Journal of Nonlinear and Convex Analysis
Volume 15, Number 1, 2014, 191–198

Yokohama Publishers
ISSN 1880-5221 ONLINE JOURNAL

GROWTH ESTIMATES FOR PSEUDO-DISSIPATIVE HOLOMORPHIC MAPS IN BANACH SPACES

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Dedicated to Prof. Simeon Reich for his 65th anniversary

Abstract. In this paper we introduce a class of pseudo-dissipative holomorphic maps which contains, in particular, the class of infinitesimal generators of semigroups of holomorphic maps on the unit ball of a complex Banach space. We give a growth estimate for maps of this class. In particular, it follows that pseudo-dissipative maps on the unit ball of (infinite-dimensional) Banach spaces are bounded on each domain strictly contained inside the ball. We also present some applications.

1. Introduction

Let $(X, \|\cdot\|)$ be a complex Banach space and let $\mathbb{B} := \{z \in X : \|z\| < 1\}$.

Definition 1.1 (see [9, 11])**.** Let $h : \mathbb{B} \mapsto X$ be a holomorphic map. One says that it has *unit radius of boundedness* if it is bounded on each subset strictly inside $\mathbb{B}$.

The problem to verify whether a holomorphic map has unit radius of boundedness arises in many aspects of infinite dimensional holomorphy (see, for example, [9, 11]) as well as in complex dynamical systems ([1, 16, 17]). In particular, it plays a crucial role in the study of nonlinear numerical range and spectrum of holomorphic maps [11], in the establishing of exponential and product formulas for semigroups of holomorphic maps [15, 16], and the study of flow invariance and range conditions in the nonlinear analysis [12, 17]. It was specifically mentioned for the class of the so-called semi-complete vector fields (or, infinitesimal generators) in the study of their numerical range and Bloch radii [12].

Note that if h is uniformly continuous on $\overline{\mathbb{B}}$, the closure of $\mathbb{B}$, then the property of h to be an infinitesimal generator is equivalent to the nonlinear dissipativeness of h [12].

In this note we consider a wider class of holomorphic maps on $\mathbb{B}$ and establish some growth estimates under weaker restrictions on its numerical range.

For $\varphi \in X^*$ we use the notation $\langle v, \varphi\rangle := \varphi(v)$. If $v \in X$, we denote by $v^* \in X^*$ any element such that

$$\mathsf{Re}\,\langle v, v^*\rangle = \|v\|^2 = \|v^*\|^2.$$

By the Hahn-Banach theorem such an element v^* exists, but in general it is not unique. However, if $(X, \langle\cdot,\cdot\rangle)$ is a Hilbert space then v^* is unique and it is defined by $v^* := \langle \cdot, v\rangle$.

2010 *Mathematics Subject Classification.* 46T25, 32K05, 40E10, 47A12.

Key words and phrases. Infinitesimal generators, growth estimates, bounded holomorphic functions.

*Partially supported by the ERC grant "HEVO - Holomorphic Evolution Equations" n. 277691.

Definition 1.2. We say that a holomorphic map $h : \mathbb{B} \mapsto X$ is *pseudo-dissipative* if there is $\varepsilon > 0$ such that the convex hull of the set

$$\Omega_\varepsilon(h) := \{\langle h(z), z^* \rangle,\ 1 - \varepsilon < \|z\| < 1\}$$

is not the whole plane $\mathbb{C}$.

Remark 1.3. If h is uniformly continuous on $\overline{\mathbb{B}}$, then its pseudo-dissipativeness actually means that the numerical range of h (see [11]) lies in a half-plane.

Remark 1.4. It is clear that the pseudo-dissipativeness of h is equivalent to the following condition: *there is $\varepsilon > 0$ such that*

$$\operatorname{Re} e^{i\theta} \langle h(z), z^* \rangle \leq a\|z\|^2 + b\left(1 - \|z\|^2\right), \quad 1 - \varepsilon < \|z\| < 1, \tag{1.1}$$

for some real θ, a and b. For some technical reasons inequality (1.1) is more convenient for our further considerations.

Let $A : X \mapsto X$ be a continuous linear operator. Then one defines

$$m(A) := \inf\{\operatorname{Re} \langle Av, v^* \rangle : \|v\| = 1\}$$

and the *numerical radius* of A as

$$V(A) := \sup\{|\langle Av, v^* \rangle| : \|v\| = 1\}.$$

The main result of this paper is the following

Theorem 1.5. *Let $\mathbb{B}$ be the unit ball in a complex Banach space X. Let F be a pseudo-dissipative holomorphic map on $\mathbb{B}$, i.e., for some real a, b, θ and $\varepsilon > 0$, inequality* (1.1) *holds for z, $1 - \varepsilon < \|z\| < 1$. Then*

(i) *inequality* (1.1) *holds with $b = \|F(0)\|$ and the same a and θ for all $z \in \mathbb{B}$;*
(ii) *$m(e^{i\theta} A) \leq a$, where $A = DF(0)$;*
(iii) *F has unit radius of boundedness. Moreover, for all $z \in \mathbb{B}$ the following estimate holds:*

$$\begin{aligned} \|F(z) - F(0)\| &\leq \|z\| \left(|a| + eV(e^{i\theta} A - a \cdot \mathsf{id})\right) \\ &\quad + 4\|z\|^2 \|F(0)\| + 8\|z\|^2 \frac{1 - \|z\| \ln 2}{(1 - \|z\|)^2} \left(a - m(e^{i\theta} A)\right) \\ &< 4\|F(0)\| \cdot \|z\|^2 + |a| \cdot \frac{\alpha \|z\|}{(1 - \|z\|)^2} + V(A) \cdot \frac{\beta \|z\|}{(1 - \|z\|)^2}, \end{aligned} \tag{1.2}$$

where $\alpha = \dfrac{8(e \ln 2 + \ln 2 + 1 - e)}{8 \ln 2 - e - 1} < 3.8$ *and* $\beta = \dfrac{8(e \ln 2 + 2 - e)}{8 \ln 2 - e} < 3.3$.

Corollary 1.6. *Let D be a domain in X, $\mathbb{B} \subset D$, and let $F : D \mapsto D$ be pseudo-dissipative on $\mathbb{B}$ such that $F(0) = 0$ and $DF(0) = A$. If operator $e^{i\theta} A - a \cdot \mathsf{id}$ is power bounded, then there is a bounded neighborhood of the origin which is invariant under F.*

Let $G : \mathbb{B} \mapsto X$ be holomorphic. The map G is called an *infinitesimal generator* if the Cauchy problem

$$\begin{cases} \overset{\bullet}{z}(t) = G(z(t)) \\ z(0) = z_0 \end{cases}$$

has a solution $[0,+\infty) \ni t \mapsto z(t)$ for all $z_0 \in \mathbb{B}$.

We will see below (see Lemma 2.3) that a map $G : \mathbb{B} \mapsto X$ is an infinitesimal generator on $\mathbb{B}$ if and only if it satisfies the inequality

$$\mathsf{Re}\,\langle G(z), z^*\rangle \leq b(1-\|z\|^2), \quad 1-\varepsilon < \|z\| < 1,$$

for some $\varepsilon > 0$ and $b \in \mathbb{R}$ (cf., formula (2.1)). Therefore, we obtain from Theorem 1.5 that that *each holomorphic generator satisfies the following inequality:*

$$\begin{aligned} \|G(z)-G(0)\| &\leq 4\|z\|^2\|G(0)\| + e\|z\|V(T) - 8m(T)\|z\|^2\frac{1-\|z\|\ln 2}{(1-\|z\|)^2} \\ &< 4\|z\|^2\|G(0)\| + \frac{\beta\|z\|}{(1-\|z\|)^2}V(T), \quad \textit{where } T = DG(0) \textit{ and } \beta < 3.3\,. \end{aligned} \tag{1.3}$$

In case $X = \mathbb{C}^n$ with some norm $\|\cdot\|$, under the conditions that $G(0) = 0$ and $m(T) < 0$, the last estimate has been proved in [7, Lemma 2.4] (generalizing the previous result for the case $T = \mathsf{id}$ in [10, Theorem 1.2]). In [12, Theorem 8] a similar growth estimate for holomorphic maps $G : \mathbb{B} \mapsto X$ with bounded numerical range and such that $G(0) = 0, DG(0) = \mathsf{id}$ is given. Note that estimate (1.3) is more precise than those mentioned above.

Also, even in finite dimensional spaces, Theorem 1.5 has some interesting applications. For instance, it can be used to give an answer to the following natural question.

Question: Let $\{G_n\}$ be a family of infinitesimal generators on the unit ball $\mathbb{B}$ of a complex Banach space. Which are the "minimal" possible conditions that guarantee that the family contains a convergent subsequence?

In [4, Lemma 2.2] it is shown that if $\mathbb{B} = \mathbb{D}$ is the unit disc in $\mathbb{C}$, then $\{G_n\}$ contains a convergent subsequence if there exist two points $z \neq w \in \mathbb{D}$ such that $\{G_n\}$ is equibounded at z and w. The argument there is strongly based on the so-called Berkson-Porta formula. It is not clear whether and how a similar statement can be prove in higher dimensions. However, Theorem 1.5 allows to prove the following:

Corollary 1.7. *Let $D \subset \mathbb{C}^n$ be a bounded balanced convex domain. Let $\{G_n\}$ be a family of infinitesimal generators on D. Suppose that there exists $C > 0$ such that*

$$\|G_n(0)\| + \|G_n'(0))\| \leq C,$$

where here we denote by $\|G_n'(0)\|$ the operator norm of the differential of G_n at 0. Then there exists a subsequence of $\{G_n\}$ which converges uniformly on compacta to an infinitesimal generator.

This work started when the first and third authors where visiting the Mittag-Leffler Institute during the program "Complex Analysis and Integrable Systems" in Fall 2011. Both authors thank the organizers and the Institute for the kind hospitality and the atmosphere experienced there.

2. Proofs

If h is a holomorphic map on $\mathbb{B}$, we will write its expansion at 0 as $h(z) = h(0) + Tz + \sum_{j\geq 2} Q_j(z)$, where $T : X \mapsto X$ is the Fréchet differential of h at 0, and Q_j is a continuous homogeneous polynomial of degree j on X (see, e.g., [13]).

We begin by recalling (see [1, Theorem p.95] or [17]) that $G : \mathbb{B} \mapsto X$ is an infinitesimal generator if and only if

$$\mathsf{Re}\,\langle G(z), z^*\rangle \leq \mathsf{Re}\,\langle G(0), z^*\rangle(1 - \|z\|^2) \qquad \text{for all } z \in \mathbb{B}. \tag{2.1}$$

Note that if the previous formula holds for some z^*, then it holds for all z^*.

The following lemma follows immediately from (2.1)

Lemma 2.1. *Let $G : \mathbb{B} \mapsto X$ be holomorphic. Then G is an infinitesimal generator if and only if for all $v \in X$ such that $\|v\| = 1$ the holomorphic map*

$$\mathbb{D} \ni \zeta \mapsto \langle G(\zeta v), v^*\rangle,$$

is an infinitesimal generator in the unit disc $\mathbb{D} := \{\zeta \in \mathbb{C} : |\zeta| < 1\}$.

In case of strongly convex domains in $\mathbb{C}^n$ the previous lemma holds for the restriction to any complex geodesic (see [3, Proposition 4.5]).

Remark 2.2. By [1, Corollary 5], a holomorphic map $g : \mathbb{D} \mapsto \mathbb{C}$ is an infinitesimal generator if and only if

$$g(\zeta) = g(0) - \overline{g(0)}\zeta^2 - \zeta q(\zeta),$$

where $\mathsf{Re}\, q(\zeta) \geq 0$ for all $\zeta \in \mathbb{D}$.

To prove our theorem we also need the following lemmata.

Lemma 2.3. *Let $F : \mathbb{B} \mapsto X$ be a pseudo-dissipative holomorphic map. Then inequality* (1.1) *holds for some suitable θ, a and b for all $z \in \mathbb{B}$. Moreover, the map $G : \mathbb{B} \mapsto X$ defined by*

$$G(z) = e^{i\theta} F(z) - az \tag{2.2}$$

is an infinitesimal generator on $\mathbb{B}$.

Proof. It follows from (2.2) and (1.1) that G satisfies the inequality

$$\mathsf{Re}\,\langle G(z), z^*\rangle \leq b\left(1 - \|z\|^2\right) \tag{2.3}$$

for all z with $1 - \varepsilon < \|z\| < 1$.

Fix any $v \in \partial\mathbb{B}$ and consider the function g defined by

$$g(\zeta) = \langle G(\zeta v), v^*\rangle, \quad \zeta \in \mathbb{D}.$$

Actually, according to Lemma 2.1, we have to show that g is an infinitesimal generator on $\mathbb{D}$. To do this, fix any $w \in \mathbb{D}$, choose $s \in \mathbb{R}$ with $\max(1 - \varepsilon, |w|) < s < 1$ and define

$$h_{r,t}(\zeta) = \zeta - t\left(w + rg(\zeta)\right), \quad r > 0,\ 0 \leq t \leq 1.$$

For all ζ on the circle $|\zeta| = s$, we have by (2.3)

$$\mathsf{Re}\left(h_{r,t}(\zeta)\bar{\zeta}\right) = |\zeta|^2 - t\mathsf{Re}\,(w\bar{\zeta}) - tr\mathsf{Re}\,\langle G(\zeta)v, (\zeta v)^*\rangle \geq s^2 - ts|w| - trb(1 - s^2).$$

It is clear that for s close enough to 1, there is $\delta > 0$ such that $\mathsf{Re}\left(h_{r,t}(\zeta)\bar{\zeta}\right) > \delta$ on the circle $|\zeta| = s$. Therefore, $h_{r,t}$ has no null points on this circle. Then by the logarithmic residue formula, the number of null points is a continuous in t function. Since this function takes natural values, it is constant. Because of $h_{r,0}(\zeta) = \zeta$, we conclude that the function $h_{r,1} = \zeta - w - rg(\zeta)$ has a unique null point in $\mathbb{D}$. Then Proposition 3.3.1 in [19] implies that g is a generator.

Now, by (2.1), we have the following inequality:

$$\mathsf{Re}\,\langle e^{i\theta}F(z), z^*\rangle \le a\|z\|^2 + \|F(0)\|(1 - \|z\|^2), \quad z \in \mathbb{B},$$

which completes the proof. □

Lemma 2.4. *Let $G : \mathbb{B} \mapsto X$ be an infinitesimal generator with expansion $G(z) = G(0) + Tz + \sum_{j\ge2} Q_j(z)$. Let $v \in X$ be such that $\|v\| = 1$ and let $v^* \in X^*$ be such that $\langle v, v^*\rangle = \|v\| = \|v^*\| = 1$. Then*

$$\mathsf{Re}\,\langle Tv, v^*\rangle \le 0. \tag{2.4}$$

Moreover, if $\mathsf{Re}\,\langle Tv, v^*\rangle = 0$ then

$$\begin{aligned} \langle Q_2(v), v^*\rangle &= -\overline{\langle G(0), v^*\rangle}, \\ \langle Q_j(v), v^*\rangle &= 0, \quad j \ge 3. \end{aligned}$$

Proof. let $g : \mathbb{D} \mapsto \mathbb{C}$ be defined as $g(\zeta) := \langle G(\zeta v), v^*\rangle$. By Lemma 2.1 the holomorphic map g is an infinitesimal generator in $\mathbb{D}$. By Remark 2.2 we can write $g(\zeta) = g(0) - \overline{g(0)}\zeta^2 - \zeta q(\zeta)$, where $\mathsf{Re}\, q(\zeta) \ge 0$ for all $\zeta \in \mathbb{D}$. From this we have

$$-q(0) = g'(0) = \langle Tv, v^*\rangle, \tag{2.5}$$

and (2.4) follows from $\mathsf{Re}\, q(0) \ge 0$.

Now, let $q(\zeta) = q(0) + \sum_{j\ge1} a_j\zeta^j$. Expanding g we see that

$$\begin{aligned} g(0) - q(0)\zeta - (a_1 + \overline{g(0)})\zeta^2 - \sum_{j\ge3} a_{j-1}\zeta^j &= g(\zeta) \\ &= \langle G(0), v^*\rangle + \langle Tv, v^*\rangle\zeta + \sum_{j\ge2}\langle Q_j(v), v^*\rangle\zeta^j, \end{aligned} \tag{2.6}$$

from which it follows that

$$\begin{aligned} \langle Q_2(v), v^*\rangle &= -\left(a_1 + \overline{\langle G(0), v^*\rangle}\right), \\ \langle Q_j(v), v^*\rangle &= -a_{j-1}, \quad j \ge 3. \end{aligned} \tag{2.7}$$

If $\mathsf{Re}\,\langle Tv, v^*\rangle = 0$ then $\mathsf{Re}\, q(0) = 0$, hence $q(\zeta) = ia$ for some $a \in \mathbb{R}$ and $a_j = 0$ for all $j \ge 1$, and the statement follows. □

Now we are in good shape to prove our main result:

Proof of Theorem 1.5. Assertion (i) is already proven in Lemma 2.3. Consider now the map $G : \mathbb{B} \mapsto X$ defined by $G(z) = e^{i\theta}F(z) - az$. Then $T(= DG(0)) = e^{i\theta}A - a\cdot\mathsf{id}$, and assertion (ii) follows immediately by Lemma 2.4.

In order to prove assertion (iii), by using Lemma 2.3, it is sufficient to prove inequality (1.3) for the same map G being an infinitesimal generator on $\mathbb{B}$.

For a fixed $v \in X$ with $\|v\| = 1$, let $v^* \in X^*$ be such that $\langle v, v^*\rangle = \|v\| = \|v^*\| = 1$. It follows from Lemma 2.4 that $\mathsf{Re}\,\langle Tv, v^*\rangle \leq 0$. Let $g : \mathbb{D} \mapsto \mathbb{C}$ be defined as $g(\zeta) := \langle G(\zeta v), v^*\rangle$. By Lemma 2.1, the holomorphic map g is an infinitesimal generator in $\mathbb{D}$. According to Remark 2.2, $g(\zeta) = g(0) - \overline{g(0)}\zeta^2 - \zeta q(\zeta)$, where $\mathsf{Re}\, q(\zeta) \geq 0$ for all $\zeta \in \mathbb{D}$. The Carathéodory inequalities ([5], see also [2]) imply that $|a_j| \leq 2\mathsf{Re}\, q(0)$ for all $j \geq 1$. Now by (2.7) and (2.5) we get

$$\begin{aligned} |\langle Q_2(v), v^*\rangle| \leq& \|G(0)\| - 2\mathsf{Re}\,\langle Tv, v^*\rangle, \\ |\langle Q_j(v), v^*\rangle| \leq& -2\mathsf{Re}\,\langle Tv, v^*\rangle, \quad j \geq 3. \end{aligned} \tag{2.8}$$

To proceed, we need L. Harris' inequalities [11]. Namely, if $P_m : X \mapsto X$ is a continuous homogeneous polynomial of degree $m \geq 1$ then $\|P_m\| \leq k_m V(P_m)$, where $k_m = m^{m/(m-1)}$ for $m \geq 2$, $k_1 = e$ and

$$V(P_m) = \sup\{|\langle P_m(v), v^*\rangle| : \|v\| = 1\}.$$

These estimates together with (2.8) imply that

$$\begin{aligned} \|Tz\| \leq& e\|z\|V(T), \\ \|Q_2(z)\| \leq& k_2\|z\|^2\|G(0)\| - 2k_2\|z\|^2 m(T), \\ \|Q_j(z)\| \leq& -2k_j\|z\|^j m(T), \quad j \geq 3. \end{aligned} \tag{2.9}$$

Now for all $z \in \mathbb{B}$ we have by (2.9)

$$\begin{aligned} \|G(z) - G(0)\| &\leq \|Tz\| + \|Q_2(z)\| + \sum_{j\geq 3} \|Q_j(z)\| \\ &\leq e\|z\|V(T) + k_2\|z\|^2\|G(0)\| - 2k_2\|z\|^2 m(T) - 2m(T)\sum_{j\geq 3} k_j\|z\|^j \\ &\leq e\|z\|V(T) + 4\|z\|^2\|G(0)\| - 2m(T)\sum_{j\geq 2} k_j\|z\|^j. \end{aligned} \tag{2.10}$$

In order to conclude, consider the function $k(x) = x^{x/(x-1)}$, $x \geq 2$. Since $k''(x) < 0$, one concludes that $k(x) \leq k'(2)(x-2) + k(2) = k'(2)x + 4 - 2k'(2)$ with $k'(2) = 4(1 - \ln 2)$. Applying this simple fact, we get from (2.10)

$$\begin{aligned} \|G(z) - G(0)\| &\leq e\|z\|V(T) + 4\|z\|^2\|G(0)\| - 2m(T)\sum_{j\geq 2}\left(k'(2)j + 4 - 2k'(2)\right)\|z\|^j \\ &\leq e\|z\|V(T) + 4\|z\|^2\|G(0)\| - 2m(T)\|z\|^2\left(\frac{k'(2)(2-\|z\|)}{(1-\|z\|)^2} + \frac{4-2k'(2)}{1-\|z\|}\right) \\ &= 4\|z\|^2\|G(0)\| + e\|z\|V(T) - 8m(T)\|z\|^2\frac{1-\ln 2\cdot\|z\|}{(1-\|z\|)^2}. \end{aligned}$$

□

Proof of Corollary 1.7. By hypothesis, D is the unit ball in $\mathbb{C}^n$ for the Minkowski norm defined by D. Therefore, Theorem 1.5 applies and Montel's theorem implies that the family $\{G_n\}$ is normal. Thus, since it cannot be compactly divergent because it is bounded at the origin, there exists a subsequence $\{G_{n_k}\}$ which converges uniformly on compacta to a holomorphic map $G : D \mapsto \mathbb{C}^n$. Applying (2.1) to each G_{n_k} and passing to the limit, we get the result. □

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Manuscript received April 24, 2013
revised June 22, 2013

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