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ITERATIVE SOLUTIONS OF NONLINEAR EQUATIONS FOR m-ACCRETIVE OPERATORS IN BANACH SPACES

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ABSTRACT. In this paper, we prove some new strong convergence theorems of the Ishikawa iterative scheme with mixed errors for m-accretive operators without the Lipschitzian and bounded range assumptions in uniformly smooth Banach spaces.

1. Introduction and Preliminaries

Let E be an arbitrary real Banach space and E^* be the dual space on E. The normalized duality mapping $J : E \to 2^{E^*}$ is defined by

$$J(x) = \{x^* \in E^* : \langle x, x^* \rangle = \|x\|^2 = \|x^*\|^2\}$$

for all $x \in E$, where $\langle \cdot, \cdot \rangle$ denotes the generalized duality pairing. It is well known that

(1) If E^* is strictly convex, then the mapping J is single-valued,
(2) $J(-x) = -J(x)$ for all $x \in E$,
(3) $J(\alpha x) = \alpha J(x)$ for all $x \in E$ and $\alpha \geq 0$,
(4) If E^* is uniformly convex, then the mapping J is uniformly continuous on any bounded subset of E.

An operator T with the domain $D(T)$ and range $R(T)$ in E is said to be accretive if, for any $x, y \in D(T)$, there exists $j(x-y) \in J(x-y)$ such that

$$\langle Tx - Ty, j(x-y) \rangle \geq 0.$$

The operator T is said to be strongly accretive if, for all $x, y \in D(T)$, there exist $j(x-y) \in J(x-y)$ and a constant $k > 0$ such that

$$\langle Tx - Ty, j(x-y) \rangle \geq k\|x-y\|^2.$$

An accretive operator T is said to be m-accretive if $R(T + \lambda I) = E$ for all $\lambda > 0$, where I denotes the identity mapping on E. Note that, if a mapping T is m-accretive then, for any given $f \in E$, the equation $x + Tx = f$ has a solution in E. Since T is accretive, $T + I$ is strongly accretive and hence the solution is unique.

On the other hand, Liu [3] and Xu [5] introduced the Ishikawa iterative schemes with errors, respectively. But we remark here that Xu's scheme with errors is a special case of Liu's scheme with errors and, further, putting $u''_n = 0$ for $n = 0, 1, 2, \cdots$ in our new Ishikawa iterative scheme (IS) with mixed error defined in our main Theorem 2.1, we obtain also Liu's scheme with errors.

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The purpose of this paper is to give some strong convergence theorems of the Ishikawa iterative scheme with mixed errors for m-accretive operators without the Lipschitzian and bounded range conditions in uniformly real Banach spaces. Our results extend and improve the corresponding results of Chidume and Osilike [1], Ding [2] and many others.

For our main results, we need the following:

Lemma 1.1. [4] *Let E be a real Banach space. Then, for all $x, y \in E$ and $j(x+y) \in J(x+y)$,*

$$\|x+y\|^2 \le \|x\|^2 + 2\langle y, j(x+y)\rangle. \tag{1.1}$$

Lemma 1.2. [3] *Let $\{a_n\}_{n=0}^\infty$, $\{b_n\}_{n=0}^\infty$ and $\{c_n\}_{n=0}^\infty$ be three nonnegative real sequences satisfying the following condition:*

$$a_{n+1} \le (1-t_n)a_n + b_n + c_n, \quad n \ge 0, \tag{1.2}$$

where $0 \le t_n < 1$, $\sum_{n=0}^\infty t_n = \infty$, $b_n = \circ(t_n)$ and $\sum_{n=0}^\infty c_n < \infty$. Then $a_n \to 0$ as $n \to \infty$.

Lemma 1.3. *Let $\{a_n\}_{n=0}^\infty$, $\{b_n\}_{n=0}^\infty$, $\{c_n\}_{n=0}^\infty$ and $\{w_n\}_{n=0}^\infty$ be four nonnegative real sequences satisfying the following condition:*

$$a_{n+1} \le (1-t_n)a_n + w_n a_n + b_n + c_n, \tag{1.3}$$

where $0 \le t_n < 1$, $b_n = \circ(t_n)$, $\sum_{n=0}^\infty t_n = \infty$, $\sum_{n=0}^\infty c_n < \infty$ and $\sum_{n=0}^\infty w_n < \infty$. Then $a_n \to 0$ as $n \to \infty$.

Proof. Let $a = \liminf_{n\to\infty}\{a_n : n \ge 0\}$. Then $a = 0$. In fact, assume that $a > 0$ and take $\epsilon = \min\{a, 1\}$. Since $b_n = \circ(t_n)$, there exists a positive integer $N > 0$ such that

$$b_n < \frac{1}{2}\epsilon t_n \le a_n t_n$$

for all $n \ge N$. From (1.3), it follows that

$$\begin{aligned} a_{n+1} &\le (1-t_n)a_n + w_n a_n + a_n t_n + c_n \\ &\le (1+w_n)a_n + c_n \\ &\le (1+w_n)((1+w_{n-1})a_{n-1} + c_{n-1}) + c_n \\ &\le \prod_{i=0}^n (1+w_i)a_0 + \sum_{j=0}^n \prod_{i=j+1}^n (1+w_i)c_j \\ &\le \prod_{i=0}^n (1+w_i)\Big(a_0 + \sum_{i=0}^n c_i\Big) \\ &\le \exp\Big(\sum_{n=0}^\infty w_n\Big)\Big(a_0 + \sum_{n=0}^\infty c_n\Big) < \infty \end{aligned} \tag{1.4}$$

for all $n \ge N$ and so $\{a_n\}_{n=0}^\infty$ is a bounded sequence. Hence we have $a \ne +\infty$. Let $a_n \le M$ for some $M > 0$. By (1.3), we have

$$a_{n+1} \le (1-t_n)a_n + (w_n M + c_n) + b_n. \tag{1.5}$$

Thus, by Lemma 1.2 and (1.5), we have $a_n \to 0$ as $n \to \infty$, which contradicts $a > 0$. Therefore there exists a subsequence $\{a_{n_j}\}_{j=0}^{\infty}$ of $\{a_n\}_{n=0}^{\infty}$ such that $a_{n_j} \to 0$ as $j \to \infty$ and so, for any $\epsilon > 0$, there exists a positive integer N_j such that

$$a_{n_j} < \epsilon, \quad \frac{b_n}{t_n} < \epsilon, \quad \sum_{i=N_j}^{\infty} w_i < \epsilon, \quad \sum_{q=N_j}^{\infty} c_q < \epsilon \tag{1.6}$$

for all $n, n_j \geq N_j$. Now, we show that $a_{N_j+m} \leq 2\epsilon e^{\epsilon}$ for all $m \geq 1$. By using (1.3) and (1.6), we have

$$\begin{aligned} a_{N_j+1} &\leq (1 - t_{N_j})a_{N_j} + w_{N_j}a_{N_j} + b_{N_j} + c_{N_j} \\ &< (1 - t_{N_j})\epsilon + w_{N_j}\epsilon + t_{N_j}\epsilon + c_{N_j} \\ &= (1 + w_{N_j})\epsilon + c_{N_j}, \end{aligned}$$

$$\begin{aligned} a_{N_j+2} &\leq (1 - t_{N_j+1})a_{N_j+1} + w_{N_j+1}a_{N_j+1} + b_{N_j+1} + c_{N_j+1} \\ &\leq (1 - t_{N_j+1} + w_{N_j+1})((1 + w_{N_j})\epsilon + c_{N_j}) + t_{N_j+1}\epsilon + c_{N_j+1} \\ &= (1 + w_{N_j+1})(1 + w_{N_j})\epsilon + (1 + w_{N_j+1})c_{N_j} - t_{N_j+1}(w_{N_j}\epsilon + c_{N_j}) + c_{N_j+1} \\ &\leq (1 + w_{N_j+1})(1 + w_{N_j})\epsilon + (1 + w_{N_j+1})c_{N_j} + c_{N_j+1}, \end{aligned}$$

$$\begin{aligned} a_{N_j+3} \leq\ & (1 + w_{N_j+2})(1 + w_{N_j+1})(1 + w_{N_j})\epsilon + (1 + w_{N_j+2})(1 + w_{N_j+1})c_{N_j} \\ & + (1 + w_{N_j+2})c_{N_j+1} + c_{N_j+2}, \end{aligned}$$

and so, by induction, we have

$$\begin{aligned} a_{N_j+m} &\leq \epsilon \prod_{i=N_j}^{N_j+m-1} (1 + w_i) + \sum_{q=N_j}^{N_j+m-1} \prod_{i=N_j+1}^{N_j+m-1} (1 + w_i)c_q \\ &\leq \epsilon \prod_{i=N_j}^{N_j+m-1} (1 + w_i) + \sum_{q=N_j}^{N_j+m-1} c_q \prod_{i=N_j}^{N_j+m-1} (1 + w_i) \\ &= \exp\Big(\sum_{i=N_j}^{N_j+m-1} w_i \Big)\Big(\epsilon + \sum_{i=N_j}^{N_j+m-1} c_i\Big) \\ &< e^{\epsilon}(\epsilon + \epsilon) = 2\epsilon e^{\epsilon}, \end{aligned}$$

which implies that $a_n \to 0$ as $n \to \infty$. This completes the proof. □

Remark 1.1. Lemma 1.3 extends and improves Lemma 1.2.

2. Main Results

Now, we give our main results in this paper.

Theorem 2.1. *Let E be a uniformly smooth real Banach space and $T : E \to E$ be an m-accretive operator such that there exists a constant $L \geq 1$ satisfying*

$$\|Tx - Ty\| \leq L(1 + \|x - y\|) \tag{2.1}$$

for all $x, y \in E$. Let $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$ be two sequences in E and $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ be two real sequences in $[0, 1]$ satisfying the following conditions:

(1) $u_n = u'_n + u''_n$ *for any sequences* $\{u'_n\}_{n=0}^\infty$, $\{u''_n\}_{n=0}^\infty$ *in* E *and* $n \geq 0$ *with* $\sum_{n=0}^\infty \|u'_n\| < \infty$ *and* $\|u''_n\| = \circ(\alpha_n)$,
(2) $\|v_n\| \to 0$ *as* $n \to \infty$,
(3) $\lim_{n\to\infty} \alpha_n = \lim_{n\to\infty} \beta_n = 0$ *and* $\sum_{n=0}^\infty \alpha_n = \infty$.

For any given $f \in E$, *define a mapping* $S : E \to E$ *by* $Sx = f - Tx$ *for all* $x \in E$ *and, for any given* $x_0 \in E$, *the Ishikawa iterative sequence* $\{x_n\}_{n=0}^\infty$ *with errors by*

$$\text{(IS)} \qquad \begin{cases} y_n = (1-\beta_n)x_n + \beta_n S x_n + v_n, \\ x_{n+1} = (1-\alpha_n)x_n + \alpha_n S y_n + u_n \end{cases}$$

for all $n \geq 0$. *Then the sequence* $\{x_n\}_{n=0}^\infty$ *defined by* (IS) *converges strongly to the unique solution of the nonlinear equation* $x + Tx = f$.

Proof. Since T is an m-accretive operator with the condition (2.1), it is well known that the equation $x + Tx = f$ has the unique solution q. Then the solution q is the unique fixed point of the mapping S. Since T is m-accretive, we have

$$\langle Sx - Sy, J(x-y)\rangle = -\langle Tx - Ty, J(x-y)\rangle \leq 0$$

for all $x, y \in E$. By using Lemma 1.1 and (IS), we have

$$\begin{aligned} \|x_{n+1} - q\|^2 &= \|(1-\alpha_n)(x_n - q) + \alpha_n(Sy_n - Sq) + u_n\|^2 \\ &\leq \|(1-\alpha_n)(x_n - q) + \alpha_n(Sy_n - Sq)\|^2 + 2\langle u_n, J(x_{n+1} - q)\rangle \\ &\leq (1-\alpha_n)^2\|x_n - q\|^2 + 2\alpha_n\langle Sy_n - Sq, J(x_{n+1} - q - u_n)\rangle \\ &\quad + 2\langle u_n, J(x_{n+1} - q)\rangle. \end{aligned} \tag{2.2}$$

Now observe that

$$2\langle u_n, J(x_{n+1} - q)\rangle \leq 2\|u_n\|\|x_{n+1} - q\| \leq \|u_n\|(1 + \|x_{n+1} - q\|^2), \tag{2.3}$$

$$\begin{aligned} &\langle Sy_n - Sq, J(x_{n+1} - q - u_n)\rangle \\ &\quad = \Big\langle \frac{Sy_n - Sq}{1 + \|x_n - q\|}, J\Big(\frac{x_{n+1} - q - u_n}{1 + \|x_n - q\|}\Big) - J\Big(\frac{x_n - q}{1 + \|x_n - q\|}\Big)\Big\rangle(1 + \|x_n - q\|)^2 \\ &\quad + \Big\langle \frac{Sy_n - Sq}{1 + \|x_n - q\|}, J\Big(\frac{x_n - q}{1 + \|x_n - q\|}\Big) - J\Big(\frac{y_n - q}{1 + \|x_n - q\|}\Big)\Big\rangle(1 + \|x_n - q\|)^2 \\ &\quad + \langle Sy_n - Sq, J(y_n - q)\rangle \\ &\quad \leq 2M_n(A_n + B_n)(1 + \|x_n - q\|^2), \end{aligned} \tag{2.4}$$

where

$$\begin{aligned} M_n &= \frac{\|Sy_n - Sq\|}{1 + \|x_n - q\|}, \\ A_n &= \Big\| J\Big(\frac{x_{n+1} - q - u_n}{1 + \|x_n - q\|}\Big) - J\Big(\frac{x_n - q}{1 + \|x_n - q\|}\Big)\Big\|, \\ B_n &= \Big\| J\Big(\frac{x_n - q}{1 + \|x_n - q\|}\Big) - J\Big(\frac{y_n - q}{1 + \|x_n - q\|}\Big)\Big\|. \end{aligned}$$

Now we show that $\{M_n\}_{n=0}^{\infty}$ is bounded and $A_n, B_n \to 0$ as $n \to \infty$. Indeed,

$$\begin{aligned}\frac{\|Sy_n - Sq\|}{1 + \|x_n - q\|} &= \frac{\|Ty_n - Tq\|}{1 + \|y_n - q\|}\frac{1 + \|y_n - q\|}{1 + \|x_n - q\|} \\ &\leq L\frac{1 + (1-\beta_n)\|x_n - q\| + \beta_n\|Tx_n - Tq\| + \|v_n\|}{1 + \|x_n - q\|} \\ &\leq L(1 + \beta_n L + \|v_n\|).\end{aligned}$$

Since $\left\{\frac{\|Sy_n - Sq\|}{1+\|x_n - q\|}\right\}, \left\{\frac{\|x_n - q\|}{1+\|x_n - q\|}\right\}_{n=0}^{\infty}$ are bounded and

$$\begin{aligned}\left\|\frac{(x_{n+1} - q - u_n) - (x_n - q)}{1 + \|x_n - q\|}\right\| &= \left\|\frac{(\alpha_n x_n - \alpha_n Sy_n)}{1 + \|x_n - q\|}\right\| \\ &= \left\|\frac{\alpha_n(x_n - q) - \alpha_n(Sy_n - Sq)}{1 + \|x_n - q\|}\right\| \\ &\leq \alpha_n\frac{\|x_n - q\|}{1 + \|x_n - q\|} + \alpha_n\frac{\|Ty_n - Tq\|}{1 + \|y_n - q\|}\frac{1 + \|y_n - q\|}{1 + \|x_n - q\|} \\ &\leq \alpha_n + \alpha_n L(1 + \beta_n + \beta_n L + \|v_n\|) \to 0\end{aligned}$$

as $n \to \infty$, we have $A_n \to 0$ as $n \to \infty$. Similarly, we have also $B_n \to 0$ as $n \to \infty$. Substituting (2.3) and (2.4) for (2.2), we have

$$\begin{aligned}\|x_{n+1} - q\|^2 \leq{}& (1-\alpha_n)^2\|x_n - q\|^2 + 2M_n\alpha_n(A_n + B_n)(1 + \|x_n - q\|^2) \\ &+ \|u_n\|(1 + \|x_{n+1} - q\|^2).\end{aligned} \tag{2.5}$$

Choosing a positive integer N so large that $1 - \|u_n\| > 0$ for all $n \geq N$, we have

$$\begin{aligned}\|x_{n+1} - q\|^2 \leq{}& \frac{(1-\alpha_n)^2 + 2M_n\alpha_n(A_n + B_n)}{1 - \|u_n\|}\|x_n - q\|^2 \\ &+ \frac{2M_n\alpha_n(A_n + B_n)}{1 - \|u_n\|} + \frac{\|u_n\|}{1 - \|u_n\|} \\ \leq{}& \frac{1 - \|u_n\| - 2\alpha_n + \alpha_n^2 + 2M_n\alpha_n(A_n + B_n) + \|u_n\|}{1 - \|u_n\|}\|x_n - q\|^2 \\ &+ \frac{2M_n\alpha_n(A_n + B_n)}{1 - \|u_n\|} + \frac{\|u'_n\| + \|u''_n\|}{1 - \|u_n\|} \\ \leq{}& \left(1 - \frac{2 - \alpha_n - 2M_n(A_n + B_n) - \|u''_n\|/\alpha_n}{1 - \|u_n\|}\alpha_n\right)\|x_n - q\|^2 \\ &+ \frac{\|u'_n\|}{1 - \|u_n\|}\|x_n - q\|^2 + \frac{2M_n\alpha_n(A_n + B_n) + \|u''_n\|}{1 - \|u_n\|} + \frac{\|u'_n\|}{1 - \|u_n\|}.\end{aligned} \tag{2.6}$$

In (2.6), put

$$a_n = \|x_n - q\|^2, \qquad w_n = \frac{\|u_n'\|}{1 - \|u_n\|},$$
$$b_n = \frac{2M_n\alpha_n(A_n + B_n) + \|u_n''\|}{1 - \|u_n\|},$$
$$t_n = \frac{2 - \alpha_n - 2M_n(A_n + B_n) - \frac{\|u_n''\|}{\alpha_n}}{1 - \|u_n\|}.$$

Then there exists a positive integer $N' > N$ such that $t_n \geq \frac{1}{2}$ for all $n > N'$, and we have $\sum_{n=N'}^{\infty} w_n < \infty$ and $b_n = \circ(\alpha_n)$. Therefore, by Lemma 1.3,

$$a_{n+1} \leq \big(1 - \frac{1}{2}\alpha_n\big)a_n + w_n a_n + b_n + w_n$$

for all $n \geq N'$ implies that $a_n \to 0$ as $n \to \infty$. This completes the proof. □

Note that the sequence (IS) in Theorem 2.1 satisfying the conditions (1)∼(3) is called the Ishikawa iterative scheme with mixed errors ([6]).

Remark 2.1. Theorem 2.1 contains a good number of the known results as its special cases. In particular, if the mapping T considered here satisfies one of the following assumptions:

(i) $T : E \to E$ is Lipschitzian,
(ii) $T : E \to E$ has the bounded range.

Then T satisfies the condition (2.1) (see [6]).

Corollary 2.1. *Let E be a uniformly smooth real Banach space and $T : E \to E$ be a Lipschitz and m-accretive mapping. Let $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ and $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$ be as in Theorem 2.1. Then the sequence $\{x_n\}_{n=0}^{\infty}$ defined by* (IS) *converges strongly to the unique solution of the equation $x + Tx = f$.*

Corollary 2.2. *Let E be a uniformly smooth real Banach space and $T : E \to E$ be an m-accretive mapping with the bounded range. Let $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ and $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$ be as in Theorem 2.1. Then the sequence $\{x_n\}_{n=0}^{\infty}$ defined by* (IS) *converges strongly to the unique solution of the equation $x + Tx = f$.*

Remark 2.2. Corollary 2.1 extends the main results of Chidume and Osilike [1] from $q-$uniformly smooth Banach spaces ($q > 1$) to the more general uniformly smooth Banach spaces and from the usual iterative sequences to the iterative sequences with errors. While Corollary 2.2 extends the main results of Ding [2] to the more general iterative sequence with errors. By setting $\|v_n\| \equiv 0$ for $n = 0, 1, 2, \cdots$, we can deduce Theorems 3.1 and 3.2 of Ding [2].

Remark 2.3. Actually, all the results mentioned above can be also restated in terms of $m-$dissipative operators.

A class of operators closely related to the class of accretive operators is the class of dissipative operators. An operator $T : D(T) \subset E \to E$ is said to be dissipative if $-T$ is accretive. The dissipative operator T is said to be $m-$dissipative if $R(I - \lambda T) = E$ for all $\lambda > 0$.

Theorem 2.2. *Let E be a uniformly smooth real Banach space and $T : E \to E$ be an $m-$dissipative operator such that there exists a constant $L \geq 1$ satisfying*

$$||Tx - Ty|| \leq L(1 + ||x - y||)$$

for all $x, y \in E$. Let $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ and $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$ be as in Theorem 2.1. For any given $f \in E$, define a mapping $S : E \to E$ by $Sx = f + Tx$ for all $x \in E$. Then the sequence $\{x_n\}_{n=0}^{\infty}$ defined by (IS) *converges strongly to the unique solution of the equation $x - Tx = f$.*

Proof. It follows from Theorem 2.1. □

Corollary 2.3. *Let E be a uniformly smooth real Banach space and $T : E \to E$ be a Lipschitzian and $m-$dissipative mapping. Let $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ and $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$ be as in Theorem 2.2. Then the sequence $\{x_n\}_{n=0}^{\infty}$ defined by* (IS) *converges strongly to the unique solution of the equation $x - Tx = f$.*

Corollary 2.4. *Let E be a uniformly smooth real Banach space and $T : E \to E$ be $m-$dissipative mapping with the bounded range. Let $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ and $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$ be as in Theorem 2.1. Then the sequence $\{x_n\}_{n=0}^{\infty}$ defined by* (IS) *converges strongly to the unique solution of the equation $x - Tx = f$.*

Remark 2.4. Corollary 2.3 extends the main results of Chidume and Osilike [1] from $q-$uniformly smooth Banach spaces ($q > 1$) to the more general uniformly smooth Banach spaces and from the usual iterative sequences to the iterative sequences with errors. While Corollary 2.4 extends the main results of Ding [2] to the more general iterative sequence with errors. By setting $\|v_n\| \equiv 0$ for $n = 0, 1, 2, \cdots$, we can deduce Theorems 3.3 and 3.4 of Ding [2].

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