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A PARAMETRIC EQUILIBRIUM PROBLEM WITH APPLICATIONS TO OPTIMIZATION PROBLEMS UNDER EQUILIBRIUM CONSTRAINTS

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Abstract. In this paper, we introduce a type of parametric vector equilibrium problem with applications to related mathematical programming with equilibrium constraint problems (in short, MPEC). The existence and the closedness of the graph of solution mappings for the parametric vector equilibrium problems are established. As extensions, we also obtain some existence of MPEC problems corresponding to the lower-level problem generated by parametric vector equilibrium problems.

1. Introduction

Let Ω_1 and Ω_2 be two nonempty subsets of Hausdorff topological vector spaces X and Y, respectively. Let Z be a Hausdorff topological vector space and $\operatorname{int} C(x) \subset Z$ be a domination structure generated by set-valued mapping $C : \Omega_1 \to 2^Z$ at $x \in \Omega_1$ such that $C(x)$ is a pointed convex cone with nonempty interior for each $x \in \Omega_1$. Suppose that the constraint map Ω is a set-valued mapping from Ω_1 to $2^{\Omega_2} \setminus \{\emptyset\}$. Let g be a vector-valued function from $\Omega_1 \times \Omega_2 \times \Omega_2$ to Z. We consider the following parametric vector equilibrium problem (PVEP): for a given $x \in \Omega_1$,

$$\text{(PVEP)} \qquad \begin{array}{l} \text{find } y^* \in \Omega(x) \text{ such that} \\ g(x, y^*, v) \notin -\operatorname{int} C(x) \text{ for all } v \in \Omega(x), \end{array}$$

where $\operatorname{int}C(x)$ denotes the interior of the set $C(x)$. The solution mapping S_E is a set-valued mapping from Ω_1 to 2^{Ω_2} defined by.

$$S_E(x) = \{y \in \Omega(x) : g(x, y, v) \notin -\operatorname{int} C(x), \text{ for all } v \in \Omega(x)\}. \tag{1}$$

If Ω, C, and g have a constant value for $x \in \Omega_1$, respectively, then the problem PVEP is reduced to an ordinary vector equilibrium problem. Liou et al. [5] introduced a weak PVVI as follows: for a given $x \in \Omega_1$,

$$\text{(PVVI)} \qquad \begin{array}{l} \text{find } y^* \in \Omega(x) \text{ such that} \\ \nabla_y \varphi(x, y^*)(y^* - v) \notin -\operatorname{int} C \text{ for all } v \in \Omega(x), \end{array}$$

where $\varphi = (\varphi_1, \ldots, \varphi_p) : \Omega_1 \times \Omega_2 \to \mathbb{R}^p$, $\varphi(x, \cdot)$ is differentiable in $\Omega(x)$ for a given $x \in \Omega_1$ and $\operatorname{int} C \subset Z$ is a domination structure generating a partial ordering on Z; see Yu [9]. It is clear that weak PVVI is a special case of PVEP.

The purpose of this paper is to establish some existence results for PVEP and give some applications of PVEP, particularly to the mathematical programs with vector equilibrium constraints. To this end, we will give some preliminaries which

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will be used for the rest of this paper in Section 2. We will establish some existence results and closedness of the graph of the solution map for PVEP In Section 3. Finally we will establish some existence results for the mathematical program with equilibrium constraints as applications of PVEP.

2. Preliminaries

We recall the cone-convexity of vector-valued functions by Tanaka [7]. Let X be a vector space, and Z also a vector space with a partial ordering defined by a pointed convex cone C. Suppose that K is a convex subset of X and that f is a vector-valued function from K to Z. The mapping f is said to be C-convex on K if for each $x_1, x_2 \in K$ and $\lambda \in [0,1]$, we have

$$\lambda f(x_1) + (1-\lambda) f(x_2) \in f(\lambda x_1 + (1-\lambda)x_2) + C.$$

As a special case, if $Z = \mathbb{R}$ and $C = \mathbb{R}_+$ then C-convexity is the same as ordinary convexity.

Definition 1 (C-quasiconvexity, [2, 6, 7])**.** Let X be a vector space, and Z also a vector space with a partial ordering defined by a pointed convex cone C. Suppose that K is a convex subset of X and that f is a vector-valued function from K to Z. Then f is said to be *C-quasiconvex on* K if it satisfies one of the following two equivalent conditions:

(i) for each $x_1, x_2 \in K$ and $\lambda \in [0,1]$,

$$f(\lambda x_1 + (1-\lambda)x_2) \in z - C, \text{ for all } z \in C(f(x_1), f(x_2)),$$

where $C(f(x_1), f(x_2))$ is the set of upper bounds of $f(x_1)$ and $f(x_2)$, i.e.,

$$C(f(x_1), f(x_2)) := \{z \in Z : z \in f(x_1) + C \text{ and } z \in f(x_2) + C\}.$$

(ii) for each $z \in Z$,

$$A(z) := \{x \in K : f(x) \in z - C\}$$

is convex or empty.

First statement is defined by Luc [6] and the second is by Ferro [2].

Remark 1 (See Tanaka [7]). Some readers recall the following Helbig's definition which is stronger than Luc and Ferro definition. When Z is a locally convex space and C is closed, the definition is equivalent to C-naturally quasiconvex defined by Tanaka [7].

Definition 2 (Helbig's C-quasiconvexity, [4, 7])**.** Let X be a vector space, and Z also a locally convex space with a partial ordering defined by a closed pointed convex cone C. Suppose that K is a convex subset of X and that f is a vector-valued function from K to Z. Then, f is said to be (Helbig's) *C-quasiconvex on* K if for every $x_1, x_2 \in X$ and $\lambda \in [0,1]$, and each $\varphi \in C^*, \varphi(f(\lambda x_1 + (1-\lambda)x_2) \leq \max\{\varphi(f(x_1)), \varphi(f(x_2))\}$, where C^* stands for the topological dual cone of C.

Example 1. $f : \mathbb{R} \to \mathbb{R}^2$ is defined by $f(x) = (x, -|x|)$ for $x \in [-1,1]$ and $C = \{(x,y) \in \mathbb{R}^2 : y \geq |x|\}$. Then we can see that f is Luc and Ferro C-quasiconvex, but not Helbig C-quasiconvex.

Definition 3 (C-continuity, [6, 8])**.** Let X be a topological space, and Z a topological vector space with a partial ordering defined by a solid pointed convex cone C. Suppose that f is a vector-valued function from X to Z. Then, f is said to be *C-continuous at* $x \in X$ if it satisfies one of the following two equivalent conditions:

(i) For any neighborhood $V_{f(x)} \subset Z$ of $f(x)$, there exists a neighborhood $U_x \subset X$ of x such that $f(u) \in V_{f(x)} + C$ for all $u \in U_x$.

(ii) For any $k \in \operatorname{int} C$, there exists a neighborhood $U_x \subset X$ of x such that $f(u) \in f(x) - k + \operatorname{int} C$ for all $u \in U_x$.

Moreover a vector-valued function f is said to be *C-continuous on* X if f is C-continuous at every x on X.

Remark 2. Whenever $Z = \mathbb{R}$ and $C = \mathbb{R}_+$, C-continuity and $(-C)$-continuity are the same as ordinary lower and upper semicontinuity, respectively. In [8, Definition 2.1 (pp.314–315)] corresponding to ordinary functions, C-continuous function is called C-lower semicontinuous function, and $(-C)$-continuous function is called C-upper semicontinuous function.

Definition 4 (see [1])**.** Let X and Y be two topological spaces, $T : X \to 2^Y$ a set-valued mapping.

(i) T is said to be *lower semicontinuous* (l.s.c. for short) at $x \in X$ if for each open set V with $T(x) \cap V \neq \emptyset$, there is an open set U containing x such that for each $z \in U$, $T(z) \cap V \neq \emptyset$; T is said to be l.s.c. on X if it is l.s.c. at all $x \in X$.

(ii) The graph of T, denoted by $\mathrm{Gr}(T)$ is the following set:
$$\{(x, y) \in X \times Y : y \in T(x)\}.$$

Definition 5 (Parameterized cone continuity)**.** Let Ω_1 and Ω_2 be two nonempty subsets of Hausdorff topological vector spaces X and Y, respectively. Let Z be also a Hausdorff topological vector space and C a set-valued mapping from Ω_1 to 2^Z such that $C(x)$ is a pointed convex cone with nonempty interior for each $x \in \Omega_1$. Suppose that Ω is a set-valued mapping from Ω_1 to $2^{\Omega_2} \setminus \{\emptyset\}$. Then a vector-valued function $g : \Omega_1 \times \Omega_2 \times \Omega_2 \to Z$ is said to be *parametarized C-continuous on* $\Omega_1 \times \Omega_2$ *with respect to* Ω, if for each $p \in \Omega_1$ and $x \in \Omega(p)$ such that
$$g(p, x, y) \in \operatorname{int} C(p) \text{ for some } y \in \Omega(p),$$
there exists a neighborhood $\mathcal{U}$ of (p, x) such that for all $(\tilde{p}, \tilde{x}) \in \mathcal{U} \cap \mathrm{Gr}(\Omega)$
$$g(\tilde{p}, \tilde{x}, \hat{y}) \in \operatorname{int} C(\tilde{p}) \text{ for some } \hat{y} \in \Omega(\tilde{p}).$$

Proposition 1. *Let Ω_1 and Ω_2 be two nonempty subsets of two normal spaces X and Y, respectively. Let Z be a normal topological vector space, and C a set-valued mapping from Ω_1 to 2^Z such that $C(x)$ is a pointed convex cone with nonempty interior for each $x \in \Omega_1$. Suppose that Ω is a set-valued mapping from Ω_1 to $2^{\Omega_2} \setminus \{\emptyset\}$, and that g is a vector-valued function from $\Omega_1 \times \Omega_2 \times \Omega_2$ to Z. Also assume the following conditions:*

(i) *g is $-C(p)$-continuous on $\Omega_1 \times \Omega_2 \times \Omega_2$;*

(ii) *Ω is l.s.c. on Ω_1;*

(iii) *the set-valued map $W(p) = Z \setminus -\operatorname{int} C(p)$ has closed graph.*

Then g is parametarized $-C$-continuous on $\Omega_1 \times \Omega_2$ with respect to Ω.

Proof. Suppose for each $\hat{p} \in \Omega_1$ and $\hat{x} \in \Omega(\hat{p})$ such that $g(\hat{p}, \hat{x}, \hat{y}) \in -\operatorname{int} C(\hat{p})$ for some $\hat{y} \in \Omega(\hat{p})$. Then there is a $\hat{z} \in -\operatorname{int} C(\hat{p})$ such that $\hat{z} - \operatorname{cl} C(\hat{p})$ is a closed neighborhood of $g(\hat{p}, \hat{x}, \hat{y})$.

On the other hand $\{\hat{p}\} \times (\hat{z} - \operatorname{cl} C(\hat{p}))$ is a closed subset of $\Omega_1 \times Z$ such that

$$\operatorname{Gr}(W) \cap (\{\hat{p}\} \times (\hat{z} - \operatorname{cl} C(\hat{p}))) = \emptyset.$$

Since $\Omega_1 \times Z$ is normal space and, by condition (iii), $\operatorname{Gr}(W)$ is a closed subset of $\Omega_1 \times Z$, there exist a neighborhood $\mathcal{V}_{\hat{p}}$ of $\hat{p}$ and a neighborhood $\mathcal{V}$ of $\hat{z} - \operatorname{cl} C(\hat{p})$ such that

$$\operatorname{Gr}(W) \cap (\mathcal{V}_{\hat{p}} \times \mathcal{V}) = \emptyset,$$

and so $\operatorname{Gr}(W) \cap (\mathcal{V}_{\hat{p}} \times (\hat{z} - \operatorname{int} C(\hat{p}))) = \emptyset$. Since $\hat{z} - \operatorname{int} C(\hat{p})$ is a neighborhood of $g(\hat{p}, \hat{x}, \hat{y})$, by condition (i), we can choose $\mathcal{U}_{\hat{p}}(\subset \mathcal{V}_{\hat{p}})$, $\mathcal{U}_{\hat{x}}$, and $\mathcal{U}_{\hat{y}}$ such that for all $(p, x, y) \in \mathcal{U}_{\hat{p}} \times \mathcal{U}_{\hat{x}} \times \mathcal{U}_{\hat{y}}$,

$$g(p, x, y) \in (\hat{z} - \operatorname{int} C(\hat{p})) - \operatorname{int} C(\hat{p}) = \hat{z} - \operatorname{int} C(\hat{p}),$$

where $\mathcal{U}_{\hat{p}}, \mathcal{U}_{\hat{x}}$, and $\mathcal{U}_{\hat{y}}$ stand for neighborhoods of $\hat{p}, \hat{x}$ and $\hat{y}$, respectively.

Next by condition (ii) noting $\Omega(\hat{p}) \cap \mathcal{U}_{\hat{y}} \neq \emptyset$, we can choose a neighborhood $\mathcal{U}'_{\hat{p}}$ of $\hat{p}$ such that

$$\Omega(p) \cap \mathcal{U}_{\hat{y}} \neq \emptyset \text{ for all } p \in \mathcal{U}'_{\hat{p}}.$$

Let $\mathcal{U} = (\mathcal{U}_{\hat{p}} \cap \mathcal{U}'_{\hat{p}}) \times \mathcal{U}_{\hat{x}}$ which is a neighborhood of $(\hat{p}, \hat{x})$. Then for each $(p', x') \in \mathcal{U} \cap \operatorname{Gr}(\Omega)$, since $p' \in U'_{\hat{p}}$, $\Omega(p') \cap \mathcal{U}_{\hat{y}} \neq \emptyset$, there exists $y' \in \Omega(p') \cap \mathcal{U}_{\hat{y}}$. Therefore for the (p', x', y'), we have

$$g(p', x', y') \in \hat{z} - \operatorname{int} C(\hat{p}),$$

and hence

$$(p', g(p', x', y')) \in \mathcal{V}_{\hat{p}} \times \mathcal{V}.$$

Consequently, $(p', g(p', x', y')) \notin \operatorname{Gr}(W)$ and hence

$$g(p', x', y') \in -\operatorname{int} C(p'). \qquad \square$$

Definition 6 (KKM-map)**.** Let X be a Hausdorff topological vector space, and K a nonempty subset of X. Suppose that F is a multifunction from K to 2^X. Then F is said to be a *KKM-map*, if

$$\operatorname{co}\{x_1, \ldots, x_n\} \subset \bigcup_{i=1}^{n} F(x_i)$$

for each finite subset $\{x_1, \ldots, x_n\}$ of X where co$A$ denotes the convex hull of the set A.

Remark 3. Obviously, if F is a KKM-map, then $x \in F(x)$ for each $x \in X$.

Lemma 1 (Fan-KKM; see [3])**.** *Let X be a Hausdorff topological vector space, K a nonempty subset of X, and F be a multifunction from K to 2^X. Suppose that F is a KKM-map and that $F(x)$ is a closed subset of X for each $x \in K$. If $GF(\hat{x})$ is compact for at least one $\hat{x} \in K$, then $\bigcap_{x \in K} F(x) \neq \emptyset$.*

3. Existence results for parametric vector equilibrium problem

Theorem 1. *Let Ω_1 and Ω_2 be two nonempty subsets of Hausdorff topological vector spaces X and Y, respectively. Let Z be also a Hausdorff topological vector space and C a set-valued mapping from Ω_1 to 2^Z such that $C(x)$ is a pointed convex cone with nonempty interior for each $x \in \Omega_1$. Suppose that Ω is a set-valued mapping from Ω_1 to $2^{\Omega_2} \setminus \{\emptyset\}$ and that g is a vector-valued function from $\Omega_1 \times \Omega_2 \times \Omega_2$ to Z. Also we assume the following conditions:*

(i) *Ω has closed convex values for each $x \in \Omega_1$;*
(ii) *$g(x, \cdot, v)$ is $-C(x)$-continuous on $\Omega(x)$ for each $x \in \Omega_1, v \in \Omega(x)$;*
(iii) *$g(x, y, \cdot)$ is $C(x)$-quasiconvex on $\Omega(x)$ for each $x \in \Omega_1, y \in \Omega(x)$;*
(iv) *$g(x, y, y) \notin -\operatorname{int} C(x)$ for each $x \in \Omega_1, y \in \Omega(x)$.*
(v) *for each $x \in \Omega_1$ there exist $\hat{v} \in \Omega(x)$ and a compact set $\mathcal{B} \subset Y$ such that $\hat{v} \in \mathcal{B}$ and*

$$g(x, y, \hat{v}) \in -\operatorname{int} C(x) \text{ for all } y \in \Omega(x) \setminus \mathcal{B}.$$

Then the problem PVEP has at least one solution for each $x \in \Omega_1$.

Proof. Let

$$G(v) := \{y \in \Omega(x) : g(x, y, v) \notin -\operatorname{int} C(x)\} \ v \in G(v),$$

for each $x \in \Omega_1$. First, we show that $G(v)$ is a KKM-map, for each $x \in \Omega_1$. Suppose to the contrary that there exists $\alpha_i \in [0, 1]$, $y_i \in \Omega(x)$ $(i = 1, \ldots, n)$ such that $\sum_{i=1}^n \alpha_i = 1$ and

$$\sum_{i=1}^n \alpha_i y_i = y \notin \bigcup_{i=1}^n G(y_i).$$

Then we have $y \in \Omega(x)$ because, by condition (i), $\Omega(x)$ is convex. Hence

$$f(x, y, y_i) \in -\operatorname{int} C(x), \ i = 1, \ldots, n.$$

This means that

$$f(x, y, \sum_{i=1}^n \alpha_i y_i) = f(x, y, y) \in -\operatorname{int} C(x),$$

because of condition (iii), and contradicts condition (iv).

Next, from conditions (i) and (ii), for each $v \in \Omega(x)$, $G(v)$ is a closed set, and by condition (iv), $G(v) \neq \emptyset$, and also from condition (v), $G(\hat{v})$ is a compact set. Thus we can apply Lemma 1, to get

$$S_E(x) = \bigcap_{v \in \Omega(x)} G(v) \neq \emptyset,$$

for each $x \in \Omega_1$, where S_E denotes the solution mapping defined by (1). □

Remark 4. We can replace condition (ii) by the following condition: For each $x \in \Omega_1, y \in \Omega(x), v \in \Omega(x)$ satisfying

$$g(x, y, v) \in -\operatorname{int} C(x),$$

there exists a neighborhood $\mathcal{U}_y$ of y such that for all $y' \in \mathcal{U}_y$

$$g(x, y', v') \in -\operatorname{int} C(x) \text{ for some } v' \in \Omega(x).$$

Theorem 2. *Let Ω_1, Ω_2, C, Ω and g be the same as those in Theorem 1. Let S_E be a set-valued mapping from Ω_1 to 2^{Ω_2} defined by (1). Also we assume the following conditions:*

(i) *Ω_1 is a closed set;*
(ii) *Ω has closed graph;*
(iii) *g is parametarized $(-C)$-continuous on $\Omega_1 \times \Omega_2$ with respect to Ω;*
(iv) *$S_E(x) \neq \emptyset$ for each $x \in \Omega_1$.*

Then the solution set $S_E(x)$ of problem PVEP has closed graph.

Proof. Let $(x_\alpha, y_\alpha) \in \mathrm{Gr}(S_E)$ with $(x_\alpha, y_\alpha) \to (x, y)$. Then by conditions (i) and (ii), $x \in \Omega_1$ and $y \in \Omega(x)$. Suppose on the contrary that $y \notin S_E(x)$. Then there exists $v \in \Omega(x)$ such that

$$g(x, y, v) \in -\operatorname{int} C(x).$$

Because of condition (iii), there is a neighborhood $\mathcal{U}$ of (x, y) with $y \in \Omega(x)$ such that for all $(\tilde{x}, \tilde{y}) \in \mathcal{U}$, there is $\tilde{v} \in \Omega(\tilde{x})$ such that $g(\tilde{x}, \tilde{y}, \tilde{v}) \in -\operatorname{int} C(\tilde{x})$. That is, there exists $\bar{\alpha}$ such that for all $\alpha \geq \bar{\alpha}$, $y_\alpha \notin S_E(x_\alpha)$. This is a contradiction. □

Theorem 3. *Let Ω_1, Ω_2, C, Ω, g and S_E be the same as those in Theorem 2. Also we assume the following conditions:*

(i) *Ω_1 is a closed set;*
(ii) *Ω has closed graph;*
(iii) *g is parametarized $(-C)$-continuous on $\Omega_1 \times \Omega_2$ with respect to Ω;*
(iv) *$g(x, y, \cdot)$ is $C(x)$-quasiconvex on $\Omega(x)$ for each $x \in \Omega_1$ and $y \in \Omega(x)$, and $g(x, y, y) \notin -\operatorname{int} C(x)$ for each $x \in \Omega_1$ and $y \in \Omega_2$;*
(v) *for each $x \in \Omega_1$ there exist $\hat{v} \in \Omega(x)$ and a compact set $\mathcal{B} \subset Y$ such that $\hat{v} \in \mathcal{B}$ and*

$$g(x, y, \hat{v}) \in -\operatorname{int} C(x) \text{ for all } y \in \Omega(x) \setminus \mathcal{B}.$$

Then the problem PVEP has at least one solution, and S_E has closed graph.

Proof. By condition (iii), g satisfies the condition of Remark 4. Then the result follows from Theorems 1 and 2. □

As an application of closedness result of solutions map for PVEP, we investigate the existence of solution for a MPEC. Consider the following MPEC:

$$\text{(MPEC)} \qquad \min\{f(x, y) : y \in S_E(x)\},$$

where $f : \Omega_1 \times \Omega_2 \to (-\infty, \infty)$ and $S_E : \Omega_1 \to 2^{\Omega_2}$ is a set-valued mapping such that for each $x \in \Omega_1$, $S_E(x)$ is the solution set of the following PVEP, consisting in finding $y \in \Omega$ such that

$$g(x, y, v) \notin -\operatorname{int} C(x) \text{ for all } v \in \Omega(x),$$

where g is a vector-valued function from $\Omega_1 \times \Omega_2 \times \Omega_2$ to Z, $C(x) \subset Z$ is a domination structure generated bu set-valued mapping $C : \Omega_1 \to 2^Z$ at $x \in \Omega_1$, and $\Omega : \Omega_1 \to 2^{\Omega_2} \setminus \{\emptyset\}$ stands for a constraint map. We have the following existence of MPEC.

Theorem 4. *Let Ω_1, Ω_2, C, Ω and g be the same as those in Theorem 1. Let S be a set-valued mapping from Ω_1 to 2^{Ω_2} defined by (1). Also we assume the following conditions:*

(i) *f is lower semicontinuous on* $\mathrm{Gr}(S_E)$*;*
(ii) Ω_1 *and* Ω_2 *are two compact sets;*
(iii) Ω *is has closed graph;*
(iv) g *is parametarized* $(-C)$*-continuous on* $\Omega_1 \times \Omega_2$ *with respect to* Ω*;*
(v) $g(x, y, \cdot)$ *is* $C(x)$*-quasiconvex on* $\Omega(x)$ *for each* $x \in \Omega_1$ *and* $y \in \Omega(x)$*, and* $g(x, y, y) \notin -\operatorname{int} C(x)$ *for each* $x \in \Omega_1$ *and* $y \in \Omega_2$*.*

Then the MPEC has at least one solution.

Proof. By Theorems 3, we have $S_E(x) \neq \emptyset$ and $\mathrm{Gr}(S_E)$ is closed. Moreover by condition (ii), $\mathrm{Gr}(S_E)$ is compact. Hence by condition (i), the MPEC has at least one solution. □

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References

[1] C. Berge, *Topological Space*, (1963) Oliver&Boyd, Edinburgh and London.
[2] F. Ferro, *A Minimax Theorem for Vector-Valued Functions*, J. Optim. Theory Appl. **60**(1), (1989), 19–31.
[3] K. Fan, *A Generalization of Tychonoff's Fixed Point Theorem*, Math. Ann. **142**, (1961), 305–310.
[4] S. Helbig, *On the connectedness of the set of weakly efficient points of a vector optimization problem in locally convex spaces*, J. Optim. Theory Appl. **65**(2), (1990), 257–270.
[5] Y. C. Liou, X. Q. Yang and J. C. Yao, *Mathematical programs with vector optimization constraints*, J. Optim. Theory Appl. **126**(2), (2005), 345–355.
[6] D. T. Luc, *Connectedness of the efficient point sets in quasiconcave vector maximization*, J. Optim. Theory Appl. **122**, (1987), 346–354.
[7] T. Tanaka, *Cone-quasiconvexity of vector-valued functions*, Sci.Rep. Hirosaki Univ. **42** (1995), 157–163.
[8] T. Tanaka, *Generalized semicontinuity and existence theorems for cone saddle points*, Appl. Math. Optim. **36** (1997), 313–322.
[9] P. L. Yu, *Cone convexity, cone extreme points, and nondominated solutions in decision problems with multiobjectives*, J. Optim. Theory Appl. **14**(3), (1974), 319–377.

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