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REACHING WIN-WIN STATE OF MIND IN GAMES, FORMULATION AND SOLUTION

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Abstract. Human psychology determines his/her action and behavior. This fact has not been fully incorporated in game theory. This paper intends to incorporate human psychology in formulating games as people play them. Using Habitual Domains and Markov chains theory, we present a new model for describing the evolution of the states of mind of players over time, the two person second order game. We introduce the concept of focal mind profile as well as the solution concept of win-win mind profile. Under some suitable assumptions, the restructuring game to reach a win-win mind profile can be formulated into a discrete optimal control problem. We illustrate the formulation with an example.

1. Introduction

The traditional normal form game theory is anchored on the notion of utility function, fixed set of strategies and the principle of rationality [13], [11]. In real game situations and conflicts, it often happens that players are not able to identify the utility function of the other players and their own. Moreover, the sets of strategies of players evolve over time according to the information the players receive from the other players and from the environment. The only extension of strategy sets considered in traditional game theory is the set of mixed strategies based on the initial set of pure strategies. During real-world games the players may drop some strategies or generate new ones in order to reach a solution. The players may change their evaluation of the strategy profiles. In other words, in real-world games, the structure of the game does not remain constant, the game may be constantly restructured according to the psychological states of players. This fact has never been considered in traditional games. Often the predictions of game theory are not accurate because it assumes that the players are rational. In fact the players play the game as they perceive it and according to their information processing ability. The Nash equilibrium [11] is the most prominent solution concept in traditional game theory. This equilibrium is immune against unilateral deviations. However, it is not immune against coalition (multilateral) deviations, which makes it unstable. Moreover, it may be dominated in some games like the prisoner dilemma game [3]. Some attempts have been made to introduce some refinements of Nash equilibrium that are immune against coalition deviations like Strong Berge equilibrium

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[4], Strong Nash equilibrium [1], but the problem of these equilibria is that they do not exist for most of the games.

Using Habitual domain (HD) theory [14]-[19] and Markov chain theory, in this article, we sketch second-order games that are based on psychological states of players and do not involve the utility functions and sets of strategies of players. This new game model overcomes most of the difficulties and paradoxes related to the traditional game theory utility function-strategy-rationality framework. For details see [9] and [10]. In addition, the present paper addresses the problem of finding optimal ways for restructuring games as to reach a win-win profile when time and cost are considered. Two models are presented. The first model deals with the case where the objective is to minimize the restructuring time. The second model deals with the case where the resolution time is given and the objective is to minimize the total cost for restructuring the game.

The paper is organized as follows. Section 2 briefly sketches the HD theory. Section 3 presents second-order games with two new examples showing how a game situation can be formulated as a second-order game. Section 4 discusses restructuring second-order games for reaching a win-win profile. Moreover, the two examples of Section 3 are solved by restructuring. In Section 5 two discrete optimal control problems for finding optimal ways of restructuring games for reaching a win-win profile are presented. The cost minimizing model is illustrated by an example. Section 6 concludes the paper.

2. Habitual domain theory

In this section we sketch briefly the related concept of Habitual Domains (HD). We refer the reader to [16], [17] and [19] for more details.

The collection of ideas and actions (including ways of perceiving, thinking, responding, acting, and memory) together with their formation, dynamics, and basis in experience and knowledge, is called Habitual Domain (HD) [17]. Over time, unless extraordinary events or purposeful effort is exerted, our HD will become stabilized within a certain domain [15]. As a consequence, each of us has habitual ways of eating, dressing, speaking, etc. Some habitually emphasize economic gains, while others pay attention to social reputation. Some habitually persist in their pursuit of goals, while others change their objectives frequently. Some are habitually positive and optimistic, while others are negative and pessimistic. Some habitually pay attention to details, others only to generalities. The concept of individual's HD can be extended to other living entities, such as companies, social organizations, and groups in general. The following are the basic elements of HD.

(i) Potential Domains(PD)- the collection of ideas and actions that can potentially be activated to occupy our attention.

(ii) Actual Domain (AD)- the set of ideas and actions that are actually activated or occupy our attention.

(iii) Activation Probability (AP)- the probabilities that ideas or actions in PD also belong to AD.

(iv) Reachable Domain (RD)- the set of ideas and actions that can be attained from a given set in AD.

The theory of HD is based on eight hypotheses H1-H8.

H1. Circuit Pattern Hypothesis. Thoughts, concepts or ideas are represented by circuit patterns of the brain. A circuit patterns will be reinforced when the corresponding ideas are repeated. Furthermore, the stronger the circuit patterns, the more easily the corresponding thoughts are retrieved in our thinking and decision making process.

H2. Unlimited Capacity Hypothesis. Practically every normal brain has the capacity to encode and store all thoughts, concepts and messages that one intends to.

H3. Efficient Restructuring Hypothesis. The encoded thoughts, concepts and messages (H1) are organized and sorted systematically as data bases for efficient retrieving. Furthermore, according to the dictation of attention, they are continuously restructured so that relevant ones can be efficiently retrieved to release charge.

H4. Analogy and Association Hypothesis. The perception of new events, subjects, or ideas can be learned primarily by analogy and/or association with what is already known. When faced with a new event, subject, or idea, the brain first investigates its features and attributes in order to establish a relationship with what is already known by analogy and/or association. Once the right relationship has been established, the whole of the past knowledge (preexisting memory structure) is automatically brought to bear on the interpretation and understanding of the new event, subject, or idea.

H5. Goal Setting and State Evaluation. Each one of us has a set of goal functions and for each goal function we have an ideal state or equilibrium to reach and maintain (goal setting). We continuously monitor, consciously or subconsciously, where we are relative to the ideal state or equilibrium point (state evaluation). Goal setting and state evaluation are dynamic, interactive and are subject to physiological forces, self-suggestion, external information forces, current data bank (memory) and information processing capacity.

H6. Charge Structure and Attention Allocation Hypothesis. Each event is related to a set of goal functions. When there is an unfavorable deviation of perceived value from the ideal, each goal function will produce various levels of charge. The totality of the charges created by all goal functions is called the charge structure and it can change dynamically. At any point in time, our attention will be paid to the event which has the most influence on our charge structure.

H7. Discharge Hypothesis. To release charges, we tend to select the action which yields the lowest remaining charge (the remaining charge is the resistance to the total discharge) and this is called the least resistance principle.

H8. Information Input Hypothesis. Humans have an innate need to gather external information. Unless attention is paid, external information inputs may not be processed.

Note that hypotheses H1-H4 describe how the brain functions and hypotheses H5-H8 describe how the mind functions. For the details on these hypotheses we refer the reader to [16] and [19].

Remark 2.1. Most aspects of hypotheses H1-H8 are not considered in the traditional game theory framework [2]. In fact, most of the models of traditional game theory do not consider the psychological states of players and their changes during the game. It is clear that an analysis of a game situation based on hypotheses H1-H8 will capture more psychological aspects than the traditional game theory does. For example, if the players do not interact between them and with the external world, they do not use hypothesis H8 at all. H2 suggests that players have unlimited capacity to learn if they are willing to. According to H5 players have goal functions and an ideal state for each of them; they continuously monitor where they are relative to the ideal states. According to H6, a charge is a precursor of a mental force to action or inaction, which could lead to drive or stress. In this paper, for simplicity, the charge structure will be called charge level. At any point in time, our attention will be paid to the event which has the most important influence on our charge level. The event or decision problem with the most significant charge commands our attention at any given moment. When our attention is allotted to an event, we use the following modes of action to deal with it: active problem solving or avoidance justification. The former tries to work actively to move the perceived states closer to the ideal states (discharge H7); while the later tries to rationalize the situation so as to lower the ideal states closer to the perceived states. With active problem solving, charge is transferred to drive, while with avoidance justification, charge may be reduced or transferred into stress.

3. Second-order games

In this section we introduce second order games. Then we show how a game situation can be described and analyzed by second-order games.

Let us consider a game situation involving two players, Player I and Player II. Denote by PD^I , AD^I and PD^{II}, AD^{II} the potential domain (PD) and the actual domain (AD) of Player I and Player II respectively. In terms of HD theory a state of mind of a player can be considered as a collection of ideas or thoughts that could be activated from his potential domain PD to the actual domain, which is a part of his reachable domain. Hence we will identify a *state of mind* of a player with its corresponding ideas or thoughts in the potential domain of the player. In applications, the players could be hostile (non cooperative) or friendly (cooperative) to each other (two states of mind). If needed, the moods or states of mind could be further decomposed into extremely hostile, hostile, neutral, friendly and extremely friendly. The number of states of mind in almost all practical cases is still finite, even if we further decompose them. Therefore, we have the following assumption.

Assumption 3.1. The number of states of mind of each player is finite regarding the game situation.

Let us assume that Player I has s states of mind and Player II has l states of mind regarding the game i.e. $S^I = \{S_1^I, S_2^I, ..., S_s^I\}$ and $S^{II} = \{S_1^{II}, S_2^{II}, ..., S_l^{II}\}$. We refer to the Cartesian product $S = S^I \times S^{II}$ as the *profile* (states) *space* and its elements as the *mind profiles* or simply the profiles of the game.

Definition 3.2. Player i, $i \in \{I, II\}$ is in the state of mind S_j^i at time t, if S_j^i is activated from his potential domain PD^i to his actual domain AD^i at time t , i.e. $S_j^i \in AD^i$ at time t.

Assumption 3.3. At any time t the actual domain of each player contains only one state of mind and only one state can be activated from the potential domain to the actual domain.

This assumption reflects the fact that, at any moment, the actual domain (attention) of each player is occupied by only one state of mind.

3.1. **Formulation of the Game as a Markov Chain.** We first formulate the game as a stochastic process. In order to simplify the presentation, in the remaining part of the paper, we will identify the set of states of mind of Player I, S^I, with $\{1, 2, ..., s\}$ and the set of states of mind of Player II, S^{II} , with $\{1, 2, ..., l\}$. Thus, any profile of the game (S_i^I, S_j^{II}) in $S = S^I \times S^{II}$ will be denoted by the couple (i, j). Since the activation process from the potential domain to the actual domain is probabilistic and the cardinality of $S = S^I \times S^{II}$ is $s \times l$, we can consider the problem as a stochastic process $\{X_n, n = 0, 1, 2, ...\}$ with a finite number, $s \times l$, of states.

Now let us define the transition probability from any profile to any other profile. Given that a profile (i, j) is activated at the present step, the probability that a profile $(i_1, j_1) \in S$ will be brought to the actual domain in the next step is given by the conditional probability $P_{(i,j)(i_1,j_1)}$. In practice, this probability can be evaluated by two ways. The first one is by subjective evaluation (by players or/and experts); the second one is by frequency approach, i.e. the frequency of activation of the profile (i_1, j_1) if the current profile is (i, j), when the game is repeated a certain number of times.

Assumption 3.4. The profile of the process in the next step depends only on the profile of the process in the present step i.e.

$$P(X_{p+1} = (i, j)/X_p = (i_p, j_p), X_{p-1} = (i_{p-1}, j_{p-1}), ..., X_0 = (i_0, j_0)) = P_{(i_p,j_p)(i,j)}.$$

Assumption 3.4 may be justified as follows. According to HD theory [15]-[19], unless extraordinary events occur or a special effort is exerted, the activation probability of ideas and actions will be stabilized over time. Thus when there is no occurring of extraordinary event or a special effort is exerted by players to restructure the game, we may assume that the transition probability of the processes has the Markov Property. Assumption 3.4 implies that the game can be considered as a Markov chain with a finite number of states. Hence the results of Markov chain theory can be used to study the considered game. The matrix

$P = (P_{(i,j)(i_1,j_1)}), ((i,j),(i_1,j_1)) \in S^I \times S^{II}$ is the transition probability matrix of the process. It is a stochastic matrix that describes the behavior of the players with respect to the game situation. According to HD theory, it tends to be stable, but it is subject to changes over time when some new relevant idea or event arrives. Thus, the game can be represented by the following model

$$< \{I, II\}, \{HD^I, HD^{II}\}, \{S^I, S^{II}\}, P > \tag{3.1}$$

where HD^I and HD^{II} are the habitual domains of Player I and Player II respectively. If the transition probability matrix P changes at some time t, then we obtain a new game of type (3.1) with a new transition probability matrix that starts at time t. Note that P is a function of time and HD^I and HD^{II}, and P captures the psychological atmosphere of the game, which is an AD of the game situation.

Definition 3.5. A profile $(i,j) \in S^I \times S^{II}$ is called win-win profile of the game (3.1) if

(i) at (i,j) both players' charge level reaches its global minimum,

(ii)(i,j) is an absorbing profile, i.e. $P_{(i,j)(i,j)} = 1$.

Definition 3.6. A profile $(i,j) \in S^I \times S^{II}$ is called a focal profile if it is a profile that has some special appeal to the players and it is desirable to make it a win-win profile.

Remark 3.7. At a win-win profile both players have their charge levels at their minimum levels, locally and globally. Thus both players have no incentive to move away from the win-win profile. Note that Nash equilibrium considers only local property (fixing one player's strategy, the other has no incentive to deviate from the equilibrium strategy). Here win-win profile implies global stability also. Even if players work together, they cannot move to a better profile.

A focal profile can be chosen based on general principles such as peace, equity, fairness, self-interest, collective interest, stability, harmony, etc. The collective and individual interests can be satisfied through re-arrangements [8].

Assumption 3.8. The game (3.1) has a focal profile.

This assumption is quite natural, since usually in any game there is some profile that has some appeal to both players. When there is no explicit focal profile, we can consider the profile where both players have a "Cooperative" state of mind as a focal profile. With cooperative spirit, the players can find a third new alternative such that both players can claim victory. Thus, the existence of a focal profile in the game is not a serious problem.

The following are challenging problems:

(i) under what conditions is it possible to activate a focal profile in a conflict situation? That is, when is it possible to reach it and how to reach it?

(ii) How to make it a win-win profile? In other words, how to make it stable once it is reached?

In [9] and [10] we have addressed these questions with the help of HD theory and the theory of Markov Chains. In Section 5, we address the problem of finding optimal ways of restructuring games for reaching a win-win profile in terms of time and/or cost.

Henceforth, without loss of generality, we assume that the profile $(1, 1)$ is a focal profile for the game (3.1). We also assume that the players adopt the principle of active problem solving behavior when their charge level is adequately high.

Remark 3.9. An immediate consequence of Definition 3.5 is that when the players are not in the win-win profile, the charge level of at least one of them is not at the lowest level. In order to lower his charge level, the player will take action. Once a win-win profile is in the actual domain of players, it does not create a significant charge level for both players and no player has incentive to move away from it. Note that the concept of win-win profile is more general than the concept of Nash equilibrium [11]. Indeed, Nash equilibrium is stable against unilateral deviation only, while a win-win profile is stable against any deviation: unilateral or multilateral. Furthermore, the former is based on strategies and utility function, while the latter is based on states of mind and charge level.

Remark 3.10. In order to avoid distraction we give the following short comparison between the traditional games in normal form and the second order game (3.1).

(i) In traditional games of normal form, the outcomes of the game are expressed in terms of a utility function. In game (3.1) the outcome of the game is not in terms of payoff, rather it is in terms of charge level (see [16], [18] and [19]). The objective of each player is to reduce his/her charge level.

(ii) In traditional games of normal form, the sets of strategies are somehow predetermined and fixed. In the game (3.1) there is no restriction or limitations on strategies. New strategies can be generated as deeper parts of HD are reached. The players' HDs can generate new strategies as the game evolves.

(iii) In traditional games of normal form, the structure of the game, the sets of strategies and the payoffs are fixed, no reframing is allowed. In second order games the structure of the game is dynamic and subject to restructuring.

(iv) In traditional games of normal form, the interaction between the players and with the external world is not fully considered. In second order games, the information input hypothesis H8 plays an important role in reaching a win-win profile.

Thus, the difference between the two models is structural and conceptual at the same time. Geanakoplos et al. [7] introduced psychological games. In these games the psychological aspect of players is partially taken into account by incorporating the beliefs of each player about the other players' strategies in the payoff functions. However, the structure of the game is still as in the traditional games. Indeed, the sets of strategies are still more or less fixed too, the outcomes are expressed in terms of utility function, the concept of solution used is Nash equilibrium, not a win-win profile, and no restructuring of the game is considered. Finally, note that the game (3.1) is beyond stochastic games [12] and [6]. Stochastic games did make a considerable progress in considering a variety of situations (states) in the game; however, the set of strategies is more or less fixed, the solution concept is basically Nash equilibrium, not a win-win profile, and no restructuring of the game is considered. The transition probability matrix is not based on psychological aspect as in game (3.1). Butnariu [5] introduced the theory of fuzzy games as a generalization of the traditional strategic games. In the Butnariu's framework the beliefs of players are expressed by fuzzy sets, payoffs are not involved. It is a considerable advance, but traditional sets of strategies of players are involved in his model and the concept of solution used is Nash equilibrium not a win-win profile. In addition, the restructuring of the game is not considered.

Now let us illustrate by two examples how the game (3.1) can be constructed.

Example 3.1. (Battle of sex , adapted from [16], [19]). Two college students Adam and Betty are dating. Adam would much rather attend a baseball game than a concert while Betty strongly prefers the concert to the baseball game. Note that the date could not be of fun if the two are not together. The payoffs may be represented by the following payoff matrix

(3.2)

	B	C
B	(10,3)	(1,1)
C	(-2,-2)	(3,10)

where Adam chooses the rows and Betty the columns and B for baseball, C for concert. Note that if both choose the baseball game, Adam is highly satisfied while Betty is not excited (she just accompanies him). The situation is represented by the payoff of (10,3). If Adam chooses the baseball game while Betty chooses the concert, then the fun of dating disappears. The situation is represented by the payoff of (1,1). If Adam against his interest has to go to the concert and Betty has to go to the baseball game, then both Adam and Betty would be unsatisfied at the concert or the baseball game. Thus the payoff is (-2,-2). If both go to the concert Betty is highly satisfied while Adam is not excited (he just accompanies her). The situation is represented by the payoff of (3,10). The following are worth mentioning:

(i) Suppose Adam announces first (preemptive move) with determination and credibility that he is going to the baseball game (row 1) no matter what happens. Then Betty, based on her self-interest, will choose baseball game to have a payoff of 3 instead of choosing the concert to have a payoff of 1. This, certainly makes Adam absolutely delighted.

(ii) Similarly, suppose Betty announces (preemptive move) with determination and credibility that she is going to the concert (column 2) no matter what happens. Then Adam, based on his self-interest, will choose the concert to have a payoff of 3 instead of choosing the baseball game to have a payoff of 1. This, certainly makes Betty absolutely delighted.

(iii) The game has two Nash equilibria (B,B) and (C,C). The outcomes will favor whoever chooses the right move first. The interaction/situation suggests that the player who takes the initiative will be better off.

(iv) If both players announce their preferred choice (Adam for the baseball game and Betty for the concert) simultaneously or sequentially, and are determined to carry out their decisions, what will happen to the payoffs? According to the payoff matrix, each would get 1 as payoff. Thus, there is a temptation for each player to change his/her mind to become agreeable with the other player. Note that whoever concedes (changes his/her mind) first, will get only 3 instead of the highest payoff of 10, and the one who persists will realize the highest payoff of 10. Because of this feature, the game (3.2) is called the chicken game.

From (i)-(iv) we note the that traditional game theory does not provide a satisfactory solution to the game situation (3.2), since the preferred Nash equilibrium can be reached only by a preemptive move of one of the players. Preemptive moves may be appropriate in some business situations and war, but in social interaction, they may worsen the atmosphere. In similar real-world game situations, most of the time, people reach a win-win situation by proposing new ideas and making mutual concession. In other words, by restructuring the game.

Let us now see how this game can be formulated as second-order game. As formulated in the framework of traditional game theory, the *actual domain* of both players is occupied by the aggressive idea of making a preemptive move. By H5 the players realize that there is no win-win situation in the game. The absence of a win-win situation increases the charge level of players H6, which leads to action by one of the players or both. This game can be formulated as a second-order game of type (3.1), where $S^I = S^{II} = \{C, N\}$, C means making concession by accepting to go where the other player wants and N means insisting on self-interest, a non cooperative behavior. Let the transition probability matrix of the game be assumably estimated as

$P=$

	(C,C)	(C,N)	(N,C)	(N,N)
(C,C)	0.1	0.4	0.4	0.1
(C,N)	0.2	0.1	0.1	0.6
(N,C)	0.15	0.1	0.1	0.65
(N,N)	0.2	0.2	0.2	0.4

In Section 4 we will see that this game can be restructured so that a win-win profile could be reached.

Example 3.2. Economic Discovery (Adapted from [16], [19]). During the Great Depression of the 1930's, the prevailing economic theory believed that the economy would eventually reach an equilibrium point at which the supply would equal the demand, and that the government should leave the economic system alone. As unemployment increased, the level of consumption decreased. This led to less profit and production, which in turn created further unemployment and deepened the depression. The perpetuated crisis dismayed almost everybody. High levels of charge existed everywhere. Yet solvable situations were not uncovered until John M. Keynes, a great economist, wisely and courageously jumped out of existing HDs and observed that government expenditure was a part of the national product. By increasing government expenditure for public construction (such as highways and dams) employment would increase. This meant increase in income and consumption, followed by an increase in production and employment, which would result in economic recovery and prosperity. This new vision, which reversed the prevailing view, was courageously and forcefully implemented by Franklin D. Roosevelt, who finally led the country to recovery and prosperity.

Note that in general, a solvable strategy may not always be acceptable to all players. It is the hard work of selling and implementing that account for the final success. Let us formulate this event as a second-order game. In abstract, there are two players, the producers (Player I) and the consumers (Player 2). For simplicity, assume that Player I has two states of mind: produce more (cooperative state, denoted by C), and produce less (non cooperation state, denoted by N). Similarly, Player II has two states of mind: C and N, C meaning consume more and N meaning consume less. The corresponding second order game of type (3.1), is defined by $S^I = S^{II} = \{C, N\}$. Let the transition probability matrix be assumably estimated as

$P=$

	(C,C)	(C,N)	(N,C)	(N,N)
(C,C)	0.1	0.2	0.2	0.5
(C,N)	0	0.1	0.1	0.8
(N,C)	0	0.1	0.1	0.8
(N,N)	0	0	0	1

Note, the profile (C, C) cannot be reached from any other profile. Because of the economic depression, it is not profitable to produce more and consume more. The transition probability from any profile to the profiles (C, N) and (N, C) are relatively small because no player has incentive to adopt a cooperative behavior i.e. to produce or consume more. The profile (N, N) is an absorbing profile. $\{(N, N)\}$ is the only recurrent class in P. All the other profiles are transient. The players are trapped in the profile (N, N). According to traditional game theory, the players will remain in the profile (N, N), which is a Nash equilibrium. This would deepen the depression and lead to a disaster. Note that in the framework of traditional game theory, this dangerous game situation cannot be solved because it does not include the possibility of external intervention as to restructure the game so that a win-win profile could be reached. Moreover, it would be difficult to evaluate the outcomes of the games, and the sets of strategies are difficult to identify because they evolve with time. The second-order games accommodate well the solution proposed by Keynes, which will be discussed in Section 4.

4. Restructuring games in practice

As P reflects an actual domain of a situation which is only a small part of the potential domain of the game, P can be changed. There are many parameters including the rule of the game, the form of interaction of the players, the information input and output and the HD of the players, that can be subject to change or manipulation. A *superior strategist* uses the changes of parameters to create a game situation in which all players can declare victory. An *ordinary strategist* tries to find an optimal strategy within a fixed set of parameters. For more details on this, see [16] and [19]. There are four important steps for effective restructuring games [16], [19]. (i) Understand the game situation, (ii) identify solvable game situations, (iii) create charge for moving towards solvable situations, and (iv) build enthusiasm for the desired actions. Each step, directly or indirectly, involves (a) expansion and enrichment of HDs of players, (b) effective suggestion of new ideas to catch the players' attention, and (c) effective integration of the new ideas with the core of the HDs of players. For (a), (b) and (c), the following tool boxes 1, 2 and 3, respectively, are important and useful. For the details and more of these tool boxes and beyond, please see [16] and [19].

Tool Box 1. Basic methods for expanding the HD. (i) Active learning, (ii) projecting from higher position, (iii) active association, (iv) changing the relevant parameters, (v) changing the environment, (vi) brain storming, (vii) retreating, and (viii) meditation.

Tool Box 2. Ideas that capture our attention. (i) Ideas that change the charge level, (ii) ideas that could trigger echoing in memory, (iii) ideas that arrive at the right time, (iv) ideas presented in the right ways, and (v) credible ideas.

Tool Box 3. Method for integrating an idea or concept with the core of a decision maker's HD. (i) Suggesting the idea, (ii) implanting the idea, (iii) nurturing the idea, (iv) habituating and repeating the idea and (v) integrating the idea with the core of existing HD.

We refer the reader to [16] and [19] for more details on the three tool boxes. The three sets of tools can also allow us to get into the depth of our potential domain as to gain creative and good ideas to solve our challenging game problems. For illustration let us consider the following second-order game [14]

$$< \{I, II\}, \{X_t^I, X_t^{II}\}, \{C_t^I, C_t^{II}\}, \{F_t^I, F_t^{II}\}, \{D_t^I, D_t^{II}\}, \{\{I_t^I, I_t^{II}\}\} > \tag{4.1}$$

where the decision elements $X_t^i, C_t^i, F_t^i, D_t^i$ and I_t^i are the set of strategies, the criteria, the payoffs measured in terms of criteria, the preference and the external information input, respectively, at time t for the ith player, $i \in \{I, II\}$. As the game evolves over time, all its components (parameters of the game) are subject to change and control. The game (4.1) is more general than the traditional normal form game since it allows the players to restructure the game by self suggestion or with the help of the external world through the information input I_t^i, $i \in \{I, II\}$. In the game (4.1) the evaluation of the situation at any time is done through the payoff functions and preferences, which implies that the charge levels of players are evaluated by payoffs. In [19] the techniques for restructuring the game (4.1) are discussed in details including: (i) reframing X_t^i, (ii) reframing C_t^i , (iii) reframing F_t^i, (iv) reframing D_t^i and (v) utilizing I_t^i.

Example 4.1. Let us see how the game of Example 3.1 can be solved by restructuring. Since there is no win-win profile, the charge level of both Adam and Betty are are high (H6). This charge creates drive for action (discharge H7). The use of H8 allows the players to communicate and exchange information between them and with the external world in order to reach a win-win profile. For example, Betty may propose to allow Adam to hold her hand during the concert. This offer will certainly affect the charge structure of Adam (H5, H6 and Tool Boxes 2) for it will be the first time he will get such a good opportunity. To reciprocate he will change his preference (H3), by accepting, with a great pleasure (discharge H7), to go to the concert with Betty. The new matrix of the game can assumably be represented in terms of payoffs by

(4.2)

	B	C
B	(-5,-5)	(-5,-5)
C	(-5,-5)	(10,10)

The payoff matrix in (4.2) reflects the fact that the charge structures are so much surged that unless both players go to concert they cannot be satisfied. All the other choices are psychologically equally bad. Clearly, in this new game, the profile (C, C) is a win-win profile. In terms of second-order games (3.1), we have

$P=$

	(C,C)	(C,N)	(N,C)	(N,N)
(C,C)	1	0	0	0
(C,N)	1	0	0	0
(N,C)	1	0	0	0
(N,N)	1	0	0	0

whatever the profile the players are in, in the next step they will shift to the win-win profile (C, C) and remain there.

Example 4.2. Let us see how the game of Example 3.2 was solved by restructuring. In the framework of second-order games, the game could be restructured by external intervention through information hypothesis H8 so that in the new game (C, C) became a reachable win-win profile. There are two important entities that could affect the structure of the game: the government and the academic community (economists). The government and the academic community were originally convinced that the government should not intervene in the economic system. This idea formed a strong circuit pattern in the brain of the decision makers and economists (circuit pattern hypothesis H1), so they were unable to solve the problem. As the situation worsened, the charge level reached a very high level (charge structure and attention allocation H6). A solution is imperative (discharge H7). Then John M. Keynes, a great economist, wisely and courageously jumped out of existing HDs and observed that government expenditure/investment was a part of the national product. By increasing government expenditure/investment for public construction (such as highways and dams), employment, income and profit would increase. It was difficult for Keynes to make his idea accepted because decision makers and economists were deeply rooted in the belief that the government should not intervene in the economic system. This idea was in the core of their HDs. Keynes had to convince them with the brand new idea. The process of making a new idea as part of the core of HD is a process [19] that needs patience, perseverance and courage (especially for games involving high stakes). This complex process utilizes the above mentioned Tool Boxes. After acceptance of the idea of Keynes, the game went through a sequence of restructuring. In the end, (C, C) became a reachable win-win profile. The final transition probability matrix of the game can be assumably estimated as

$P=$

	(C,C)	(C,N)	(N,C)	(N,N)
(C,C)	1	0	0	0
(C,N)	1	0	0	0
(N,C)	1	0	0	0
(N,N)	1	0	0	0

whatever the profile the players are in, in the next step they will shift to the win-win profile (C, C) and remain there.

5. A discrete optimal control model for reaching a win-win profile

In [10] we have presented two procedures for restructuring second-order games for reaching a win-win profile, but we did not address the problem of optimality of these procedures in terms of time and cost. Assume that there is a mediator (or authority) that wants to restructure the game so that the players can reach a win-win profile. In this section we formulate the problem of finding optimal ways of restructuring second-order games for reaching a win-win profile as discrete optimal control problems. We provide two models. The first one is to minimize the cost of restructuring with time limitation; the second is when the restructuring party has a cost constraint and it wants to minimize the time of restructuring.

5.1. **Minimizing Restructuring Cost Model.** Assume that the mediator wants to minimize the cost of restructuring with a time limitation T. Then the problem of finding the optimal procedure for restructuring the second-order game (3.1) may reduce to the resolution of the following discrete optimal control problem with proper linearity assumptions.

$$Min \sum_{t=0}^{T-1} \sum_{(i,j)(p,q)} (P_{(i,j)(p,q)}(t+1) - P_{(i,j)(p,q)}(t))\alpha_{(i,j)(p,q)}(t) \tag{5.1}$$

subject to

$$\sum_{(p,q)} P_{(i,j)(p,q)}(t) = 1,\ i = \overline{1,s},\ j = \overline{1,l},\ t = \overline{1,T-1} \tag{5.2}$$

$$P_{(i,j)(p,q)}(t) \geq 0,\ i = \overline{1,s},\ j = \overline{1,l},\ p = \overline{1,s},\ q = \overline{1,s},\ t = \overline{1,T-1} \tag{5.3}$$

$$P(t+1) = f(P(t), u(t)),\ t = \overline{0,T-1} \tag{5.4}$$

$$a_{(i,j)(p,q)}(t) \leq u(t)_{(i,j)(p,q)} \leq b_{(i,j)(p,q)}(t), \tag{5.5}$$
$$\text{for } i = \overline{1,s},\ j = \overline{1,l},\ p = \overline{1,s},\ q = \overline{1,l},\ t = \overline{0,T-1}$$

$$P(0) = P_{Initial} \tag{5.6}$$

$$P(T) = P_{Win} \tag{5.7}$$

where $\alpha_{(i,j)(p,q)}(t)$ is the cost for increasing the transition probability $P_{(i,j)(p,q)}(t)$ from the profile (i, j) to the profile (p, q) by one unit from the period t to the period $t + 1$, here one probability unit may be taken, for example, as 0.1. The sum in the objective function (5.1) is the total cost for bringing the game from the initial transition probability matrix $P(0) = P_{Initial}$ to the final transition probability matrix $P(T) = P_{Win}$; $u(t) = (u_{(i,j)(p,q)}(t)), i = \overline{1,s}, j = \overline{1,l}, p = \overline{1,s}, q = \overline{1,l}, t = \overline{0,T-1}$ is a $s \times l$ control matrix; $P_{Initial}$ in (5.6) is the initial transition probability matrix that has to be estimated by the mediator who wants to restructure the game; the matrix P_{Win} in (5.7) is the final transition probability matrix of the game, this matrix should have a reachable win-win profile. In [10] a number of ways of transforming a focal profile into a win-win profile is provided. Once the players reach P_{Win} the win-win profile will be reached. In [9] we provide the average number of steps that are needed to reach the win-win profile and how to improve this average. The constraints (5.2)-(5.3) express the fact that the transition probability matrices $P(t)$ are stochastic. Equation (5.4) expresses the way $P(t+1)$ is obtained from $P(t)$ by restructuring the game via the control $u(t)$; constraints (5.5) represent the boundaries of the control $u(t)$.

5.2. **Minimizing Restructuring Time Model.** Assume there is no time constraint for restructuring. The mediator wants to reach a win-win profile as soon as possible with a cost constraint. The corresponding discrete optimal control problem can be formulated, similarly to (5,1)-(5,7), with proper linearity assumption as follows

$$Min\, T \tag{5.8}$$

subject to

$$\sum_{(p,q)} P_{(i,j)(p,q)}(t) = 1, i = \overline{1,s}, j = \overline{1,l}, t = \overline{1,T-1} \tag{5.9}$$

$$P_{(i,j)(p,q)}(t) \geq 0,\ i = \overline{1,s},\ j = \overline{1,l},\ p = \overline{1,s},\ q = \overline{1,l},\ t = \overline{1,T-1} \tag{5.10}$$

$$\sum_{t=0}^{T-1} \sum_{(i,j)(p,q)} (P_{(i,j)(p,q)}(t+1) - P_{(i,j)(p,q)}(t))\alpha_{(i,j)(p,q)}(t) \leq K \tag{5.11}$$

$$P(t+1) = f(P(t), u(t)),\ t = \overline{0,T-1} \tag{5.12}$$

$$a_{(i,j)(p,q)}(t) \leq u(t)_{(i,j)(p,q)} \leq b_{(i,j)(p,q)}(t), \tag{5.13}$$

$$\text{for } i = \overline{1,s},\ j = \overline{1,l},\ p = \overline{1,s},\ q = \overline{1,l}, t = \overline{0,T-1}$$

$$P(0) = P_{Initial} \tag{5.14}$$

$$P(T) = P_{Win} \tag{5.15}$$

where T is the terminal time and (5.11) is the cost constraint with the upper bound K for the total cost.

Remark 5.1. In [9] we have identified the types of forms of P for which a win-win/focal profile can be reached. Particularly, it is possible to reach the win-win profile when the matrix P has a unique recurrent class and the win-win profile is in this class. Thus, the matrix P_{Win} in the problems (5.1)-(5.7) and (5.8)-(5.15) can be any matrix of the identified types.

Let us now illustrate the problem (5.1)-(5.7) by an example.

Example 5.1. Because of family feud, Robert (Player I) and his wife Ana (Player II) get into vicious emotional cycle (the reachable domain (RD) gets trapped). Their mutual love has evaporated as to think about divorce. They visit a marriage counselor (a mediator of the game) for their final decision-divorce or not. After careful listening and assessing the situation, the counselor is confident that the marriage can be saved by a sequence of counseling. He sets a period of three months through three meetings to solve the problem. He formulates the situation into a second-order game. There are two players, Robert (Player I) and his wife (Player II). The states of mind of Player I and Player II are $S^I = \{C, N\}$, and $S^{II} = \{C, N\}$, respectively, where C= cooperate for reconciliation, N= reject reconciliation (non cooperation). He has identified the initial probability matrix as

$P(0) = P_{Initial} =$

	(C,C)	(C,N)	(N,C)	(N,N)
(C,C)	0.1	0.1	0.1	0.7
(C,N)	0	0.1	0.1	0.8
(N,C)	0	0.1	0.1	0.8
(N,N)	0	0	0	1

According to the discrete optimal control problem (5.1)-(5.7), the counselor also estimates $\alpha_{(i,j)(p,q)}(t)$, the cost for increasing the transition probability $P_{(i,j)(p,q)}(t)$ from the profile (i, j) to the profile (p, q) by one unit from the period t to the period $t + 1$, here one unit means 0.1. He obtains the following matrices

$\alpha(0) =$

	(C,C)	(C,N)	(N,C)	(N,N)
(C,C)	0	5	5	20
(C,N)	10	0	20	10
(N,C)	10	20	0	10
(N,N)	25	15	15	0

$\alpha(1) =$

	(C,C)	(C,N)	(N,C)	(N,N)
(C,C)	0	3	4	22
(C,N)	15	0	20	8
(N,C)	8	20	0	7
(N,N)	23	17	17	0

The matrix $\alpha(0)$ can be interpreted as follows. Increasing the the transition probability from any profile to itself is not interesting for the mediator and requires no significant effort, therefore, the corresponding cost is zero. Increasing the probability transition from any profile $(i, j) \neq (C, C)$ to (C, C) is costly because it requires efforts to change the current mind profile of Roberts and/or Ana from hostility to cooperation, especially from (N, N). This is generally true in conflicts situations. Increasing the transition probability from (C, N) to (N, C) or from (N, C) to (C, N) is assigned a high cost of 20, because it is required to change the current mind profile of players to the opposite. The cost for increasing the transition probability from (C, C) to (N, C) or (C, N) is small because unilaterally worsening or escalating conflict is easy in general. Increasing the transition probability from (N, N) to (N, C)

or (C,N) means that the mediator focuses on one of the spouses and tries to change his/her mind for more cooperation. It is relatively difficult, because people tend to reciprocate in their behavior. The entries of $\alpha(1)$ can be interpreted similarly.

Let us help the counselor solve the problem. We have three periods, $t = 0, 1, 2$, the final period is $T = 2$. In order to reach the reconciliation, the final transition probability matrix in (5.1)-(5.7) should be

$P(2) = P_{Win}=$

	(C,C)	(C,N)	(N,C)	(N,N)
(C,C)	1	0	0	0
(C,N)	1	0	0	0
(N,C)	1	0	0	0
(N,N)	1	0	0	0

For the restructuring of the transition probability matrix in equation (5.4), we will assume that it is linear $P(t+1) = P(t) + u(t),\ t = 0, 1$. In the constraints (5.5) we assume that the control is bounded as follows $-0.5 \leq u_{(i,j)(p,q)}(t) \leq 0.5,\ i, j, p, q = 1, 2,\ t = 0, 1$. Note that, here in our case, the control may take negative values as well, because we may reduce or increase transition probabilities for restructuring. Then the problem (5.1)-(5.7) becomes

$$Min\ C(u) = \sum_{t=0}^{1} \sum_{(i,j),(p,q)} u_{(i,j)(p,q)}(t)\ \alpha_{(i,j)(p,q)}(t) \tag{5.16}$$

subject to

$$\sum_{(p,q)} P_{(i,j)(p,q)}(1) = 1,\ p, q = 1, 2, \tag{5.17}$$

$$P_{(i,j)(p,q)}(1) \geq 0,\ i, j, p, q = 1, 2, \tag{5.18}$$

$$P(1) = P(0) + u(0),\quad P(2) = P(1) + u(1) \tag{5.19}$$

$$-0.5 \leq u_{(i,j)(p,q)}(t) \leq 0.5,\ i, j, p, q = 1, 2,\ t = 0, 1, \tag{5.20}$$

$$P(0) = P_{Initial} \tag{5.21}$$

$$P(2) = P_{Win} \tag{5.22}$$

Note that the constraint (5.20) means that the mediator is limited in terms of the quantity by which he can vary the transition probabilities. This constraint is realistic in the sense that, in general, conflicts need progressive and gradual restructuring so that a win-win profile could be reached. By solving the discrete optimal control problem (5.16)-(5.22), we provide the counselor with the changes he has to make on the matrix P at each period (month). The changes or restructuring can be done by using the practical techniques of restructuring games presented and discussed in Section 4, [10] and [19]. By solving the problem (5.16)-(5.22), we get the following optimal control

$$u(0) = \begin{pmatrix} 0.4 & -0.1 & -0.1 & -0.2 \\ 0.5 & -0.1 & -0.1 & -0.5 \\ 0.5 & -0.1 & -0.1 & -0.5 \\ 0.5 & 0 & 0 & -0.5 \end{pmatrix}, \; u(1) = \begin{pmatrix} 0.5 & 0 & 0 & -0.5 \\ 0.5 & 0 & -0.2 & -0.3 \\ 0.5 & 0 & -0.2 & -0.3 \\ 0.5 & 0 & 0 & -0.5 \end{pmatrix},$$

the transition probabilty matrix

$$P(1) = \begin{pmatrix} 0.5 & 0 & 0 & 0.5 \\ 0.5 & 0 & 0.2 & 0.3 \\ 0.5 & 0 & 0.2 & 0.3 \\ 0.5 & 0 & 0 & 0.5 \end{pmatrix},$$

and the minimum cost $Min\ C(u) = 11$.

Let us now interpret these results in terms of restructuring. After a first meeting the mediator has estimated the atmosphere of the conflict as $P(0)$. According to the solution found, the optimal restructuring procedure for reaching the win-win profile (C, C) is described by $u(0)$ and $u(1)$. At the first period the mediator has to act according to the control $u(0)$. On one hand he has to decrease the transition probabilities from all the profiles to the profile (N, N). As a result Robert and Ana will reduce the intensity of rejection of the reconciliation. The mediator can do this by invoking the consequences of divorce on their individual life and the future of their children and asking them to think deeply about this. On the other hand he has to increase the transition probability from any profile to the profile (C, C). This means that he has to invoke the positive aspects of reconciliation and returning to normal family life. Here the restructuring techniques of Section 4 are very useful. At the end of the first period, the matrix $P(1)$ is obtained. In the second period the mediator has to restructure the game according to the control $u(1)$. Here also he has to work on both reducing the transition probabilities to the profile (N, N) and on increasing the transition probabilities to the profile (C, C). He may use the same techniques as in the first period to achieve these changes.

6. Conclusion

In [9] and [10] a HD theory based model for two person games has been formulated and presented as the second-order games. In such games, sets of strategies and utility functions are not involved; they are based on the states of mind of players and their charge level. The concept of win-win profile has been introduced as a concept of solution to such games. Thus, the new theory could significantly enlarge the scope of applications of game theory. Two procedures for restructuring second order games for reaching a win-win profile were presented in [10]. In these procedures, no criteria such as time and cost were taken into account. When the mediator (the party that restructures the game) considers criteria of time and cost, the question of optimality arises. In [9],[10] we left the question open. In this paper, we address this question. The problem of finding optimal ways of restructuring a game as to reach a game that has a reachable win-win profile can be formulated as a discrete optimal control problem. Two models are presented: (5.1)-(5.7) and

(5.8)-(5.15). The first deals with the case where the mediator wants to minimize the cost of restructuring the game within a fixed duration. The second concerns the problem of minimizing the restructuring time, when there is a cost constraint. The first model is illustrated with an example.

Many research problems remain open. For instances, in mathematical analysis for optimal restructuring, instead of two criteria: time and cost, there are other criteria, such as minimizing the frustration and anxiety of interaction among the players, that can be considered. This may lead to multiple criteria discrete optimal control problems. In addition, the transition of psychological states may involve uncertainty and fuzziness. How do we deal with this kind of problems in formulation? Also, how do we incorporate those methods mentioned in Section 4 in restructuring the games so that the win-win profile could be reached in a most effective way remains an important problem practically and academically.

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