BEM) is used to discretize the mechanical problem. Then the elastostatic, frictional contact, rolling problem is formulated as a compact LCP using the boundary element method and the rigid displacement approach.

2. MODELLING, DISCRETIZATION AND CONDENSATION OF THE PROBLEM

Following the approach used in [1] the rolling problem is reduced to a plane strain state (see, e.g., [6, 7, 8]) where the Coulomb's law describes the friction.

The friction is assumed to follow the dry Coulomb's law where normal and tangential tractions on the boundaries of the contact zone are related via a coefficient of friction. Under this assumption, two points belonging to the cylinder A and the rigid body B fall in three different states relative to each other (Fig. 1):

(1) <u>Stick state</u>

$$(1) \qquad \begin{aligned} r_N^A \leq 0, \quad r_N^B \leq 0, \quad r_N^A = r_N^B, \quad \delta_n = 0, \\ r_T^A = r_T^B, \qquad\qquad s_t = 0, \end{aligned}$$
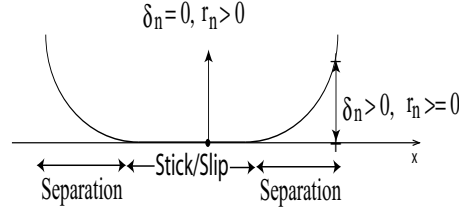

FIGURE 1. Potential contact zone.

(2) Slip state

$$(2) \quad \begin{array}{cccc} r_N^A \le 0, & r_N^B \le 0, & r_N^A = r_N^B, & \delta_n = 0, \\ |r_T^A| = \mu |r_N^A|, & |r_T^B| = \mu |r_N^B|, & sgn(s_t) = -sgn(r_T^A), \end{array}$$

(3) Separation state

$$(3) \quad r_N^A = 0, \quad r_N^B = 0, \quad r_T^A = 0, \quad r_T^B = 0, \quad \delta_n \ge 0.$$

Here, μ is the constant coefficient of friction, r_N^A, r_N^B, r_T^A and r_T^B are the normal and tangential tractions of cylinder A and rigid body B, respectively, at contacting points, u_N and u_T are the normal and the tangential cylinder surface displacements, s_t is the slip velocity and δ_n is the normal separation given by:

$$(4) \quad \delta_n = \delta_{n0} - u_N.$$

The initial separation, δ_{n0}, for a cylinder and a flat body in contact can be approximated by

$$(5) \quad \delta_{n0} = \frac{x^2}{2R},$$

where R is the cylinder radius and x is the Eulerian coordinate along the contact zone used to position each pair of points at each time relative to a rigid body position of the cylinder.

When applying an Eulerian description of particles moving through the contact area, the relative tangential slip velocity of each cylinder surface point is defined as

$$(6) \quad s_t = \dot{\delta}_t = \frac{d\delta_t(x,\tau)}{d\tau}$$

where τ is the time coordinate, $x = x(\tau)$ is the Cartesian coordinate of each point relative to fixed axes and varying time τ, and δ_t is the tangential separation given by

$$(7) \quad \delta_t = (x^A - x^B) + u_T^A + u_T^B$$

where x^A and x^B are the cartesian coordinates of contacting points of the cylinder and the rigid respectively. u_T^A and u_T^B describe the tangential displacements of at contacting points of the cylinder and the rigid body, respectively.

Substituting (7) into (6) leads to

$$(8) \quad s_t = V^A - V^B + V^A u_{T,x}^A + V^B u_{T,x}^B + u_{T,\tau}^A + u_{T,\tau}^B$$

where V^α is the velocity of a point of the rigid body $\alpha(\alpha = A, B)$ in the x-direction. $u^A_{T,x} = \frac{\partial u^A_T}{\partial x}$. Under steady-state rolling conditions, the variation with time vanishes, so

$$(9) \qquad\qquad s_t = V^A - V^B + V^A u^A_{T,x} + V^B u^B_{T,x}$$

which is usually approximated [9] by

$$(10) \qquad\qquad s_t = |V|\left(\xi + sgn(V)(u^A_{T,x} + u^B_{T,x})\right)$$

where V is given by $(V^A + V^B)/2$, and ξ is the normalized relative rigid slip velocity (creepage), defined as $\xi = \dfrac{V^A - V^B}{|V|}$. In fact, we consider s_t, in practice, as dimensionless variable denoted by s^*_t [8]:

$$(11) \qquad\qquad s^*_t = \xi + sgn(V)(u^A_{T,x} + u^B_{T,x})$$

When the displacement derivatives are approximated using a finite difference scheme, the tangential slip velocity for a point located at the coordinate x_i is, under steady-state rolling conditions, expressed as

$$(12) \qquad\qquad s^*_t(x_i) = \xi + sgn(V)\left(\frac{u_T(x_{i+1}) - u_T(x_i)}{h_i}\right)$$

where h_i denotes the distance between two adjacent boundary points x_{i+1} and x_i.

To analyze coupled 2D rolling contact between two cylinders, Abascal et al. have formulated the problem considering the slip velocity as unknown variable. Indeed, in [8], the NORM-TANG iteration [16] to solve two complementarity problems. In [6], the problem is analyzed by minimizing a function representing the equilibrium equation and the contact restrictions. In our case, the static contact-friction problem between an elastic cylinder and a flat rigid body is first solved and, subsequently, the slip velocity is computed explicitly using the above formula. The model is discretized using the boundary element method which is more suitable for this class of problem where the nonlinearity is limited to the boundary of the body. The here adopted formulation follows the lines of [2], [3], [11], [12], [17] and [18] where the matrix formulation of displacement boundary element in elastostatics is given by [4]:

$$(13) \qquad\qquad\qquad\qquad Hu = Gt.$$

Here, u is the vector of the nodal boundary displacements, t is the vector of element boundary tractions, and H, G are the appropriate matrices.

The number of equations in (13) depends on the number of nodes on the discretized boundary. Let us note that the number of boundary element tractions (i.e., the dimension of t) depends on the nature of the boundary elements used.

The classical approach for the solution of the bilaterally constrained structures goes through the specification of appropriate, known boundary displacements or tractions, the rearrangement of the system (13), and finally, the formulation of a nonsymmetric system of equations, after reordering:

$$(14) \qquad\qquad\qquad\qquad Ay = b.$$

The 2n-dimensional vector y contains all the unknown boundary displacements or tractions of the problem. In the contact area, both displacements and tractions are unknown. They must be kept in the formulation and connected with the inequality and complementarity relations of the unilateral contact mechanism. After solving the arising LCP, one knows which of these variables vanishes. Thus, we proceed with condensation and, then, formulate the linear complementarity problem.

Let us consider u_c and t_c as being the boundary nodal displacements and tractions, respectively, at the frictional unilateral contact boundary of the cylinder. After partitioning the boundary of the cylinder, the equation (14) gives:

$$(15) \qquad \begin{bmatrix} H_{ff} & H_{fc} \\ H_{cf} & H_{cc} \end{bmatrix} \begin{bmatrix} u \\ u_c \end{bmatrix} = \begin{bmatrix} f_f \\ f_c \end{bmatrix} + \begin{bmatrix} G_{fc} \\ G_{cc} \end{bmatrix} \begin{bmatrix} t_c \end{bmatrix}$$

When n_c is the number of the nodes at the unilateral contact boundary, then u_c and t_c have $2n_c$ elements, H_{cc} and G_{cc} have $2n_c \times 2n_c$ elements, H_{ff} has $2(n-n_c) \times 2(n-n_c)$ elements, G_{fc}, H_{fc} and H_{cf} have $2(n-n_c) \times 2n_c$ elements. u denotes the nodal boundary displacements of the cylinder outside the contact area. The next step is to perform a local coordinate transformation so that normal and tangential to the unilateral boundary quantities appear in the formulation. Therefore, let us consider w and r as the natural local coordinates (normal and tangential coordinates) of the displacements and the tractions in the contact boundary, respectively:

$$(16) \qquad u_c^i = \begin{bmatrix} u_{cx}^i \\ u_{cy}^i \end{bmatrix}, \quad t_c^i = \begin{bmatrix} t_{cx}^i \\ t_{cy}^i \end{bmatrix}, \quad w^i = \begin{bmatrix} u_N^i \\ u_T^i \end{bmatrix}, \quad r^i = \begin{bmatrix} r_N^i \\ r_T^i \end{bmatrix}.$$

The transformation for a single unilateral boundary node i reads:

$$(17) \qquad C_i u_c^i = w^i, \quad -C_i t_c^i = r^i$$

with

$$(18) \qquad C_i = \begin{bmatrix} \cos\phi_i & \sin\phi_i \\ -\sin\phi_i & \cos\phi_i \end{bmatrix}.$$

Since $C_i^{-1} = C_i^T$, relations (17) can be inverted:

$$(19) \qquad u_c^i = C_i^T w, \quad t_c^i = -C_i^T r.$$

Taking into account these transformations, one has

$$(20) \qquad \begin{bmatrix} H_{ff} & H_{fc}C^T \\ H_{cf} & H_{cc}C^T \end{bmatrix} \begin{bmatrix} u \\ w \end{bmatrix} = \begin{bmatrix} f_f \\ f_c \end{bmatrix} - \begin{bmatrix} G_{fc}C^T \\ G_{cc}C^T \end{bmatrix} \begin{bmatrix} r \end{bmatrix}.$$

Then, the problem will be condensed, i.e., in order to get the flexibility matrix, the unknown variable u will be reduced by considering the relation (20):

$$(21) \qquad u = H_{ff}^{-1}f_f - H_{ff}^{-1}G_{fc}C^T r - H_{ff}^{-1}H_{fc}C^T w$$

Then,

$$(22) \quad \left[H_{cc}C^T - H_{cf}H_{ff}^{-1}H_{fc}C^T \right] w = \left[f_c - H_{cf}H_{ff}^{-1}f_f \right]$$
$$- \left[G_{cc}C^T - H_{cf}H_{ff}^{-1}G_{fc}C^T \right] r$$

Thus, w can be written as:

$$(23) \qquad\qquad w = w_0 + Fr$$

where

$$w_0 = \left[H_{cc}C^T - H_{cf}H_{ff}^{-1}H_{fc}C^T \right]^{-1} \left[f_c - H_{cf}H_{ff}^{-1}f_f \right]$$

and

$$F = - \left[H_{cc}C^T - H_{cf}H_{ff}^{-1}H_{fc}C^T \right]^{-1} \left[G_{cc}C^T - H_{cf}H_{ff}^{-1}G_{fc}C^T \right],$$

or

$$(24) \qquad \begin{bmatrix} w_n \\ w_t \end{bmatrix} = \begin{bmatrix} w_{n0} \\ w_{t0} \end{bmatrix} + \begin{bmatrix} F_{nn} & F_{nt} \\ F_{tn} & F_{tt} \end{bmatrix} \begin{bmatrix} r_N \\ r_T \end{bmatrix}.$$

3. The frictional contact formulation

We follow the formulation of [13] and [14]. Let the normal forces and the friction forces be assembled in vectors $\mathbf{r}_N = \{r_{N1}, \ldots, r_{Nn}\}^T$ and $\mathbf{r}_T = \{r_{T1}, r_{T2}, \ldots, r_{Tn}\}^T$, respectively, where, e.g., r_{Ti} is the frictional force of the i-th contact node.

Coulomb's law of dry friction connects the tangential (frictional) forces with the normal (contact) forces by the relation

$$(25) \qquad \gamma_i = \mu|r_{Ni}| - |r_{Ti}|, \ \ i = 1, \ldots, n, \ \ \gamma_i \geq 0.$$

Here $|.|$ denotes the absolute value and μ is the friction coefficient. The friction mechanism is considered to work in the following way: If $|r_{Ti}| < \mu|r_{Ni}|$ (i.e., $\gamma_i > 0$), the slipping value γ_{iT} must be equal to zero, and if $|r_{Ti}| = \mu|r_{Ni}|$ (i.e., $\gamma_i = 0$), then we have slipping in the opposite direction of r_{Ti}:

$$(26) \qquad \begin{aligned} &\text{if } \gamma_i > 0, \text{ then } y_{Ti} = 0 \\ &\text{if } \gamma_i = 0, \text{ then there exists } \sigma > 0 \text{ such that } y_{Ti} = -\sigma r_{Ti}. \end{aligned}$$

By assembling the contributions of all (n) unilateral nodes, relation (26) reads:

$$(27) \qquad \gamma = \mathbf{T}_N^T \mathbf{r}_N + \mathbf{T}_T^T \mathbf{r}_T$$

with the matrices $\mathbf{T}_T$ and $\mathbf{T}_N$

$$(28) \qquad \mathbf{T}_T = \text{diag} \left[\mathbf{T}_T^1, \mathbf{T}_T^2, \ldots, \mathbf{T}_T^n \right], \quad \mathbf{T}_N = \text{diag} \left[\mathbf{T}_N^1, \mathbf{T}_N^2, \ldots, \mathbf{T}_N^n \right].$$

These matrices are obtained from the linearized friction law considered in 2D [2, 13].

$$(29) \qquad \mathbf{T}_T^j = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{T}_N^j = \left[\mu, \mu \right].$$

Finally, the slip value in (25), (26) is written as

$$(30) \qquad \mathbf{y}_T = \mathbf{T}_T \lambda, \quad \lambda \geq 0$$

where λ is a vector of nonnegative slipping parameters. Then, γ and λ fulfil the following complementarity condition:
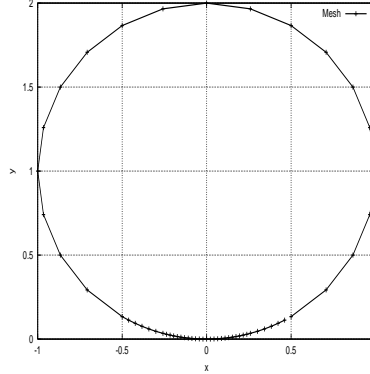
$$(31) \qquad \gamma^T \lambda = 0$$

FIGURE 2. The mesh around the cylinder contour.

3.1. **The Linear Complementarity Problem formulation.** To formulate the linear complementarity problem the rigid body displacements approach is adopted. The slipping value λ and the tangential displacements $\mathbf{u}_T$ are then related by the compatibility relation:

$$(32) \qquad \mathbf{T}_T\lambda - \mathbf{u}_T = \mathbf{d}_T$$

where $\mathbf{d}_T$ denotes the initial tangential distance.

The general decomposition scheme with slack variables can be found in more details for two-dimensional friction problems in [14].

The structure is assumed linear elastic which, on the assumption that everything outside of the frictional contact interfaces has been condensed out, reads:

$$(33) \qquad \tilde{\mathbf{u}} = \tilde{\mathbf{F}}\tilde{\mathbf{r}}$$

where

$$\tilde{\mathbf{u}} = \begin{bmatrix} \mathbf{u}_N \\ \mathbf{u}_T \end{bmatrix}, \quad \tilde{\mathbf{F}} = \begin{bmatrix} \mathbf{F}_{NN} & \mathbf{F}_{NT} \\ \mathbf{F}_{TN} & \mathbf{F}_{TT} \end{bmatrix}, \quad \tilde{r} = \begin{bmatrix} \mathbf{r}_N \\ \mathbf{r}_T \end{bmatrix}.$$

Here, $\tilde{\mathbf{F}}$ is the symmetric flexibility matrix where $\mathbf{F}_{NN}$ is an $n \times n$ nonsingular matrix with the mechanical meaning of being the normal flexibility matrix, $\mathbf{F}_{TT}$ is a $2n \times 2n$ nonsingular matrix (the tangential flexibility) and $\mathbf{F}_{NT}$, $\mathbf{F}_{TN}$ are the corresponding couple flexibility matrices.

By using the previous relations, the unilateral kinematic conditions normal and tangential to the interface, take the form:

$$(34) \qquad \begin{aligned} \mathbf{y}_N - \mathbf{F}_{NN}\mathbf{r}_N - \mathbf{F}_{NT}\mathbf{r}_T &= \mathbf{d}_N, \\ \mathbf{T}_T\lambda - \mathbf{F}_{TN}\mathbf{r}_N - \mathbf{F}_{TT}\mathbf{r}_T &= \mathbf{d}_T. \end{aligned}$$

A standard LCP formulation is derived by means of the following change of variables. First, from the second relation in (34), $\mathbf{r}_T$ is expressed as follows:

$$(35) \qquad \mathbf{r}_T = -\mathbf{F}_{TT}^{-1}\mathbf{F}_{TN}\mathbf{r}_N + \mathbf{F}_{TT}^{-1}\mathbf{T}_T\lambda - \mathbf{F}_{TT}^{-1}\mathbf{d}_T$$

Then, by eliminating $\mathbf{r}_T$ from equations (34), we obtain

$$(36) \qquad \begin{aligned} \mathbf{y}_N - \left(\mathbf{F}_{NN} - \mathbf{F}_{NT}\mathbf{F}_{TT}^{-1}\mathbf{F}_{TN}\right)\mathbf{r}_N - \mathbf{F}_{NT}\mathbf{F}_{TT}^{-1}\mathbf{T}_T\lambda &= \mathbf{d}_N - \mathbf{F}_{NT}\mathbf{F}_{TT}^{-1}\mathbf{d}_T \\ \gamma + \left(\mathbf{T}_T^T\mathbf{F}_{TT}^{-1}\mathbf{F}_{TN} - \mathbf{T}_N^T\right)\mathbf{r}_N - \mathbf{T}_T^T\mathbf{F}_{TT}^{-1}\mathbf{T}_T\lambda &= -\mathbf{T}_T^T\mathbf{F}_{TT}^{-1}\mathbf{d}_T \end{aligned}$$
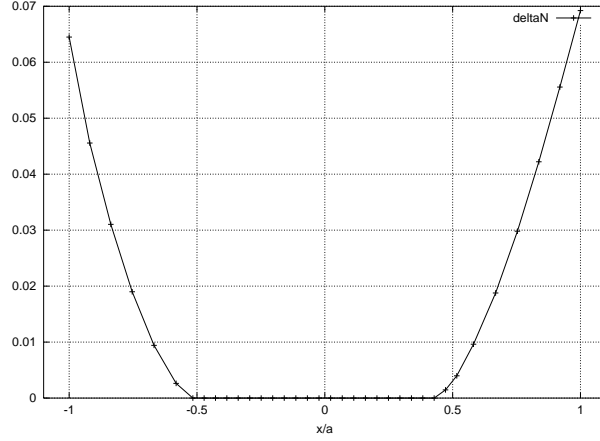
FIGURE 3. Distribution of stick/slip and normal separation in the potential contact zone.

Finally, a standard LCP is obtained from equations (36):

$$(37) \qquad \mathbf{z} \geq \mathbf{0}, \quad \mathbf{Mz} + \mathbf{b} \geq \mathbf{0}, \quad (\mathbf{Mz} + \mathbf{b})^T \mathbf{z} = 0$$

with

$$(38) \qquad \begin{aligned} \mathbf{z} &= \begin{bmatrix} \mathbf{r}_N \\ \lambda \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} \mathbf{d}_N - \mathbf{F}_{NT}\mathbf{F}_{TT}^{-1}\mathbf{d}_T \\ -\mathbf{T}_T^T\mathbf{F}_{TT}^{-1}\mathbf{d}_T \end{bmatrix}, \\ \mathbf{M} &= \begin{bmatrix} (\mathbf{F}_{NN} - \mathbf{F}_{NT}\mathbf{F}_{TT}^{-1}\mathbf{F}_{TN}) & \mathbf{F}_{NT}\mathbf{F}_{TT}^{-1}\mathbf{T}_T \\ -(\mathbf{T}_T^T\mathbf{F}_{TT}^{-1}\mathbf{F}_{TN} - \mathbf{T}_N^T) & \mathbf{T}_T^T\mathbf{F}_{TT}^{-1}\mathbf{T}_T \end{bmatrix}, \\ \mathbf{Mz} + \mathbf{b} &= \begin{bmatrix} \mathbf{y}_N \\ \gamma \end{bmatrix} \end{aligned}$$

Note also that (37) can be written as the variational inequality:

$$(39) \qquad \mathbf{z} \geq \mathbf{0}: \ (\mathbf{Mz} + \mathbf{b})^T(\mathbf{v} - \mathbf{z}) = 0, \ \forall \ \mathbf{v} \geq \mathbf{0}$$

4. NUMERICAL RESULTS

To discretize the rolling problem with BEM (Fig. 2), constant elements are used. In fact, Karami [10] and Stavroulakis et al. [18] remarked the existence of oscillations in the traction behaviour with quadratic elements. Young modulus and Poisson coefficient are taken to be 94500 and 0.1, respectively. The rigid body normal and tangential displacement used were $d_N = R/2$ and $d_T = 0.1 d_N$ where R is the radius of the cylinder. The LCP (37 is solved using Lemke's algorithm [15] The results shown in the Figures 3, 4, 5, and 6 are obtained by applying a normal and a tangential rigid body displacements approach. Figure 3 shows the existence of a separation area in the potential contact zone. The stick/slip area is not centered because of the tangential effect. In the Figures 4 and 5, the normal and the tangential tractions in the frictional contact case are plotted. Figure 6 shows the velocity behaviour only in the effective contact zone.
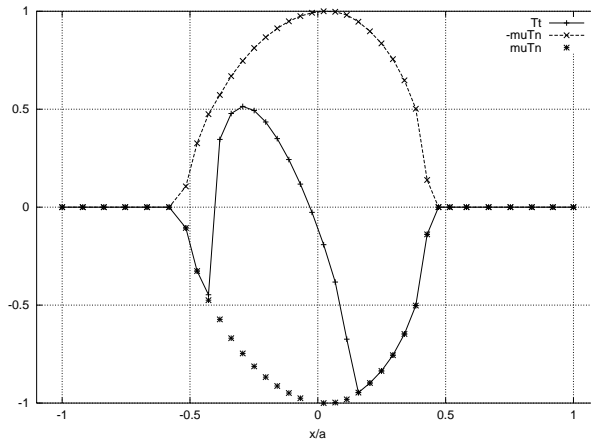
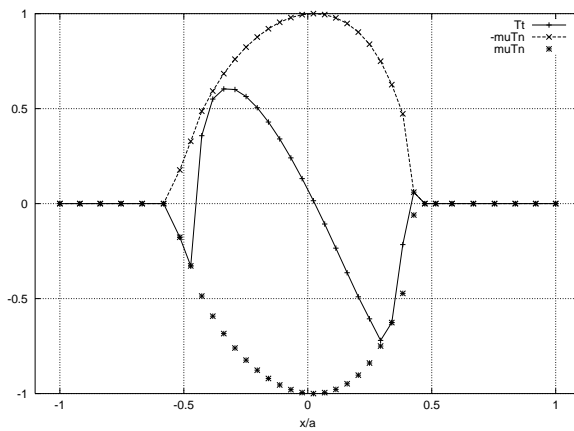FIGURE 4. Normal and tangential tractions in the frictional contact case ($\mu = 0.1$).

FIGURE 5. Normal and tangential tractions in the frictional contact case ($\mu = 0.3$).

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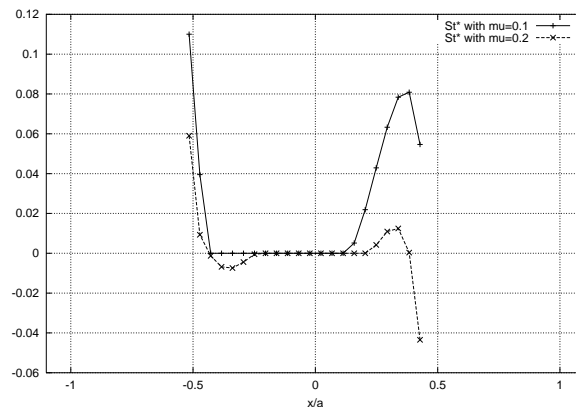
FIGURE 6. Tangential velocity in the effective contact zone: Dependence on the friction coefficient

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