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FIXED POINTS OF MULTIMAPS IN THE BETTER ADMISSIBLE CLASS

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Abstract. Since the author established fixed point theorems for the admissible class $\mathfrak{A}_c^\kappa$ of multimaps, there have appeared another classes: the 'better' admissible class $\mathfrak{B}$, the KKM-class, the S-KKM class, and others. In this paper, we unify fixed point theorems for such classes of compact closed multimaps on convex subsets of topological vector spaces, and obtain a fixed point theorem for a new class $\mathfrak{B}^p$.

1. Introduction

Recently, there have appeared a number of fixed point theorems for new classes of multimaps defined on convex subsets of topological vector spaces. Apparently motivated by the author's previous works, some other authors obtained a number of new classes of multimaps and seemingly new fixed point theorems. In this paper, we unify fixed point theorems for such classes of compact closed multimaps defined on convex subsets of topological vector spaces, and obtain a fixed point theorem for a new class $\mathfrak{B}^p$.

Section 2 deals with the almost fixed point property of a multimap $T : X \multimap X$, where X is a nonempty convex subset of a topological vector space, and the so-called s-KKM class of multimaps having the almost fixed point property. We obtain a generalization (Theorem 2.6) of results in [4,5,8].

In Section 3, we are mainly concerned with fixed point theorems for upper semicontinuous maps in the admissible class $\mathfrak{A}_c^\kappa$ and the 'better' admissible class $\mathfrak{B}$ due to the author for topological vector spaces more general than locally convex ones. Recently, some authors dealt with fixed point theorems for seemingly new and broad classes of multimaps. We clarify that some of them are consequences of the author's earlier result.

Finally, in Section 4, we obtain a general fixed point theorem for a new 'better' admissible class $\mathfrak{B}^p$ of multimaps, which encompasses almost all of other known results in this paper.

2. The almost fixed point property

A t.v.s. means a Hausdorff topological vector space and $\mathcal{V}$ denotes a fundamental system of open neighborhoods of the origin 0 of a t.v.s. A *multimap* (simply, a *map*) $T : X \multimap Y$ is a function from X into $2^Y \setminus \{\emptyset\}$. All terminology is standard. The closure operation is denoted by cl or $\overline{\quad}$.

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For topological spaces X and Y, a multimap $T : X \multimap Y$ is said to be *upper semicontinuous* (u.s.c.) if for any open subset $A \subset Y$, the set $T^+(A) :=\{x \in X \mid T(x) \subset A\}$ is open in X. T is said to be *closed* if it has the closed graph $\mathrm{Gr}(T) \subset X \times Y$, and *compact* if its range $T(X)$ is contained in a compact subset of Y. Recall that a compact closed multimap is u.s.c. and compact-valued; and an u.s.c. multimap with closed values is closed.

Let X be a convex subset of a t.v.s. E. A multimap $T : X \multimap X$ is said to have the (*convexly*) *almost fixed point property* if, for every (convex) neighborhood U of the origin 0 of E, there exists a point $x_U \in X$ such that $x_U \in T(x_U) + U$ or $T(x_U) \cap (x_U + U) \neq \emptyset$.

The following is well-known:

Theorem 2.1. *Let X be a subset of a t.v.s. E. Then any compact closed multimap $T : X \multimap X$ having the almost fixed property has a fixed point.*

We have the following fixed point theorem:

Theorem 2.2. *Let X be a compact subset of a t.v.s. E and $T : X \multimap E$ a multimap such that*

(i) *T has the almost fixed point property;*
(ii) *T has closed values; and*
(iii) *T satisfies the following equality:*

$$\bigcap_{U\in\mathcal{V}} \{x \in X \mid x \in T(x) + U\} = \bigcap_{U\in\mathcal{V}} \mathrm{cl}\{x \in X \mid x \in T(x) + U\}.$$

Then T has a fixed point.

Proof. For any $U \in \mathcal{V}$, let

$$F_U := \{x \in X \mid x \in T(x) + U\}.$$

By (i), there is an $x_U \in X$ such that $x_U \in T(x_U)+U$. Then $F_U \neq \emptyset$ for each $U \in \mathcal{V}$. It is clear that $\{F_U \mid U \in \mathcal{V}\}$ has the finite intersection property. Since each $\mathrm{cl}\, F_U$ is a closed subset of the compact space X, we have $\bigcap_{U\in\mathcal{V}} \mathrm{cl}\, F_U \neq \emptyset$. Therefore, by (iii), there exists an $\widehat{x} \in X$ such that

$$\widehat{x} \in \bigcap_{U\in\mathcal{V}} F_U = \bigcap_{U\in\mathcal{V}} \mathrm{cl}\, F_U \neq \emptyset,$$

and hence, we have

$$\widehat{x} \in \bigcap_{U\in\mathcal{V}} (T(\widehat{x}) + U) = \mathrm{cl}\, T(\widehat{x}) = T(\widehat{x})$$

by (ii). This completes our proof. □

Recall that multimaps satisfying a condition similar to (iii) were studied in [8,9,19].

Let $\langle X \rangle$ be the set of all nonempty finite subsets of a set X.

Let X be a convex subset of a vector space and Y a topological space. Motivated by an earlier work [13] of the author, in 1996, Chang and Yen [5] defined

$T \in \mathrm{KKM}(X, Y) \iff T : X \multimap Y$ is a multimap such that the family $\{S(x) \mid x \in X\}$ has the finite intersection property whenever $S : X \multimap Y$ has closed values and $T(\mathrm{co}\, N) \subset S(N)$ for each $N \in \langle X \rangle$.

Moreover, Chang and Yen [6] introduced the class of S-KKM maps and gave a characterization of such maps and an s-KKM theorem. This was extended to an S-KKM theorem by Lin and Chang [10] with additional results. This is followed by Chang, Huang, Jeng, and Kuo [4], Agarwal and O'Regan [1,2], and Shahzad [21].

Let X be a nonempty set, Y a nonempty convex subset of a vector space and Z a topological space. If $S : X \multimap Y$, $T : Y \multimap Z$ and $F : X \multimap Z$ are three multimaps satisfying

$$T(\mathrm{co}\, S(A)) \subset F(A)$$

for any $A \in \langle X \rangle$, then F is called a generalized S-KKM map with respect to T. If the multimap $T : Y \multimap Z$ satisfies that for any generalized S-KKM map F with respect to T the family $\{\overline{F(x)} \mid x \in X\}$ has the finite intersection property, then T is said to have the S-KKM property. The class S-KKM(X, Y, Z) is defined to be the set $\{T : Y \multimap Z \mid T$ has the S-KKM property$\}$; see [6,10].

As shown in [4], when $X = Y$ and S is the identity map 1_X, then S-KKM(X, Y, Z) reduces to the class KKM(X, Z) introduced by Chang and Yen in [5], and moreover, KKM(Y, Z) is contained in S-KKM(X, Y, Z) for any $S : X \multimap Y$ and generally this inclusion is proper.

If S is a single-valued function $s : X \to Y$, then we can consider the class s-KKM(X, Y, Z).

Lemma 2.3. *Let I be a nonempty set, X a convex subset of a t.v.s. E (not necessarily Hausdorff), $s : I \to X$, $T \in s$-KKM (I, X, X) a multimap, and U a convex open subset of E. If*

$$\overline{T(X)} \subset \bigcup_{z \in A} (s(z) + U) \quad \text{for some} \quad A \in \langle I \rangle, \tag{1}$$

then there exists an $x_U \in X$ such that $T(x_U) \cap (x_U + U) \neq \emptyset$.

Proof. Let $F : I \multimap X$ be a map defined by

$$F(z) := \overline{T(X)} \backslash (s(z) + U) \quad \text{for } z \in I.$$

Since $\overline{T(X)} \subset \bigcup_{z \in A} (s(z) + U)$,

$$\bigcap_{z \in A} F(z) = \overline{T(X)} \backslash \bigcup_{z \in A} (s(z) + U) \subset \overline{T(X)} \backslash \overline{T(X)} = \emptyset,$$

and hence $\{F(z)\}_{z \in I}$ does not have the finite intersection property. Since F has closed values and $T \in s$-KKM(I, X, X), there exists a $B \in \langle I \rangle$ such that

$$T(\mathrm{co}\, s(B)) \not\subset F(B).$$

Hence there exists $y_0 \in T(\mathrm{co}\, s(B)) \subset T(X)$ such that

$$y_0 \notin F(z) = \overline{T(X)} \backslash (s(z) + U) \quad \text{for all } z \in B.$$

Therefore, $y_0 \in s(z) + U$ or $s(z) \in y_0 - U$ for all $z \in B$. Since $s(B) \subset y_0 - U$ and $y_0 - U$ is convex, we have $\mathrm{co}\, s(B) \subset y_0 - U$. On the other hand, $y_0 \in T(\mathrm{co}\, s(B))$

implies $y_0 \in T(x_U)$ for some $x_U \in \operatorname{co} s(B) \subset y_0 - U$. Hence, $y_0 \in x_U + U$ and $T(x_U) \cap (x_U + U) \neq \emptyset$. This completes our proof. □

Corollary 2.4. *Let I, X, E, s, and T be the same as in Lemma 2.3. If $\overline{T(X)}$ is totally bounded and $T(X) \cap s(I)$ is dense in $T(X)$, then T has the convexly almost fixed point property.*

Proof. For any convex open neighborhood U of 0 in E, if $T(X) \cap s(I)$ is dense in $T(X)$ and $\overline{T(X)}$ is totally bounded, then condition (1) holds. □

Note that if $X = I$ and $\overline{T(X)} \subset s(I)$, then Corollary 2.4 reduces to Chang et al. [4, Lemma 3.1].

From Corollary 2.4 and Theorem 2.2, we immediately have the following:

Theorem 2.5. *Let X be a compact convex subset of a t.v.s. E, I a nonempty set, $s : I \to X$, and $T \in s$-KKM(I, X, X) a closed-valued map such that $T(X) \cap s(I)$ is dense in $T(X)$. If T satisfies condition*

$$\bigcap_{U \in \mathcal{V}} \{x \in X \mid x \in T(x) + U\} = \bigcap_{U \in \mathcal{V}} \operatorname{cl}\{x \in X \mid x \in T(x) + \operatorname{co} U\},$$

then T has a fixed point.

Note that if $X = I$ and $\overline{T(X)} \subset s(I)$, then Theorem 2.5 reduces to Huang and Jeng [8, Theorem 2.2], and if $X = I$ and $s = 1_X$, then Theorem 2.5 originates from Park [19, Theorem 1]. If $T : X \multimap X$ is u.s.c. with convex values, then $T \in$ KKM$(X, X) \subset s$-KKM(X, X, X); see Huang and Jeng [8]. Therefore, Theorem 2.5 also generalizes Huang and Jeng [8, Corollary 2.6].

From Theorem 2.1 and Corollary 2.4, we immediately have the following:

Theorem 2.6. *Let X be a convex subset of a locally convex t.v.s., I a nonempty set, $s : I \to X$, and $T \in s$-KKM(I, X, X) a compact closed map such that $T(X) \cap s(I)$ is dense in $T(X)$. Then T has a fixed point.*

If $X = I$ and $\overline{T(X)} \subset s(I)$, then Theorem 2.6 reduces to Chang et al. [4, Theorem 3.2] and further if $s = 1_X$, then to Chang and Yen [5, Theorem 2], and if X itself is compact, then to Huang and Jeng [8, Corollary 2.4].

3. Fixed points for new classes of maps

Let us recall some history of analytical fixed point theory; for details, see [18].

Since 1992, we introduced and supplied a lot of examples of the class $\mathfrak{A}_c^\kappa$ in [12,13,20]. Up to now, many authors used the class $\mathfrak{A}_c^\kappa$, but no one could find any new example of maps in that class, and some authors mistook $\mathfrak{A}$ for $\mathcal{U}$.

In 1996, Chang and Yen [5] introduced the KKM class (see Section 2) and obtained the following:

Theorem 3.1 ([5])**.** *Let X be a convex subset of a locally convex t.v.s. E. Then every compact closed map $T \in$ KKM(X, X) has a fixed point.*

It was known that $\mathfrak{A}_c^\kappa(X, X) \subset$ KKM(X, X) in [13], but, any significant example of compact closed maps $T \in$ KKM(X, X) such that $T \notin \mathfrak{A}_c^\kappa(X, X)$ is not yet found.

Note that Theorem 3.1 is a consequence of Theorem 2.1. In fact, the map $T \in \mathrm{KKM}(X, X)$ in Theorem 3.1 has the convexly almost fixed point property by Corollary 2.4, but not conversely.

A *polytope* P in a subset X of a t.v.s. E is a nonempty compact convex subset of X contained in a finite dimensional subspace of E.

In 1996, the author introduced the 'better' admissible class $\mathfrak{B}$ of multimaps as follows:

$F \in \mathfrak{B}(X, Y) \iff F : X \multimap Y$ is a map such that, for each polytope P in X and for any continuous function $f : F(P) \to P$, the composition $f(F|_P) : P \multimap P$ has a fixed point,

where Y is a topological space. We noticed that $\mathfrak{A}_c^\kappa \subset \mathfrak{B}$ and that, in the class of compact closed maps, two subclasses $\mathfrak{B}$ and KKM coincide. Recall that some variants of Theorem 3.1 were given in [15]. One of the simplest results is the following restatement of Theorem 3.1:

Theorem 3.2 ([15])**.** *Let X be a convex subset of a locally convex t.v.s. E. Then every compact closed map $F \in \mathfrak{B}(X, X)$ has a fixed point.*

Subclasses of $\mathfrak{B}$ are classes of continuous functions $\mathbb{C}$, the Kakutani maps $\mathbb{K}$ (u.s.c. with nonempty compact convex values and codomains are convex sets), the Aronszajn maps $\mathbb{M}$ (u.s.c. with R_δ values), the acyclic map $\mathbb{V}$ (u.s.c. with compact acyclic values), $\mathbb{K}_c$ (finite compositions of Kakutani maps), the Powers maps $\mathbb{V}_c$ (finite compositions of acyclic maps), the O'Neill maps $\mathbb{N}$ (continuous with value of one or m acyclic components, where m is fixed), the Fan-Browder maps (codomains are convex sets, nonempty convex values and open fibers), the approachable maps $\mathbb{A}$, admissible maps of Górniewicz, σ-selectionable maps of Haddad and Lasry, permissible maps of Dzedzej, the class $\mathbb{K}_c^+$ of Lassonde, the class $\mathbb{V}_c^+$ of Park et al., approximable maps of Ben-El-Mechaiekh and Idzik, and others. Those subclasses are examples of the admissible class $\mathfrak{A}_c^\kappa$ due to the author. Some examples of maps in $\mathfrak{B}$ not belonging to $\mathfrak{A}_c^\kappa$ were known. For details, see [12-18].

Recall that a nonempty subset X of a t.v.s. E is said to be *admissible* (in the sense of Klee) provided that, for every nonempty compact subset K of X and every neighborhood V of the origin 0 of E, there exists a continuous function $h : K \to X$ such that $x - h(x) \in V$ for all $x \in K$ and $h(K)$ is contained in a finite dimensional subspace L of E. Examples of admissible subsets can be seen in [17,18].

In 1997, we obtained the following:

Theorem 3.3 ([17,18])**.** *Let E be a t.v.s. and X an admissible* (*in the sense of Klee*) *convex subset of E. Then any compact closed map $F \in \mathfrak{B}(X, X)$ has a fixed point.*

In 1998, this was restated as follows:

Theorem 3.4 ([19])**.** *Let X be an admissible convex subset of a t.v.s. Then any compact closed map $T \in \mathfrak{K}(X, X)$ has a fixed point, where $\mathfrak{K}$ denotes* KKM*.*

Moreover, in [18], we listed more than sixty papers in chronological order, from which we could deduce particular forms of Theorem 3.3. Further examples of maps in the class $\mathfrak{B}$ were given in [16]. In 1999, Lin and Yu [11] published Theorem 3.4 and generalized versions of the present author's other results.

Since then generalizations of KKM class to S-KKM or s-KKM classes follow. One of the main targets of such works was to try to generalize Theorems 3.1 and 3.4. However, the main results in [4,5,8] are generalized and unified by Theorem 2.6 in this paper.

For the s-KKM class, we need the following:

Lemma 3.5. *Let X be a convex subset of a t.v.s., I a nonempty set, $s : I \to X$ a surjection, and $T \in s$-KKM(I, X, X). If T is closed and compact, then $T \in \mathfrak{B}(X, X)$.*

Proof. Let P be a polytope in X, $T' := T|_P$, and $f : T(P) \to P$ a continuous function. Let $I' := s^{-1}(P) \subset I$ and $s' := s|_{I'} : I' \to P$.

Claim 1. $T \in s'$-KKM(I', X, X).

Let $F' : I' \multimap X$ be closed-valued such that

$$T(\operatorname{co} s'(A)) \subset F'(A) \quad \text{for all } A \in \langle I' \rangle. \tag{2}$$

Define $F : I \multimap X$ by $F(z) := F'(z)$ for $z \in I'$ and $F(z) := X$ for $z \in I \backslash I'$. Then F is closed-valued with respect to the relative topology of X. Now, from (2), we have

$$T(\operatorname{co} s(A)) \subset F(A) \quad \text{for all } A \in \langle I \rangle.$$

Since $T \in s$-KKM(I, X, X), $\{F(z)\}_{z \in I}$ has the finite intersection property, and hence, so does $\{F'(z)\}_{z \in I'}$.

Claim 2. $f \circ T' \in s'$-KKM(I', P, P).

Let $F' : I' \multimap P$ be closed-valued such that

$$(f \circ T')(\operatorname{co} s'(A)) \subset F'(A) \quad \text{for all } A \in \langle I' \rangle.$$

Then we have

$$T(\operatorname{co} s'(A)) = T'(\operatorname{co} s'(A)) \subset (f^{-1} \circ F')(A) \quad \text{for all } A \in \langle I' \rangle.$$

Since $f^{-1} \circ F'$ is closed-valued and $T \in s'$-KKM(I', X, X), $\{(f^{-1} \circ F')(z)\}_{z \in I'}$ has the finite intersection property, and hence, so does $\{F'(z)\}_{z \in I'}$.

Claim 3. $f \circ T' : P \multimap P$ has a fixed point.

Since T is closed and compact, it is u.s.c. with closed values, and hence, so is T'. Moreover, $f \circ T'$ is u.s.c. and closed-valued, and hence, it is closed and compact. Since $f \circ T' \in s'$-KKM(I', P, P) by Claim 2 and $(f \circ T')(P) \subset P = s(I')$, by Theorem 2.6, $f \circ T'$ has a fixed point.

Therefore $f \circ (T|_P)$ has a fixed point and hence $T \in \mathfrak{B}(X, X)$. This completes our proof. □

Remark. If $s : I \to X$ is not a surjection, then Lemma 3.5 may not hold. For example, let $X = [0, 1]$ and $s : X \to X$ be defined by $s(x) := x/2$ for $x \in X$. Let $T : X \multimap X$ be defined by $T(x) := \{1\}$ if $x \in [0, 1/2)$, $T(x) := \{0, 1\}$ if $x = 1/2$, and $T(x) := \{0\}$ if $x \in (1/2, 1]$. Then $T \in s$-KKM(X, X, X) by noting $1 \in T(s(x))$ for any $x \in X$; see [4, p.219]. Note that T is closed and compact, but $T \notin \mathfrak{B}(X, X)$.

From Lemma 3.5 and Theorem 3.3, we have the following:

Theorem 3.6. *Let E be a t.v.s. and X an admissible (in the sense of Klee) convex subset of E, I a nonempty set, $s : I \to X$ a surjection, and $T \in s$-KKM(I, X, X). If T is closed and compact, then T has a fixed point.*

Note that if I is a nonempty subset of X, then Theorem 3.6 reduces to Chang, Huang, and Jeng [3, Theorem 3.1].

It should be noticed that the main fixed point theorems in [3,5,11] and others are disguised forms of our Theorem 3.3. Most of other results in those papers are also formally generalized (but not practical) or disguised forms of earlier works of the author on the classes $\mathfrak{A}_c^\kappa$ or $\mathfrak{B}$ of multimaps.

4. A new general fixed point theorem

In this section, we obtain a generalized version of our previous result [18] by switching the admissibility of domain of the multimap to the Klee approximability (defined below) of codomain.

Let X be a subset of a t.v.s. E. A compact subset K of X is said to be *Klee approximable in* X if for any $V \in \mathcal{V}$, there exists a continuous function $h : K \to X$ such that $x - h(x) \in V$ for all $x \in K$ and $h(K)$ is contained in a polytope in X.

We give some examples of Klee approximable sets as follows:

(1) A subset X of E is admissible (in the sense of Klee) if and only if every compact subset K of X is Klee approximable in X.
(2) Any polytope in a subset of a t.v.s.
(3) Any compact subset K of a convex subset X in a locally convex t.v.s.
(4) Any compact subset K whenever $\operatorname{co} K$ is an admissible subset of X.
(5) Any compact convex subset of a metrizable t.v.s. is admissible.

Remark. For a compact subset K of E, it is well-known that $\operatorname{co} K$ is σ-compact and paracompact. Therefore, if E is metrizable, then the following is supplementary to (4) and (5):

Proposition (Dobrowolski [7]). *For a σ-compact convex set C in a metrizable t.v.s. the following conditions are equivalent:*

(a) $C \in$ AR,
(b) $C = \bigcup_1^\infty A_n$ *with* $A_n = \overline{A}_n \in$ AE(compact) *for* $n = 1, 2, \dots,$
(c) C *is admissible.*

Here AR denotes an absolute retract and AE(compact) the absolute extension property for compacta.

For a topological space Y, we define a new class of multimaps as follows:

$F \in \mathfrak{B}^p(X, Y) \iff F : X \multimap Y$ is a multimap such that for any polytope P of X, there is a map $\Gamma \in \mathfrak{B}(P, Y)$ such that $\Gamma(x) \subset F(x)$ for each $x \in P$.

Note that any map $F \in \mathfrak{B}(X, Y)$ belongs to $\mathfrak{B}^p(X, Y)$. Recently it is known that, for the subclass of $\mathfrak{B}$ consisting of approximable multimaps, any u.s.c. map with nonempty compact contractible values or nonempty compact values having trivial shape belongs to $\mathfrak{B}^p(X, Y)$.

Now, [18, Theorem 1] can be generalized as follows:

Theorem 4.1. *Let X be a subset of a t.v.s. E and $F \in \mathfrak{B}^p(X, X)$ a compact closed multimap. If $\overline{F(X)}$ is a Klee approximable subset of X, then F has a fixed point.*

Proof. Let $V \in \mathcal{V}$. Since $K := \overline{F(X)}$ is compact and Klee approximable in X, there exist a continuous function $h : \overline{F(X)} \to X$ and a polytope P in X such that $x - h(x) \in V$ for all $x \in \overline{F(X)}$ and $h(\overline{F(X)}) \subset P$. Note that $h : \overline{F(X)} \to P$ and $F|_P : P \multimap \overline{F(X)}$. Since $F \in \mathfrak{B}^p(X, X)$, there is a map $\Gamma \in \mathfrak{B}(P, X)$ such that $\Gamma(x) \subset F(x)$ for each $x \in P$. Since $\Gamma(P) \subset F(P) \subset \overline{F(X)}$ and $h : \overline{F(X)} \to P$, the composition $h\Gamma : P \multimap P$ has a fixed point $x_V \in h\Gamma(x_V)$. Let $x_V = h(y_V)$ for some $y_V \in \Gamma(x_V) \subset F(x_V) \subset \overline{F(X)}$. We have $y_V - h(y_V) = y_V - x_V \in V$. Since $\overline{F(X)}$ is compact, we may assume that the net y_V converges to some $\hat{x} \in \overline{F(X)}$. Then the corresponding net x_V in X also converges to $\hat{x}$. Since the graph of F is closed and $(x_V, y_V) \in \mathrm{Gr}(F)$, we have $\hat{x} \in F(\hat{x})$. This completes our proof. □

Corollary 4.2. *Let X be a subset of a t.v.s. E and $F \in \mathfrak{B}(X, X)$ a compact closed map. If $\overline{F(X)}$ is a Klee approximable subset of X, then F has a fixed point.*

We note that Corollary 4.2 generalizes and unifies Theorems 2.6, 3.1–3.4, and 3.6.

Corollary 4.3. *Let X be a convex subset of a t.v.s. E and $F \in \mathfrak{B}^p(X, X)$ a closed map. If $F(X)$ is contained in a finite dimensional compact subset of X, then F has a fixed point.*

Proof. Note that $\overline{F(X)}$ is contained in a polytope in X and hence Klee approximable. □

Recall that [18, Theorem 1] was incorrectly stated and it should be replaced by our new Theorem 4.1.

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